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Broadband optical gain via interference in the free electron laser: Principles and proposed realizations

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We propose experimentally simplified schemes of an optically dispersive interface region between two coupled free electron lasers (FELs), aimed at achieving a much broader gain bandwidth than in a conventional FEL or a conventional optical klystron composed of two separated FELs. The proposed schemes can *universally* enhance the gain of FELs, regardless of their design, when operated in the short pulsed regime.

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I. INTRODUCTION

Free electron lasers (FELs) convert part of the kinetic energy of nearly free relativistic electrons into coherent radiation [1–4]. They can be of two types: (1) devices wherein the electrons are accelerated in a spatially periodic magnetostatic, electrostatic, or electromagnetic field, called a wiggler or an undulator; (2) devices wherein the electrons are unperturbed, but instead the laser wave is subject to dispersion, as in Cherenkov transition radiation.

In the classical description of FELs based on wigglers, the combined effect of the wiggler and laser fields yields a ponderomotive potential that causes bunching of the electrons along the wiggler axis. This bunching is associated with the gain or loss of energy by electrons, or, equivalently, their axial acceleration or deceleration, depending on the phase between their transverse motion and the laser wave. The oscillation of electrons in the ponderomotive potential is described by the pendulum equation, which can yield near-resonant gain, provided the electron velocity allows it to be nearly synchronous, i.e., to maintain a slowly changing phase, with this potential. The near-synchronism condition is for the electron velocity to be near the resonant velocity

$$v_r = \frac{\nu}{k_L + k_W}, \tag{1}$$

where ν is the laser frequency, and $k_{L(W)}$ are the laser (wiggler) wave vectors. The electrons whose velocities are above v_r contribute on average to the small-signal gain (radiation emission), and those whose velocities are below v_r contribute on average to the corresponding loss (radiation absorption).

This results in an antisymmetric dependence of the small-signal standard gain G_{st} on the deviation of the electron velocity v from the resonant velocity v_r. Such dependence has been thought to be a fundamental consequence of the Madey gain-spread theorem [1–3], which states that the gain line shape is antisymmetric, since it is proportional to the derivative of the symmetric spontaneous-emission line shape, which is a sinc^2 function of $(v - v_r)$. This gain line shape allows for net gain only if the initial velocity distribution is centered above v_r, which is often called *momentum population inversion*. In other words, this gain line shape restricts the momentum spread to values comparable with the width of the positive (gain) part of G_{st} in order to achieve net gain. This width severely limits the FEL gain performance at short wavelengths [2–4].

In a variant of the FEL, composed of two wigglers separated by a drift region between them (see Fig. 1) first suggested by Vinokurov [5], known as an "optical klystron" [6], the first wiggler serves to "bunch" the electron phases, which then acquire favorable values in the drift region between the wigglers, and finally yield enhanced gain in the second wiggler. However, in this case the width of the gain region is proportionally narrower, and this makes the restrictions on the velocity spread even more severe.

In an attempt to overcome the adverse effect of momentum spread on FEL gain, we have put forward ideas inspired by lasing without inversion (LWI) [7] in atomic systems, namely, the cancellation of absorption by interference in the gain medium. The analogous schemes proposed by us for FELs [8–11] involve two wigglers coupled by a specially designed drift region, which yields a gain curve that differs substantially from that of an optical klystron. In an optical klystron, the electrons drift between the wigglers in a dispersive region and thereby acquire a phase shift relative to the ponderomotive potential that *increases* with increasing deviation of the electron velocity from v_r. This is in contrast to our schemes [11], in which electrons in the drift region are magnetically deflected so as to acquire a dispersion that is just opposite to the one for the optical klystron: The introduced phase shift $-(\mathbf{k}+\mathbf{k}_W)(\mathbf{v}+\mathbf{v}_r)T$ (T being the mean interaction time in the wiggler) *decreases* with increasing deviation of the electron velocity from v_r. This cancels the interaction phase picked up in the first wiggler, i.e., bunching is reversed. In addition, electrons with initial velocities v

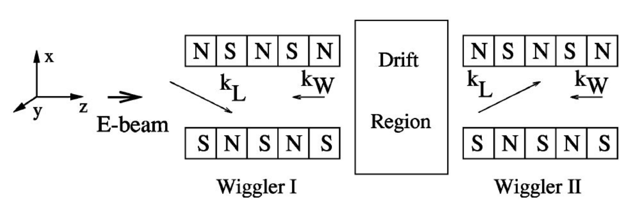

FIG. 1. Two coupled wigglers separated by a drift region.

