Microwave-induced control of free-electron-laser radiation

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The dynamical response of a relativistic bunch of electrons injected in a planar magnetic undulator and interacting with a counterpropagating electromagnetic wave is studied. We demonstrate a resonance condition for which the free-electron-laser (FEL) dynamics is strongly influenced by the presence of the external field. It opens up the possibility of control of short wavelength FEL emission characteristics by changing the parameters of the microwave field without requiring change in the undulator's geometry or configuration. Numerical examples, assuming realistic parameter values analogous to those of the TTF-FEL, currently under development at DESY, are given for possible control of the amplitude or polarization of the emitted radiation.

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I. INTRODUCTION

Since their first experimental realization [1], free-electron lasers (FELs) have been one of the most promising sources of coherent electromagnetic radiation [2,3]. The physics of FEL emission is radically different from that of any other laser source. In particular, the tunability over a broad range of frequencies and the brightness of its output are difficult to achieve in other lasing schemes. On the other hand, its polarization, pulse shape, etc., are strongly connected with the geometry of the undulator and hence are inconvenient to modify. At the same time, modifications in the typical undulator's physical structure may induce new features, e.g., FELs with two magnetic wigglers of different spatial frequencies may increase the radiation at higher harmonics [4], or suppress the sidebands [5], and may allow the radiation spectrum [6] to be controlled. However, a systematic experimental exploration of these possibilities is very awkward, if not precluded, due to the difficulty of engineering and constructing the modified undulators for every such experiment. It is therefore worthwhile to explore theoretically the possibility of achieving control of the amplitude and polarization of the emitted radiation, specially at very short wavelengths, without having to alter the undulator geometry.

The basic dynamics of the interaction of free electrons with electromagnetic waves has been studied in many circumstances in the past. They include pioneering studies of the radiation of a single electron driven by an electromagnetic wave [7,8], interaction of relativistic electrons under general initial conditions with such radiation [9], charged particle acceleration by simultaneous interaction with an electromagnetic wave and a static electric field [10,11], etc. Another important application of the FEL principle is particle acceleration by the inverse mechanism. Particle acceleration by the inverse free-electron-laser principle has been demonstrated both theoretically [12] and experimentally [13,14]. In contrast, much less seems to be known about the complementary geometry, in which the electron bunch interacts with a *counterpropagating* electromagnetic wave, perhaps because of the absence of acceleration schemes for this case. In this paper we explore the possibility of modifying the electron dynamics in the amplification stage of FEL, by means of interaction with a counterpropagating microwave field. It will be shown that, under certain conditions, the counterpropagating wave can strongly influence the dynamics of the electrons inside the undulator. Thus, by a careful choice of the wave parameters control of the dynamics can be achieved that could lead to desired FEL radiation properties without requiring geometrical changes in the undulators.

II. ELECTRON DYNAMICS AND PHASE MATCHING CONDITION

Let us consider a modified FEL configuration as depicted in Fig. 1: a free electron is injected axially into a linearly polarized magnetic undulator, where an electromagnetic wave also propagates axially, in the opposite direction, inside a waveguide. The evolution of the electron motion, in the combined steady magnetic field of the undulator and the electromagnetic wave, is governed by the Newton-Lorentz equation

$$\frac{d}{dt}\vec{p}=q\left[\vec{E}_0+\frac{1}{c}\,\vec{v}\times(\vec{B}_u+\vec{B}_0)\right] \tag{1}$$

where $\vec{B}_u=f(x)B_u\sin(k_u x)\vec{e}_z$ is the undulator's magnetic field. The explicit form of the counterpropagating field depends on the waveguide geometry, as well as on the choice of a particular transverse mode. Due to the small transverse dimensions of the electron bunch used in a FEL (of about some tens of micrometers), only the field at the central axis of the waveguide is relevant. The following discussion, therefore, may be applied to any waveguide mode of any geometry, provided it has a nonvanishing linearly polarized

