PHYSICAL REVIEW E, VOLUME 64, 035201(R)

Spiral wave dynamics under feedback derived from a confined circular domain

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(Received 16 March 2001; published 28 August 2001)

Spiral waves are induced in a thin layer of the light-sensitive Belousov-Zhabotinsky reaction and controlled by a time-dependent uniform illumination. The intensity of the illumination is taken to be proportional to the average wave activity observed within a circular domain of the reaction layer. Stabilization and destabilization of spiral waves, as well as the existence of additional types of attractors, are demonstrated. A simple mathematical description is proposed in order to explain the existence of the attractors and the dependence of their size on the time delay in the feedback loop.

DOI: 10.1103/PhysRevE.64.035201 PACS number(s): 05.45.−a, 05.65.+b, 82.40.Bj, 47.54.+r

Spiral wave dynamics has been studied in many different excitable media such as the vortices of electrical activity in cardiac tissue [1], the CO oxidation on platinum surfaces [2], and the chemical waves in the Belousov-Zhabotinsky (BZ) reaction [3,4]. Controlling the dynamics of spiral waves by parametric forcing, as demonstrated in experiments of the light-sensitive BZ reaction [5–8], presents a wide variety of spatiotemporal patterns. For instance, a resonance drift of spiral waves can be induced, if the forcing frequency is chosen to be close to the eigenfrequency of the rotating spiral waves [9]. This frequency can be determined from a local (one-channel) measurement of the momentary wave activity in the medium [10]. Under such a one-channel feedback control a broad spectrum of different dynamical regimes for spiral waves has been found in experiments, numerical computations, and theoretical considerations [6–8,10,11].

In extension of this local feedback control, one can chose the feedback signal to be proportional to the integral of the wave activity taken over the whole medium with certain assumptions on the confined geometry of the system, the so-called global feedback. A theoretical analysis of the global feedback shows that stabilization and destabilization of the rigid rotation of a spiral wave occur depending on intensity, sign, and/or time delay in the feedback loop [12].

In this Rapid Communication we study experimentally the properties of a type of feedback control which occupies an intermediate case between the local and global feedback mentioned above. We are using the photosensitive BZ reaction where the wave activity is characterized by the absorption of transmitted light corresponding to the concentration of the oxidized catalyst. The intensity of a time-dependent uniform illumination is taken to be proportional to the integral absorption observed within a circular domain S that covers only a part of the reaction layer in which the spiral wave propagates. The role of feedback parameters such as the sign of the gain and the time delay is investigated.

The experimental setup, in extension of that used in our previous studies [6–8], includes a petri dish (diameter, 7 cm) with a thin gel layer. The light sensitive $Ru(bpy)_3^{2+}$ catalyst is immobilized in the gel at a concentration of 4.2 mM [13]. The BZ solution without catalyst is poured on top of the gel. After equilibrium between liquid and gel is established, the following concentrations are reached: 0.2 M $NaBrO_3$, 0.17 M malonic acid, 0.39 M H_2SO_4, and 0.09 M NaBr. The experiments are carried out at a temperature of 22 $\pm 1\,°C$.

The petri dish is illuminated from below by a video projector (Panasonic PT-L555E) controlled by a computer via a frame grabber (Data Translation, DT 2851). The illuminating light is filtered with a bandpass filter (BG6, 310–530 nm). The pictures of the oxidation waves appearing in the transmitted light are detected by a charge-coupled-device camera (Hamamatsu H 3077) and digitized online for immediate processing by the computer. In parallel, the images are stored on a video recorder (Sony EVT 301).

A single spiral wave, which constitutes the initial condition for all the experiments, is created in the center of the dish by breaking a wave front with a cold intense light spot [8]. The spiral wave tip is defined as the intersection of contour lines ($0.6\times$amplitude) of two consecutive frames of the digitized movie (time step 3.12 s) and its motion is followed automatically by the computer [14].