$<v_r$, which contribute on average to loss (absorption), are given a phase shift of π, in order to cause destructive interference with electrons that contribute to loss in the second wiggler. In the resulting gain curve, the usual absorptive part (below resonance) is eliminated, whereas the gain part above resonance is doubled [11]. This implies that net gain is obtained in such schemes even from beams with a very broad (inhomogeneous) momentum spread, whence we named them FELs without inversion (FELWI), analogously to atomic LWI.

The previously proposed FELWI schemes [8–11] may open perspectives for short-wavelength FELs, provided that the technical challenges associated with magnetic field designs for the drift region are adequately met. In this paper we propose a considerably simpler variant of such schemes, aimed at extending the optical gain bandwidth in FELs. The proposed setup (Fig. 1) is equivalent to the previously proposed extension of momentum spread capable of gain, by phase shift manipulations, yet it involves only optical (laser) phase shifts in the drift region (Sec. II). These phase shifts are much easier to manipulate, since they require only linear optical elements—prisms or Bragg mirrors, etc. (Sec. III). We conclude that the proposed scheme is universal, i.e., applicable to FELs regardless of their wiggler design. It may substantially enhance the FEL performance in the pulsed regime (Sec. IV).

II. PRINCIPLES OF THE BROADBAND GAIN MECHANISM IN FELS

A. General formula for small-signal gain in two interfering wigglers

The dynamics of an electron interacting with the laser field in wiggler I or II is expressed by the pendulum equations [2]

$$\frac{d\psi}{dt}=\Omega,\quad \frac{d\Omega}{dt}=a\sin\psi, \tag{2}$$

where

$$\Omega=q_z(v_z-v_r),\qquad \psi=-\Delta\nu t+q_z z+q_x x+\phi,$$

$$q_x=k_L\sin\theta,\qquad q_z=k_L\cos\theta+k_W. \tag{3}$$

The coupling constant

$$a=\left(k_x^2+\frac{q_z^2}{\gamma_r^2}\right)\frac{2e^2A_WA_L}{p_z^2}. \tag{4}$$

is proportional to the laser field amplitude and will be used as the perturbation parameter in the small-signal regime.

The dynamics and resulting gain depend on the detuning Ω, which is a function of the laser frequency and the initial velocity v_z. A small change in the laser frequency by $\delta\omega$ or in the electron velocity by δv_z produces a similar effect on Ω, and thus on the FEL dynamics and gain [Fig. 2(a),(b)]. Here we assume that the electron beam has a narrow distribution of electron velocities v_z, and are mainly interested in