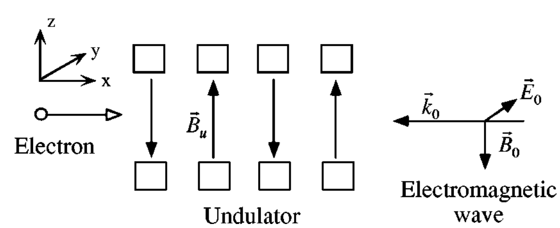

FIG. 1. Schematic diagram of the modified FEL amplifier configuration used throughout this paper. An electron bunch is injected into the linearly polarized magnetic undulator in the presence of a counterpropagating electromagnetic wave.

field along the central axis, which can be regarded as constant over the whole bunch's section. The choice of particular waveguide parameters will influence the quantitative values where the interference condition considered below is attained. For concreteness, let us assume a TE_{n0} [15] mode propagating in a rectangular waveguide. The explicit forms for the electric and magnetic field now read as follows:

$$\vec{E}_0 = -E_0 g(k_w x + \omega_0 t) \frac{k_0}{k_c} \cos(k_c z) \sin(k_w x + \omega_0 t + \phi_0) \vec{e}_y, \quad (2)$$

$$\vec{B}_0 = -E_0 g(k_w x + \omega_0 t) \sin(k_c z) \cos(k_w x + \omega_0 t + \phi_0) \vec{e}_x + E_0 g(k_w x + \omega_0 t) \frac{k_w}{k_c} \cos(k_c z) \sin(k_w x + \omega_0 t + \phi_0) \vec{e}_z, \quad (3)$$

$k_w = \sqrt{k_0^2 - k_c^2}$ being the wave number of the traveling wave, $k_0 = \omega_0/c$, and $k_c = n\pi/a$ the cutoff wave number of the waveguide (a being the width of the waveguide). $f(x)$ and $g(k_w x + \omega_0 t)$ are considered slowly varying envelopes.

Without loss of generality (by shifting the time coordinate) we may assume that the electron is initially at $x = 0$, moving along the x axis with velocity v_0. Before solving the equation of motion numerically, we may gain qualitative insight into the problem by first considering the dynamics in a new reference frame in which the electron is initially at rest. In the new frame, the undulator's magnetic field becomes a counterpropagating time-varying electromagnetic field with

$$\vec{E}'_u = -\gamma \beta f(k'_u x' + \omega'_u t') B_u \sin(k'_u x' + \omega'_u t') \vec{e}_y, \quad (4)$$

$$\vec{B}'_u = \gamma f(k'_u x' + \omega'_u t') B_u \sin(k'_u x' + \omega'_u t') \vec{e}_z, \quad (5)$$

where $k'_u = k_u \gamma$, $\omega'_u = k'_u v_0$, and $\gamma = 1/\sqrt{1-\beta^2}$ is the Lorentz factor ($\beta = v_0/c$). Note that this electromagnetic field has two peculiarities: one, the magnetic and the electric field amplitudes do not coincide in their strengths, the electric field being smaller, and, two, it propagates with a velocity $v_0 = \omega'_u / k'_u < c$. A more fruitful way is to reinterpret this field as an electromagnetic wave propagating in vacuum with a space dependent phase,

$$\vec{E}'_u = -\gamma \beta B_u f(\kappa'_u x' + \omega'_u t' + \phi_u(x')) \times \sin[\kappa'_u x' + \omega'_u t' + \phi_u(x')] \vec{e}_y, \quad (6)$$

$$\vec{B}'_u = \gamma B_u f(\kappa'_u x' + \omega'_u t' + \phi_u(x')) \times \sin[\kappa'_u x' + \omega'_u t' + \phi_u(x')] \vec{e}_z, \quad (7)$$

$$k'_u x' + \omega'_u t' = \kappa'_u x' + \omega'_u t' + \phi_u(x'), \quad (8)$$

with $\kappa'_u = \omega'_u / c$ and $\phi_u(x') = \omega'_u (1/v_0 - 1/c) x'$.