The illumination intensity of the feedback control is expressed as

$$I(t)=I_0+k_{fb}[B(t-\tau)-B_0], \tag{1}$$

where I_0 is a background intensity, and

$$B(t)=\frac{1}{S}\int_s w\,ds \tag{2}$$

is the time-dependent integral of the absorption, w, observed in the circular area S with a diameter about one wavelength of the rotating spiral wave. The intensity of this signal $I(t)$ is controlled by the gain k_{fb} and proportional to the value of the integral $B(t-\tau)$, where τ is the time delay. The constant B_0 is the value of this integral for the case of a centrally symmetric rotation corresponding to the background intensity I_0.

The integral absorption $B(t)$ is determined online from a digitized image of the spiral wave as the average of the pixel gray level in the circular integration area. This image is measured during a short observation light pulse (duration, 80 ms;

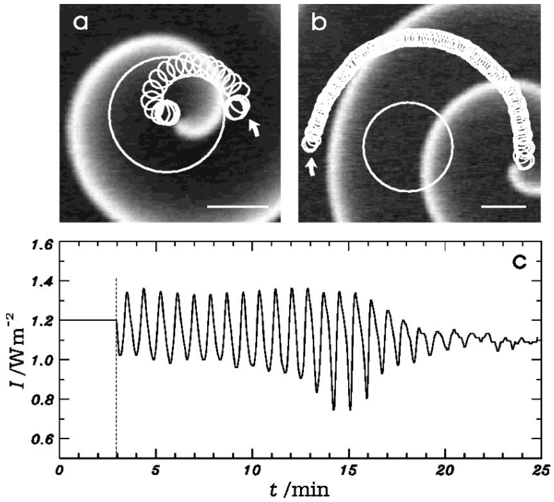

FIG. 1. (a,b) Trajectories of a spiral wave tip subjected to the feedback control Eq. (1), with $k_{fb}=-0.5$ and $\tau=0$ for different initial locations of the spiral wave core (arrows). In (a) the tip motion is stabilized at the center of the integration area (circle), in (b) the spiral wave core drifts along a circular attractor. (c) Feedback illumination intensity $I(t)$ corresponding to the trajectory in (a). Dashed line: feedback is switched on. Scale bar: 1 mm.

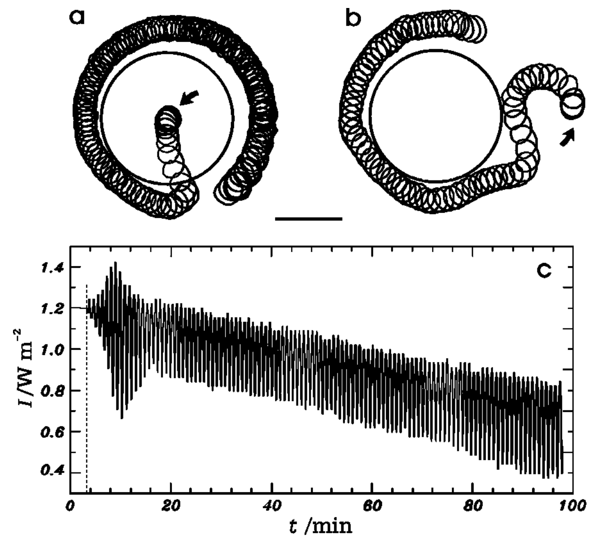

FIG. 2. (a,b) Trajectories of the spiral wave tip under positive feedback ($k_{fb}=0.5$, $\tau=0$) observed for different initial locations of the spiral wave core (arrows): (a) at the center and (b) outside of integration area (circle). (c) Feedback illumination intensity $I(t)$ corresponding to the trajectory in (a). Dashed line: feedback is switched on. Scale bar: 1 mm.

light intensity, 1.2 W m^{-2}; period, 1.48 s corresponding to the time step needed for the feedback loop), in order to eliminate the dependence of the measured brightness distribution on the alternating feedback illumination $I(t)$. To exclude the rather high pixel noise of the image background, the part of the image where the pixel gray level is smaller than 0.3× amplitude of the intensity profile of the spiral wave image is omitted.