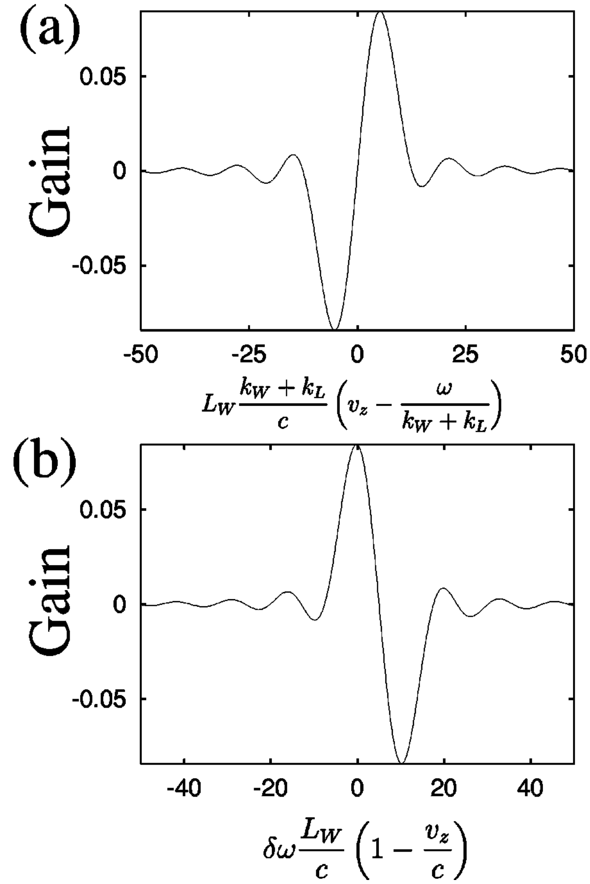

FIG. 2. Gain (arbitrary units) dependence on detuning Ω as a function of (a) $\delta v_z=v_z-\omega/(k_W+k_L)$ and (b) laser frequency variation $\delta\omega$, for the ordinary free electron laser. The frequency variation is normalized by the interaction time in the wiggler $T=L_W/c(1-v_z/c)$.

the gain dependence on the laser frequency variation $\delta\omega$. The detuning then depends on laser frequency as

$$\Omega=q_zv_z-\delta\omega\left(1-\frac{v_z}{c}\right). \tag{5}$$

Equations (2)–(4) are the basis of our consideration of uniform wigglers. In the ultrarelativistic limit, small changes of the energy, momentum, detuning, and velocity are proportional to each other, so that in order to calculate the gain we need only calculate the change in the detuning of the electrons upon averaging over the initial phase. Equations (2) are effectively one dimensional, but $\vec{q}$ and Ω are two-dimensional (2D) parameters and this 2D dependence will prove to be of vital importance.

In order to enhance the bandwidth of the FEL gain, we consider the setup of two identical wigglers of equal length L_W with a specially designed drift region between them, as described below. Since the change of electron energy inside the first and second wigglers is given by the same set of equations (2)–(4), we obtain, by taking into account the phase shift in the drift region and averaging over the random initial phases, the following expression for gain in the entire two-wiggler setup [11,12]:

$$(\mathrm{Gain})\sim\langle\Delta\gamma\rangle\sim-\langle\Delta\Omega\rangle$$
$$=\frac{1}{\Omega^3}[2\Omega T\sin\Omega T+4\cos\Omega T-4+2\Omega T\sin(2\Omega T+\Delta\psi)-2\Omega T\sin(\Omega T+\Delta\psi)+2\cos\Delta\psi+2\cos(2\Omega T+\Delta\psi)-4\cos(\Omega T+\Delta\psi)], \tag{6}$$

where $T=L_W/c(1-v_z/c)$ is the mean interaction time in the wiggler.

For $\Delta\psi=0$ the two-wiggler gain coincides with the result for one wiggler of twice the length. The resulting gain dependence on the detuning Ω and laser frequency variation $\delta\omega$ is depicted in Figs. 2(a) and 2(b), respectively. The average of this antisymmetric gain over detuning vanishes, in accordance with the Madey gain-spread theorem, which is the main restriction on gain in short-wavelength FELs [2].

The electron oscillates coherently in the ponderomotive potential, and therefore its oscillations in the two sequential wigglers exhibit interference with a phase $\Delta\psi$ which depends on the path (or time) difference between the two regions. In an optical klystron, the phase shift, produced in a free space of length L between the wigglers, is equal to

$$\Delta\psi_{\mathrm{klystron}}=k_W L+\frac{ck_L q_z L}{\omega^2}\Omega. \tag{7}$$

The gain dependence of the optical klystron on the detuning Ω and laser frequency $\delta\omega$ is depicted in Fig. 3(a). The maximum gain exceeds that of the ordinary FEL, but the restriction on the spread of Ω [and therefore on $\delta\omega$ in Eq. (5)] becomes more stringent, because of rapid oscillations of the gain dependence on the detuning [Fig. 3(a)].