On the other hand, the counterpropagating electromagnetic wave in the new reference frame becomes

$$\vec{E}'_0 = -\gamma E_0 \frac{k_0 + k_w \beta}{k_c} \cos(k_c z') g(k'_w x' + \omega'_0 t') \times \sin(k'_w x' + \omega'_0 t' + \phi_0) \vec{e}_y \quad (9)$$

$$\vec{B}'_0 = -E_0 \sin(k_c z') g(k'_w x' + \omega'_0 t') \cos(k'_w x' + \omega'_0 t' + \phi_0) \vec{e}_x + \gamma E_0 \frac{k_0 \beta + k_w}{k_c} \cos(k_c z') g(k'_w x' + \omega'_0 t') \times \sin(k'_w x' + \omega'_0 t' + \phi_0) \vec{e}_z \quad (10)$$

with $\omega'_0 = \gamma(\omega_0 + k_w \beta c)$ and $k'_w = \gamma(k_w + k_0 \beta)$. Note that in the strong relativistic case (large γ) the effective field observed by the electron can be considered as a TEM wave. In addition, the phase velocity of this field approaches c. These two facts, together with the condition $k_c z' \simeq 0$, which is ensured by the reduced dimensions of the electron bunch, permit us to ascribe the effective field acting on the electron in its rest frame to a plane wave.

By inspection of Eqs. (6)–(10), one sees that it is possible to derive a phase matching condition in which both fields can be seen to have the same frequency in the moving frame,

$$\left.\begin{array}{l} k'_w = \kappa'_u \\ \omega'_0 = \omega'_u \end{array}\right\} \rightarrow \frac{\omega_0}{c} = \frac{k_u^2 + k_c^2}{2k_u}, \quad (11)$$

provided that the waveguide has a transverse dimension greater than half the undulator's wavelength, $a > \lambda_u/2$. Note that condition (11) has been calculated for the case of strongly relativistic electrons, $\beta \approx 1$. Although this result is derived for a rectangular waveguide, it is worth stressing that this is independent of the particular geometry (which is described by the appropriate form of k_c). Note also that the dependence of the frequency of the electromagnetic wave on k_c permits us to attain the same phase matching condition for a variety of electromagnetic waves, only by modifying the waveguide geometry.

As given by Eq. (11), the phase matching condition is defined only for the temporal oscillation. Since the undulator field has a spatial phase dependence, the corresponding wave number matching will hold only over a certain coherence length such that $\phi_u(\ell'_{coh}) = \pi$,

$$\ell'_{coh} = \frac{\lambda_u/2}{\gamma(1-\beta)}. \quad (12)$$

Before proceeding further, we may point out that the nature of the electron motion can strongly depend on this coherence length and show an interesting disordered behavior when the coherence length becomes comparable to the undulator's wavelength. However, here we are concerned with the condition in which the coherence length is greater than the total undulator length. This condition is easily fulfilled by very high energy electrons. In this situation the motion of the electron remains regular.