The value $B=B_0$, determined at the begining of each experiment for a spiral wave carefully placed as close as possible to the center of S, was found to decrease linearly by about 50% in the range from $I_0=0.8$ to 1.6 W m^{-2}. On the other hand, the effect of aging in the closed BZ system leads to slight changes in B_0 in the course of measurements. In order to compensate the aging effect the background intensity I_0 is permanently reduced with time (0.02 W m^{-2} every 4 min) during about 2 h of experiment.

The initial value of the background intensity is set at $I_0 = 1.2\,\mathrm{W\,m^{-2}}$ for all experiments. At this illumination level the tip moves on a circular path with a rotation period $T = 52\,\mathrm{s}$ and a spiral wave pitch of $\lambda = 2.0\,\mathrm{mm}$.

In the experiment shown in Fig. 1(a) the spiral wave core is placed initially near the boundary of the integration area. After about three rotations, the feedback with negative gain $k_{fb}=-0.5$ and time delay $\tau=0$ was switched on. The modulation of the light intensity induces a drift of the spiral wave core towards the center of the integration area where finally a stable rigid rotation of the spiral wave is achieved [see Fig. 1(a)]. The intensity of feedback signal [Fig. 1(c)] oscillates around the background level, with the same period as the spiral rotates. The oscillation amplitude decreases, when the spiral tip moves closer to the symmetry center until it becomes approximately constant (except for small fluctuations). If the spiral wave core is placed far away from the domain center, its drift describes a circular pathway around the center, as shown in Fig. 1(b).

The results of the feedback control with a positive feedback gain are shown in Fig. 2. Even if the spiral wave core is placed at the center of the integration domain, the application of positive feedback causes a destabilization of the rigid rotation [see Fig. 2(a)]. It follows a drift of the core first towards the boundary of the integration area, and finally along a circular pathway with a radius of 0.65λ. If the spiral wave core was placed outside the integration domain, as shown in Fig. 2(b), the spiral wave tip after some transient process, approaches again the same asymptotic circular pathway as in the previous case. Both experiments suggest that this asymptotic trajectory can be considered as an attractor for the studied dynamical system under such a feedback. Figure 2(c) shows the feedback intensity corresponding to the experiment in Fig. 2(a), as a function of time. It fluctuates with a small amplitude at the beginning, and its amplitude increases during the transient process, then approaches a constant when the spiral wave core drifts along the circular trajectory. Note that application of the compensation mechanism leads to a linear decrease of the average feedback intensity.

The dynamics of spiral wave can be drastically changed by introducing a time delay into the feedback loop. In the example shown in Fig. 3(a) a negative feedback ($k_{fb}=-0.5$) with time delay $\tau=0.28T$ results in a destabilization of the rigid rotation around the center of the integration domain. A further increase of the time delay, $\tau=0.57T$ [Fig. 3(b)], causes a drift of the spiral wave core along a circular path, similar to the experiment with $k_{fb}=0.5$ and $\tau=0$ in Fig. 2(a). On the other hand, a positive feedback ($k_{fb}=0.5$) with time delay $\tau=0.28T$ [Fig. 3(c)], results in a drift of the spiral wave core on a smaller circular path, compared

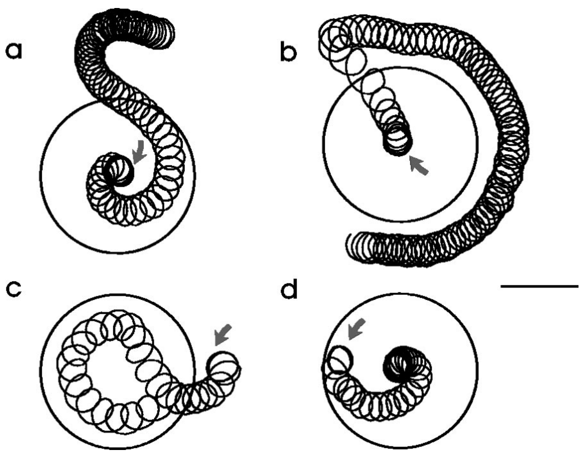

FIG. 3. Trajectories of the spiral wave tip observed for different signs of the feedback gain and time delays: (a) $k_{fb}=-0.5$, $\tau=0.28T$, (b) $k_{fb}=-0.5$, $\tau=0.57T$, (c) $k_{fb}=0.5$, $\tau=0.28T$, (d) $k_{fb}=0.5$, $\tau=0.57T$, where T is the rotation period. Integration domains and the initial spiral core locations are indicated by circles and arrows, respectively. Scale bar: 1 mm.