B. Broadband gain by drift-region optical dispersion

In order to overcome the limitations of FELWI phase-shift implementation for electrons in magnetic fields discussed in Sec. I, we consider the alternative phase shift produced by an optically dispersive drift region, where the light path depends on the deviation $\delta\omega$ of the laser frequency as follows:

$$\Delta\psi=\Delta\psi_0+\delta\omega\left(\frac{s_L(\omega)+\omega[ds_L(\omega)/d\omega]}{c}-\frac{s_e}{v}\right). \tag{8}$$

Here $s_L(\omega)$ is the optical path depending on the laser frequency in the drift region, s_e is the electron path passing through the drift region as before, ω_0 is the mean laser frequency, and the corresponding phase shift is equal to

$$\Delta\psi_0=\omega_0\left(\frac{s_L(\omega_0)}{c}-\frac{s_e}{v}\right). \tag{9}$$

The spectral dispersion of the drift region in Eq. (8) allows us to manipulate the dependence of the gain (6) on detuning. As a result, *broadband gain* appears, as shown in Fig. 3(b). The explicit condition that the dispersion (8) must satisfy in order to obtain broadband gain is

$$\frac{s_L(\omega)+\omega[ds_L(\omega)/d\omega]}{c}\simeq\frac{s_e}{v}. \tag{10}$$

The gain dependence on the laser frequency deviation $\delta\omega$ exhibits a broadband in Fig. 3(b) for the same parameters as in Fig. 3(a): the contrast between our design and an optical klystron is striking indeed, and demonstrates the crucial effect of optical dispersion on gain.

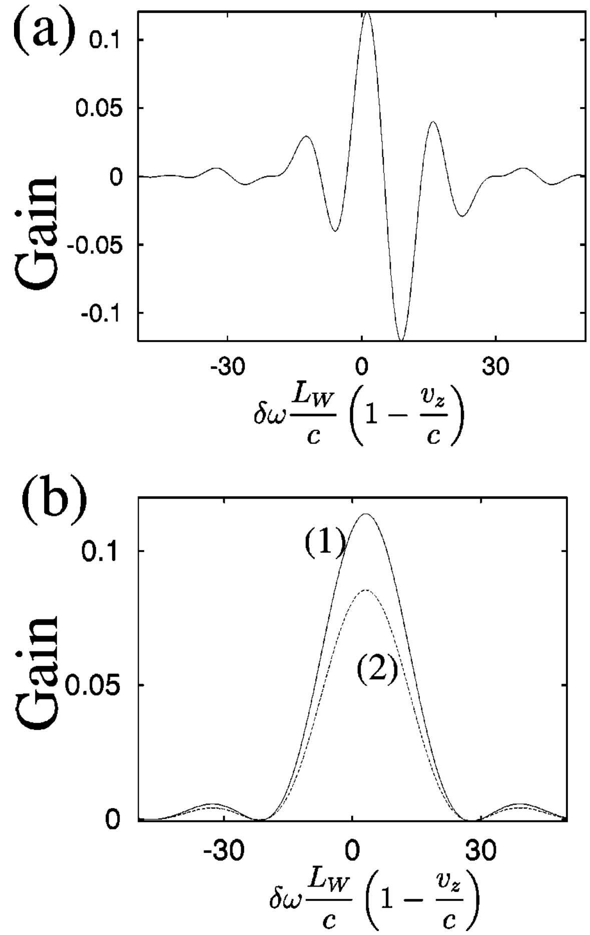

FIG. 3. Gain (arbitrary units) dependence on laser frequency for two coupled wigglers. (a) As is clearly seen, in comparison with Fig. 2, the maximum gain for an optical klystron configuration has been increased by adding a proper phase shift via the drift region but without adjustment of dispersion for the laser beam. The gain dependence on the laser frequency experiences fast oscillations because of interference between waves emitted in the first and in the second wiggler. (b) By proper adjustment of the phase shift for every laser frequency via an appropriately designed dispersive drift region, satisfying Eqs. (8)–(10), a broadband gain has been obtained. For the case (b) two curves are shown: (1) without electron momentum spread and (2) with electron velocity spread $\Delta v_z=5c/L_W(k_L+k_W)$. Note the broad gain bandwidth, which allows for ultrashort pulsed FEL operation ($\delta\omega\sim20c/L_W$) (normalization by T as in Fig. 2).