III. NUMERICAL INTEGRATION

As indicated above, our objective is to study the possibility of modifying the FEL emission characteristics induced by the counterpropagating microwave field, in a configuration similar to that being developed at DESY [16,17]. Hence the initial conditions consist of a relativistic electron *bunch* entering the undulator in the presence of a very weak *seed* of FEL radiation field, which is assumed to be generated from vacuum noise in the first stage of the FEL laser. Since the bunch injection energy is high, the dynamics encloses two very different space-time scales, namely, that of the undulator's field and that corresponding to the output radiation, which differ typically by a factor γ^2. This disparity becomes a limiting difficulty for the numerical integration of the evolution equations, which is usually overcome by using the appropriate slowly varying envelope approximations, along with the projections on the field cavity modes [18–20]. In this work we have chosen an alternative procedure [21] which computes the radiated field from the superposition of the *Liénard-Wiechert* fields [22] emitted from every pseudoparticle (see below) of the bunch. Furthermore, we have preferred to integrate the equations in the initial rest frame of the bunch. This allows us to avoid the problem associated with the disparity of scales since in this frame the undulator and radiated field have similar frequencies. Moreover, in the chosen frame, it becomes readily evident that the bunch density is decreased by a factor $\gamma \gg 1$, allowing us to neglect self-fields. The large number of electrons per bunch (in our case $\simeq 10^9$) in a realistic situation forces us to define *pseudo*particles, each of which includes a few thousands of electrons that are assumed to move together. Note that this is the same conceptual philosophy as employed in the successful particle-in-cell codes for the simulation of plasma dynamics [23,24]. The modulations of the charge density in the system can be modeled either by considering the spatially variable distribution of equally charged pseudoparticles, or by a spatially uniformly distributed set of variably charged pseudoparticles. For convenience, we have chosen the latter approach in the present investigation. To simulate the velocity and acceleration of the pseudoparticles, we have used a relativistic Boris algorithm [24] and to calculate the resulting emitted field of the electrons in the forward direction we have used the well-known formula of the far field radiation field amplitude of an accelerated charged particle [22]:

$$\vec{E}_{rad}(t) = \frac{q}{c}\,\frac{\dot{\beta}_y(t') - \beta_x(t')\dot{\beta}_y(t') - \beta_y(t)\dot{\beta}_x(t)}{[R - x(t')][1 - \beta_x(t')]^3}\Bigg|_{t' = t - [R - x(t')]/c}\vec{e}_y \tag{13}$$

where R is assumed to be large enough. Once the integration is performed, we Lorentz-transform the computed quantities to the laboratory reference system.

IV. COHERENT CONTROL OF FEL RADIATION

In this section we will demonstrate theoretically the possibility of controlled FEL radiation through the external electromagnetic wave. The key idea is to consider a counterpropagating wave resonant with the undulator field, in the sense discussed in Sec. II. The frequency of the wave depends, therefore, on the spatial periodicity of the undulator's magnetic field and on the particular geometry of the waveguide. In our case, we take the 2.73 cm undulator wavelength of TTF-FEL at DESY [17], and a TE_{10} mode of a rectangular waveguide of size 1.5 cm, which is similar to the size of the beam pipe of the FEL at DESY. Equation (11) defines the resonant condition for a counterpropagating electromagnetic wave in the microwave region with $\lambda = 2.99$ cm when propagating in free space.

In addition to a resonant frequency, the microwave control of the FEL amplification is more effective for the case in which the amplitude of this field in the bunch's rest frame equals the amplitude of the electromagnetic wave associated with the undulator's field. At present, microwave fields in the gigahertz range are available with powers up to 100 MW [25]. Although this is already close to the value needed to optimally control the radiation of the TTF-FEL at DESY, we prefer to be conservative and to consider in this paper a tapered undulator to reduce the undulator's magnetic field to 25 mT. With this value it should be possible to demonstrate the microwave control experimentally with current technology. On the other hand, the state of the art of microwave generation by the FEL concept allows one to foresee the availability of brighter sources in the near future [26].

Unless stated otherwise explicitly, the results are based on calculations for an electron bunch (of 300 MeV) injected into a 4.5 m magnetic undulator, whose characteristics have been commented upon above. Our numerical tests show *a posteriori* that the assumption of an initially cold bunch is acceptable. The bunch is described by a spatial $\sin^2$ distribution of 20 000 particles, 250 μm long. Small changes of this number and/or the choice of bunch shape are found not to affect the conclusions drawn from the simulations.

In the following, we consider two cases of microwave control of free-electron-laser emission. First, we analyze the possibility of suppressing the FEL output by microwave interaction, opening ways to control the pulse of the FEL radiation by modulation of the microwave amplitude. Second, we consider control of the polarization angle of the FEL radiation by changing the microwave polarization. These possibilities are particularly interesting in view of the lack of convenient optical elements at very short wavelengths to manipulate these characteristics of FEL radiation once they are extracted from the source.