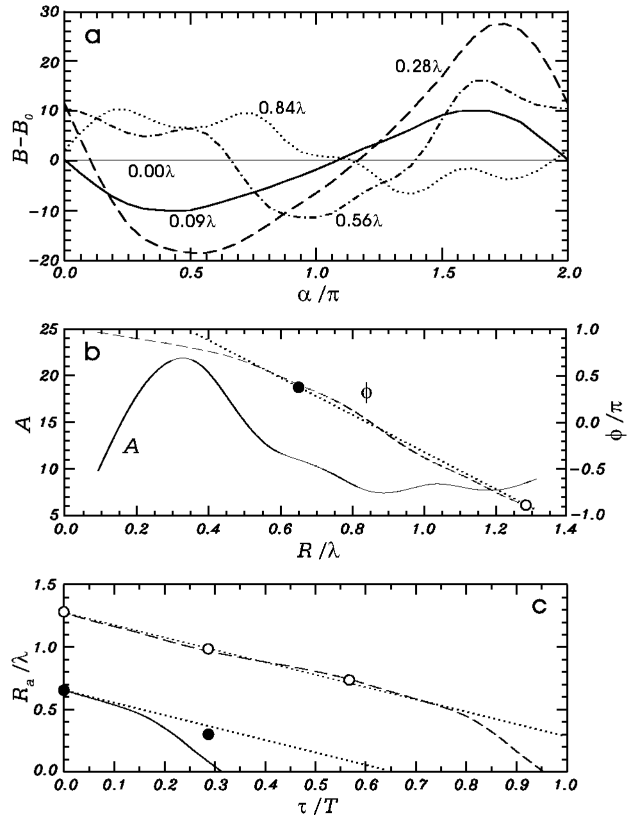

FIG. 4. Analysis of the feedback mechanism. (a) Integral value $B(\alpha)$ as a function of the rotation angle α computed for a spiral wave located at different radical displacements R from the center of integration domain (indicated by numbers close to each curve). (b) Amplitude A (solid) and phase ϕ (dashed), of the first harmonic in the Fourier expansion of $B(\alpha)$ as a function of radial displacement R. The attractor size R_a as a function of the time delay τ computed from Eq. (3) for a positive (solid) and negative (dashed) gain. The dotted lines correspond to a linear approximation of $\phi(R)$ valid for an Archimedean spiral. Open (closed) circles represent the data of the experimental measurements with the negative (positive) gain.

with the experiment without time delay in Fig. 2(a). A stabilization of the rigid rotation for positive feedback is obtained by using the time delay $\tau=0.57T$ [Fig. 3(d)].

In order to explain the main features of the observed phenomena let us compute the integral B for a given shape of a spiral wave as a function of its orientation, expressed by the rotation angle α. This function is depending on the radial displacement R from the center of the integration domain as shown by different curves in Fig. 4(a). The integral B, computed for a centrally symmetric spiral, does not depend on the rotation angle α [horizontal line in Fig. 4(a)]. A small displacement ($R=0.09\lambda$) in radial direction induces an asymmetry and leads to a practically harmonic function [Fig. 4(a)], that is, a rotation of the spiral displaced in radial direction will result in a periodic variation of the integral B and, hence, a generation of a periodic feedback signal in accordance with Eq. (1). Such a periodic forcing with a frequency exactly equal to the eigenfrequency of the spiral will induce a resonant drift [5,9]. In the case of a positive gain this drift occurs almost in radial direction as shown in Fig. 2(a) and leads to a destabilization of the centrally symmetric rotation. It is clear that by use of a negative gain the feedback signal will create a radial drift towards the center of the domain; that is, a stabilization of the centrally symmetric rotation should follow [see Fig. 1(a)]. The stabilization and the destabilization of a spiral wave illustrated by Figs. 3(d) and 3(b), respectively, occur due to a time delay that amounts to roughly half of the rotation period that should be equivalent to the change of the sign in the feedback loop.