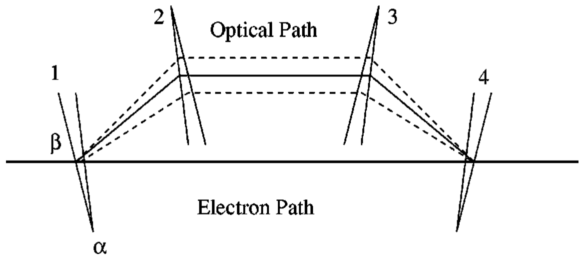

FIG. 4. Scheme of the drift region with prisms for a broadband FEL (or optical phased klystron).

The extension of this treatment to situations wherein the effects of δv_z and $\delta\omega$ are comparable involves the averaging of Eqs. (6) and (8) over a thermal spread of electron velocities. For moderate spreads the broadband character of the gain persists, as seen in Fig. 3(b).

Before we discuss implementations of the drift region for broadband gain, let us note here that, although the current and the FELWI concepts are both based upon interference of radiation emitted by electrons moving in the first and the second wigglers via the phase shift created by the drift region, there is an important difference. As has been shown in [12], the motion in the FELWI drift region has to be 2D (otherwise, the phase density is conserved in accordance with Liouville's theorem, and, for an electron distribution having large spread of momenta, the gain is zero). To obtain broadband gain, it is not necessary to have the setup of the drift region 2D (for electron motion), but rather the optical dispersion of the drift region should satisfy Eq. (10).

III. DRIFT-REGION DESIGNS FOR BROADBAND GAIN

We shall now discuss two possible experimental implementations of optically dispersive drift regions created by optical elements in order to get broadband gain: (a) diffraction by prisms; (b) Bragg reflectors.

A. Drift-region dispersion based on prisms

Let us consider the setup of the drift region depicted in Fig. 4. After the first wiggler the electron beam, having passed through free space of length s_e, enters the second wiggler. The laser beam is diffracted and guided by the set of prisms 1, 2, 3, and 4, which are adjusted to have *vanishing total dispersion*. The phase shift introduced by this setup for the laser field is given by

$$\Delta\psi = k_L\left[2x_0\left(\frac{1}{\cos\alpha}-1\right)-s_e\left(\frac{c}{v_e}-1\right)\right], \tag{11}$$

where x_0 is the distance between prisms 1 and 2, α is the angle of diffraction for the laser field, $\pi/2$ is the tip angle of the prism, and s_e is the distance between prisms 1 and 4, which is also the electron path in the drift region. For a small deviation of the laser frequency $\delta\omega$, the phase shift is given by

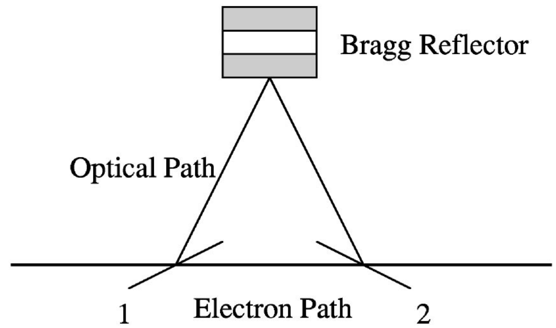

FIG. 5. Scheme of the drift region with a Bragg reflector for a broadband FEL.

$$\Delta\psi = \Delta\psi_0 + \frac{1}{c}\left[s_e\left(1-\frac{c}{v_e}\right)+2x_0\left(\frac{1}{\cos\alpha}-1\right) + \frac{2x_0\omega(d\alpha/d\omega)\tan\alpha}{\cos\alpha}\right]\delta\omega, \tag{12}$$

where the phase shift for the central frequency ω_0 is given by

$$\Delta\psi_0 = \frac{\omega_0}{c}\left[s_e\left(1-\frac{c}{v_e}\right)+2x_0\left(\frac{1}{\cos\alpha}-1\right)\right], \tag{13}$$

$d\alpha/d\omega$ being the angular dispersion of the prism.