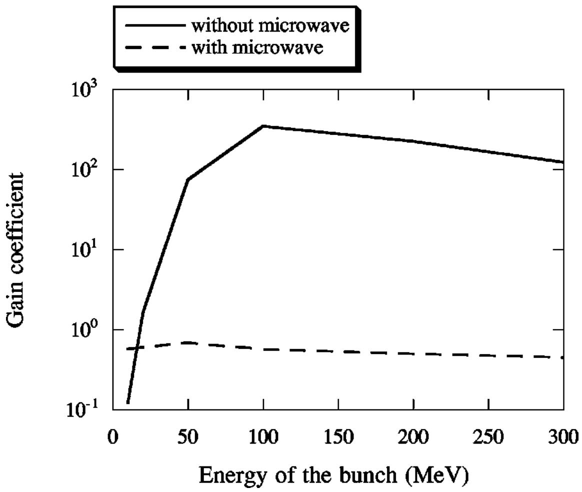

FIG. 2. Dependence of the FEL amplification factor on the initial bunch energy, in the presence of the microwave field (dashed line) and in its absence (solid line).

A. Coherent suppression of radiation

Let the counterpropagating microwave field be linearly polarized, with the polarization vector perpendicular to the direction of the undulator's magnetic field. The undulator field and the microwave field may then be made to interfere destructively when they have their phases properly matched at a critical value of the amplitude of the microwave field,

$$E_{crit}=\frac{k_c}{k_0\beta+k_w}B_u\,. \tag{14}$$

Note that [like the phase matching condition, Eq. (11)] the critical field becomes almost independent of the energy of the electron in the highly relativistic case, $\beta\approx 1$. Thus, a nearly complete destructive interference can occur for the sum magnetic field, or $B_T'=B_u'+B_0'=0$, for all time, in the

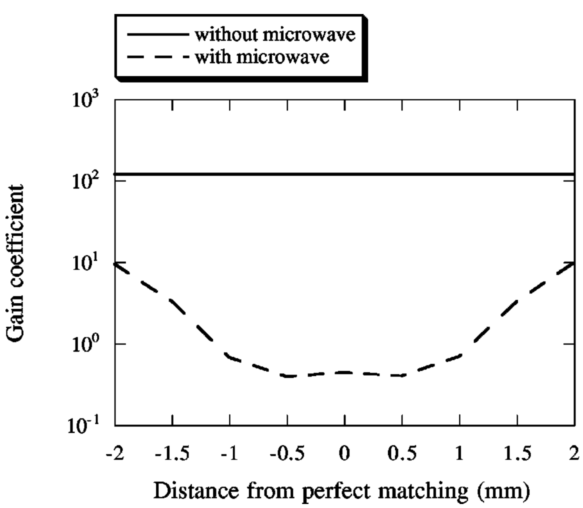

FIG. 3. Dependence of the amplification factor on the initial coordinate of the bunch; $x=0$ corresponds to the initial position for which the undulator and the microwave fields in the rest frame of the bunch have opposite phases.