An increase of the displacement R destroys the harmonic shape of the function $B(\alpha)$ [see curves with $R=0.56\lambda$ and $R=0.84\lambda$ in Fig. 4(a)]. Here the drift velocity is determined by the amplitude A and the phase ϕ of the first harmonic in the Fourier expansion of the function $B(\alpha)$ shown in Fig. 4(b). An increase of the displacement up to $R=0.3\lambda$ results in an increase of the amplitude and changes the phase $\phi(R)$ [cf. curve with $R=0.28\lambda$ in Fig. 4(a)]. Further increase of the displacement leads to a decrease of the amplitude and a monotonic change of the phase shift $\Delta\phi=\phi(R)-\phi(0)$. This phase shift results in a turn of the direction of the induced drift by the angle $\Delta\phi$. At the radial distance $R=0.65\lambda$, the phase shift achieves the value $\Delta\phi\approx-0.5\pi$ [closed circle in Fig. 4(b)]. Instead of the drift in the radial direction away from the center of the domain in Fig. 2(a), the core should now move in the tangential direction. Consequently, the core of the spiral wave forced by the oscillating signal, shown in Fig. 2(c), will describe a circular trajectory with radius 0.65λ. This agrees well with the trajectory of the attractor observed in Fig. 2(a).

In the presence of a time delay τ the first Fourier component of the feedback signal can be represented as $F(t)=A(R)\sin[\omega t-\omega\tau+\phi(R)]$. Here the direction of the induced drift should depend on the total phase $[\phi(R)-\omega\tau]$. For all attractors observed for different time delays, this phase should remain the same. This determines the dependence of the attractor size on the time delay $R_a(\tau)$ as a solution of the equation

$$\phi[R_a(\tau)]=\phi[R_a(0)]+\omega\tau. \quad (3)$$

The dependence $R_a(\tau)$ can be easily presented, since the function $\phi(R)$ was already computed and is shown in Fig. 4(b). The open and closed circles in Fig. 4(b) indicate the phase $\phi[R_a(0)]$ corresponding to the attractors in Figs. 2(b) and 3(a), respectively. By use of these data the dependence $R_a(\tau)$ is derived from Eq. (3) and plotted in Fig. 4(c) as a solid (dashed) curve for positive (negative) gain. The circles in Fig. 4(c) represent the experimental measurements of the size of the attractors with time delay $\tau>0$ shown in Figs. 3(a)–(c) and demonstrate a good agreement with the theoretical estimate of Eq. (3). Moreover, Fig. 4(b) illustrates that for relatively large displacements R the phase $\phi(R)$ can be approximated by a straight line, $\phi(R)=\phi_0-2\pi R/\lambda$. This is a direct consequence of the Archimedian shape of the spiral far from the core. Here, the dependence $R_a(\tau)$ becomes linear [see Fig. 4(c)],

$$R_a(\tau)=R_a(0)-\lambda\tau/T. \quad (4)$$

Thus, the spiral wave dynamics under a feedback signal computed by integrating the wave activity over a circular domain exhibits very interesting properties. These include simultaneously the characteristics of a global [12] and a one-channel feedback [8]. Indeed, the stabilization and the destabilization of a rigidly rotating spiral is observed under a global feedback, whereas the resonance attractor is a specific property of a one-channel feedback. On the other hand, the existence of an attractor outside the integration domain is, of course, not possible under a global feedback. The fast drift inside the integration domain, as shown in Figs. 2(a) and 3(b), is a qualitatively new property with respect to the one-channel feedback.

Finally, it should be stressed that feedback forcing is a powerful tool to control the dynamics of excitable systems, e.g., to suppress cardiac alternas [10,12,15,16]. Our results demonstrate that the considered control mechanism is highly efficient, when the integration domain has the size of the wavelength λ. A variation of this size to optimize the proposed control and to study a smooth transition between the one-channel and the global feedback are interesting challenges for future work.

O.K. thanks the Deutscher Akademischer Austauschdienst for financial support. C.K.C. acknowledges support from the Alexander von Humboldt Foundation.

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