A properly chosen dispersion of the prisms, in accordance with Eq. (10), allows us to adjust the drift region so as to have broadband optical gain [see Fig. 3(b)]. The choice, which cancels the $\delta\omega$ term in Eq. (12), is

$$\frac{d\alpha}{d\omega} = \frac{s_e(c/v_e-1)+2x_0(1-1/\cos\alpha)}{2x_0\omega\tan\alpha}\cos\alpha. \tag{14}$$

B. Drift-region dispersion based on a Bragg reflector

A system of prisms is not the only way to create a proper phase shift. We may use a Bragg reflector instead, depicted in Fig. 5. The phase shift introduced by a Bragg reflector is given by

$$\psi = \tan^{-1}\left(\frac{1-R}{1+R}\,\frac{1}{\tan(kl/2\cos\theta)}\right), \tag{15}$$

where R is the reflectance of the Bragg structure, l is its length, and θ is the angle of incidence. The broadband gain conditions for this setup are $\cos\theta > v/c$ and

$$\frac{d\psi}{d\omega} = \frac{1-R^2}{\sin^2(kl/2\cos\theta)(1+R)^2+\cos^2(kl/2\cos\theta)(1-R)^2} \times \frac{l}{2c\cos\theta} = L_e\left(\frac{1}{v}-\frac{1}{c\cos\theta}\right), \tag{16}$$

where L_e is the distance between mirrors 1 and 2 (Fig. 5).

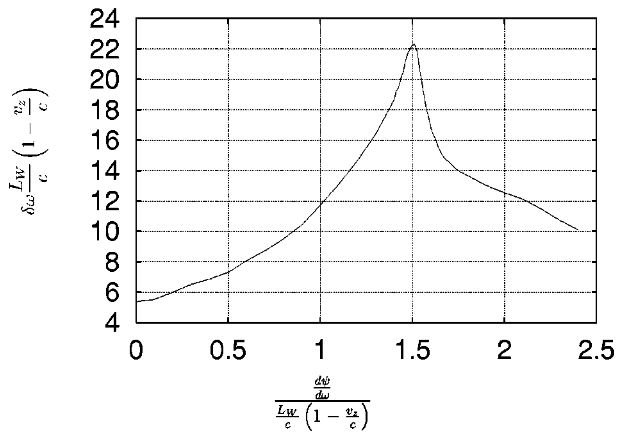

FIG. 6. Dependence of the FEL gain width on the dispersion parameter $d\psi/d\omega$. It is seen that gain exists for all FEL frequencies at which strong interaction with the ponderomotive potential occurs. The same type of dependence appears for drift regions shown in Figs. 4 and 5. The parameter space is different, but in both cases there is a parameter region where broadband gain exists (normalization by T as in Figs. 2 and 4).

It is instructive to calculate the dependence of gain width on the parameters of the drift regions. In Fig. 6, we show the gain-width dependence on the dispersion of the drift region, $d\psi/d\omega$. The figure demonstrates that the gain width can be made as large as the spectral range wherein the ponderomotive interaction between the electrons and the laser field does not vanish [as seen in Fig. 3(a), there is no spectral range where the gain is negative]. Clearly, the parameters that provide broadband gain are different for the drift regions shown in Fig. 4 and Fig. 5, but the gain-width dependence on the total dispersion $d\psi/d\omega$ is *universal*.

IV. CONCLUSIONS

We have shown in this paper that the small-signal gain in a FEL comprised of two coupled wigglers can exhibit a broadband character as a function of the laser frequency, if their interface (the region wherein electrons and light drift without interaction) is endowed with appropriate optical dispersion. This design is based on the same principles as in [10,11], except that in [11] specially designed magnetic fields are proposed for manipulating the drift-region phase shifts, whereas here linear optical elements, such as prisms (Sec. III A) or a Bragg reflector (Sec. III B) suffice. The present dispersive scheme, similarly to [11], achieves the cancellation of absorption in a broad spectral range by destructive interference of frequencies that contribute on average to loss in the two wigglers, and the reinforcement of emission by constructive interference of frequencies that contribute to gain. Remarkably, the resulting broadband gain exhibits universal dependence on the optical dispersion, and is compatible with existing FELs, regardless of their design. The broadband character of the gain persists for moderate spreads of electron velocities.

The proposed scheme may allow effective FEL operation using femtosecond optical pulses, which correspond to a wide spectral band: a gain bandwidth of $\sim 20c/L_W$ (Fig. 3) may attain 10^{14} Hz values for optical wigglers with submicrometer periods [2–4].

ACKNOWLEDGMENTS

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