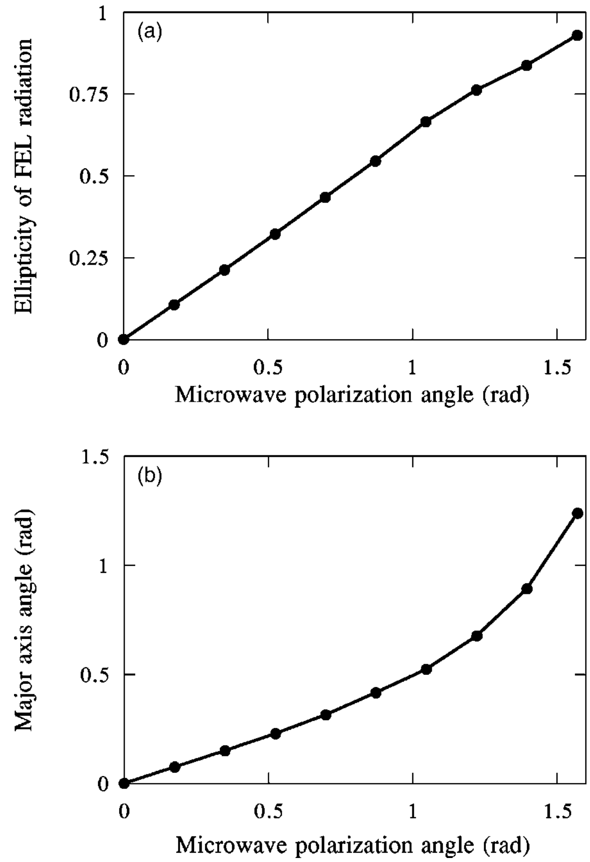

FIG. 4. (a) Ellipticity of the amplified FEL radiation versus the relative angle between the planes of polarization of the microwave and undulator fields. (b) Tilt angle of the major axis of the polarization ellipse of the FEL radiation with respect to the polarization plane of the undulator field, for the same cases as considered in (a).

moving frame. In contrast, because of the asymmetry between the magnetic and electric field amplitudes of the (Lorentz-transformed) undulator field, the total electric field in the moving frame does not vanish exactly. A small residual electric field $E_{res}'=\gamma(1-\beta)B_u$ remains, which, however, diminishes greatly as the electron energy increases. Thus for $\beta\approx 1$, $E_{res}'\simeq B_u/\gamma^2$ the interference condition for the total electric field becomes almost exactly fulfilled.

Figure 2 shows the resulting suppression of FEL radiation calculated for different initial bunch energies. Note that, as expected, the amplification gain is dramatically reduced as the bunch energy increases. This is because the residual electric field E_{res}' vanishes with increasing energy of the bunch. Note that, for the higher energies, the microwave field reduces the gain by nearly three orders of magnitude.

For the destructive interference mechanism to be effective in practice, a constant π phase difference between the undulator and the microwave fields is required, as experienced in the electron bunch's reference system. Note that ensuring an initial π dephasing requires a certain control of the bunch's conditions before injection, since the bunch must enter the undulator when the microwave phase is opposite to the undulator's. The required constancy is ensured by Eq. (12) since the coherence length is greater than the undulator's dimension for the range of bunch energies assumed here (e.g., the TTF-FEL).

To analyze the sensitivity of the coherent suppression effect against fluctuations in the initial field dephasing, we have performed a series of calculations in which the bunch's initial position against the undulator vertex is changed. The shift in the initial position is directly related to the time delay of the bunch in reaching the undulator and, therefore, to the initial dephasing between the undulator and microwave fields (in the rest frame of the bunch). The results of calculations with a 300 MeV, 250 μm bunch are presented in Fig. 3 which shows the amplification vs fluctuations in the bunch position. It can be seen that the gain suppression effect is robust against fluctuations smaller than 1 mm, which is well above the usual experimental uncertainty.

B. Control of polarization of FEL radiation

Polarization control is of particular interest for very short wave FEL radiation. This can be achieved in the same configuration by rotating the polarization of the microwave by a certain angle from the undulator's plane of polarization. From the theoretical analysis above we expected that, in general, the emitted radiation would be elliptically polarized. The ellipticity would depend on the initial dephasing of the fields as well as their relative amplitudes. We show in Fig. 4(a) the calculated change of ellipticity of FEL output vs the relative angle between the polarization plane of the undulator and that of the microwave (chosen to be linearly polarized). The amplitudes of the undulator and microwave fields in the rest frame of the bunch are chosen to be comparable while the initial dephasing is set to $\pi/2$. The resulting ellipticity of the emitted radiation is found [Fig. 4(a)] to change from the linear to the circular polarization. The results of the various cases presented in this figure are summarized in Fig. 4(b) in terms of the tilt angle of the major axis of the polarization ellipse of the emitted radiation.

V. CONCLUSION

Coherent modifications of FEL radiation induced by a *counterpropagating* electromagnetic wave interacting with an electron bunch in a magnetic undulator are studied. A phase matching condition between the undulator field and an external microwave field in the rest frame of a relativistic electron bunch is derived. This condition is found to depend only on the geometry of the problem. It is found that possible control of both the amplitude and the polarization of the FEL radiation (including very short wavelengths) could be achieved without having to alter the undulator's geometry, by simply varying the incident microwave field. Results of concrete numerical simulations assuming realistic FEL parameters (corresponding to those of TTF-FEL, currently under development at DESY) are given, and their robustness against small fluctuations in initial conditions is illustrated.

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[1] L.R. Elias *et al.*, Phys. Rev. Lett. **36**, 717 (1976).
[2] For a review of the fundamentals of free-electron lasers, see, e.g., G. Dattoli, L. Giannessi, A. Renieri, and A. Torre, Prog. Opt. **31**, 321 (1993).
[3] A collection of state-of-the-art papers on free-electron lasers, is *Free Electron Lasers 1999*, edited by J. Feldhaus and H. Weise (Elsevier Science, Amsterdam, 2000).
[4] M.J. Schmitt and C.J. Elliott, IEEE J. Quantum Electron. **23**, 1552 (1987).
[5] D. Iracane and P. Bamas, Phys. Rev. Lett. **67**, 3086 (1991).
[6] M.G. Kong, X. Zhong, and A. Vourdas, Nucl. Instrum. Methods Phys. Res. A **445**, 7 (2000).
[7] L.S. Brown and T.W.B. Kibble, Phys. Rev. A **133**, 705 (1965).
[8] E.S. Sarachik and G.T. Schappert, Phys. Rev. D **1**, 2738 (1970), and references therein.
[9] Y.I. Salamin and F.H.M. Faisal, Phys. Rev. A **54**, 4383 (1996).
[10] M.S. Hussein, M.P. Pato, and A.K. Kerman, Phys. Rev. A **46**, 3562 (1992).
[11] Y.I. Salamin and F.H.M. Faisal, Phys. Rev. A **58**, 3221 (1998).
[12] E.D. Courant, C. Pellegrini, and W. Zakowicz, Phys. Rev. A **32**, 2813 (1985).
[13] I. Wernick and T.C. Marshall, Phys. Rev. A **46**, 3566 (1992).
[14] A. van Steenbergen, J. Gallardo, J. Sandweiss, and J.-M. Fang, Phys. Rev. Lett. **13**, 2690 (1996).
[15] S. Ramo, J.R. Whinnery, and T. Van Duzer, *Fields and Waves in Communication Electronics* (Wiley & Sons, New York, 1994).
[16] E.L. Saldin, E.A. Schneidmiller, and M.V. Yurkov, Nucl. Instrum. Methods Phys. Res. A **445**, 178 (2000).
[17] R. Treusch, http://www.hasylab.desy.de/
[18] G. Dattoli *et al.*, J. Appl. Phys. **80**, 6589 (1996).
[19] M. Goto *et al.*, Nucl. Instrum. Methods Phys. Res. A **445**, 45 (2000).
[20] P. Sprangle, A. Ting, and C.M. Tang, Phys. Rev. A **36**, 2773 (1987).
[21] L.R. Elias and I. Kimel, Nucl. Instrum. Methods Phys. Res. A **393**, 100 (1997).
[22] J.D. Jackson, *Classical Electrodynamics* (Wiley & Sons, New York, 1998).
[23] J.M. Dawson, Rev. Mod. Phys. **55**, 403 (1983).
[24] C.K. Birdsall and A.B. Langdon, *Plasma Physics via Computer Simulation*, Plasma Physics Series (IOP Publishing, Bristol, 1991).
[25] H. Hanjo and Y. Nakagawa, J. Appl. Phys. **70**, 1004 (1991).
[26] Wang Pingsham, Lei Fangyan, Huang Hua, Gan Yanqing, Wang Wendou, and Gu Binglin, Phys. Rev. Lett. **80**, 4594 (1998).