Nonlinear collisional absorption in dense laser plasmas

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(Received 3 April 2001; published 27 July 2001)

Collisional absorption of dense, fully ionized plasmas in strong laser fields is investigated starting from a quantum kinetic equation with non-Markovian and field-dependent collision integrals in dynamically screened Born approximation. This allows to find rather general balance equations for the energy and the current. For high-frequency laser fields, quantum statistical expressions for the electrical current density and the cycle-averaged electron-ion collision frequency in terms of the Lindhard dielectric function are derived. The expressions are valid for arbitrary field strength assuming the nonrelativistic case. Numerical results are presented to discuss these quantities as a function of the applied laser field and for different plasma parameters. In particular, nonlinear phenomena such as higher harmonics generation and multiphoton emission and absorption in electron-ion collisions are considered. The significance to include quantum effects is demonstrated comparing our results for the collision frequency with previous results obtained from classical theories.

DOI: 10.1103/PhysRevE.64.026414 PACS number(s): 05.30.−d, 52.25.Dg

I. INTRODUCTION

In recent years the laser-matter interaction has become a field of increasing interest. This is due to the impressive progress in laser technology that makes short-pulse lasers of high intensity available in laboratory experiments [1]. During the interaction of such laser pulses with solid targets, dense plasmas can be created relevant for astrophysical research and for inertial confinement fusion. Especially, at high intensities the quiver velocity $v_0 = eE_0/m_e\omega$ can be large compared to the thermal velocity $v_{\rm th} = \sqrt{k_B T_e/m_e}$, and interesting nonlinear effects have to be expected.

Of special importance for modeling intense laser-matter interaction is to understand the transfer processes of energy from the laser field to matter. They determine the created nonequilibrium plasma state and the various high-field phenomena such as multiphoton ionization, higher harmonics generation, and x-ray emission. One of the important mechanisms of energy deposition is inverse bremsstrahlung that considers laser light absorption via collisional processes between the plasma particles. In strongly ionized plasmas, this absorption process is essentially governed by the electron-ion interaction usually described in terms of the electron-ion collision frequency.

In several papers, various approaches were used to calculate the electron-ion collision frequency and the dynamic conductivity, respectively, for laser plasmas under different conditions. Dawson and Oberman [2] evaluated the high-frequency conductivity for fully ionized plasmas in lowest order of $v_0/v_{\rm th}$ in the frame of classical dielectric theory based on the Vlasov-Poisson set of equations. Recently, their model was extended to the strong field case in a paper of Decker *et al.* [3]. Expressions for the electron-ion collision frequency for arbitrary ratios $v_0/v_{\rm th}$ were already derived by Silin in 1964 [4] and by Klimontovich in 1975 [5] starting from kinetic equations for classical plasmas with non-Markovian Landau and Lenard-Balescu collision terms, respectively. A ballistic model was recently considered by Mulser *et al.* [6] in order to study the time-dependent electron-ion collision frequency in laser plasmas. The effect of non-Maxwellian distribution functions during the absorption process was predicted by Langdon [7], and it was investigated more in detail by several authors in subsequent papers (see, e.g., [8–10,3]). Inverse Bremsstrahlung absorption calculations for classical strongly coupled plasmas applying a tree code simulation were performed by Pfalzner and Gibbon [11].

The theories mentioned so far consider classical plasmas. Consequently, they cannot be applied to situations where quantum effects become important. Quantum effects in dense plasmas can be expected (i) if the Landau length $l = e^2/4\pi\epsilon_0 k_B T_e$ is comparable with the thermal wavelength $\lambda = (2\pi\hbar^2/m_e k_B T_e)^{1/2}$, i.e., $l/\lambda \leqslant 1$, (ii) for $\hbar\omega/k_B T_e > 1$ with ω being the laser frequency, and (iii) if the electrons with number density n_e have to be described by Fermi statistics in degenerate plasmas, i.e., $n_e\lambda^3 > 1$. Quantum mechanical treatments were first given by Rand [12] and by Schlessinger and Wright [13] determining the nonlinear absorption rate by averaging the Born cross section of photon emission and absorption. A quantum approach to calculate the electron-ion collision frequency in strong fields was also developed by Silin and Uryupin [14]. Cauble and Rozmus [15] investigated the inverse bremsstrahlung absorption coefficient in the linear regime using a semiclassical memory function kinetic formalism including lowest order quantum effects by a quantum pair potential. Here, the correlations between the particles were treated by applying classical integral-equation methods. In papers of Röpke *et al.* [16] and Reinholz *et al.* [17], a quantum statistical approach to the dynamical conductivity was presented using linear-response theory. A quantum mechanical dielectric model to calculate the electron-ion collision frequency for arbitrary field strength was recently presented by Kull and Plagne [18].

A rigorous kinetic approach to the inverse bremsstrahlung absorption in dense plasmas including all the quantum effects mentioned above was still missing until recently. Kremp *et al.* [19] derived a quantum kinetic equation for dense plasmas in strong laser fields using nonequilibrium

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Green's function techniques. In this approach, the different interaction processes, e.g., electron-electron, electron-ion, and electron-atom scattering, can be taken into account by appropriate approximations of the generalized field-dependent scattering rates. In the paper mentioned above, the theory was applied to fully ionized nonrelativistic plasmas. The non-Markovian collision term was evaluated in statically screened Born approximation including nonlinear field effects such as multiphoton processes and higher harmonics generation. Subsequently, quantum statistical expressions for the electron-ion collision frequency were derived, and time-dependent phenomena were studied by numerical solution of this equation [20–22]. It turns out that the quantum treatment avoids automatically the well-known divergencies occurring in the classical weak coupling theories. Generalized quantum expressions for the collision term and the electron-ion collision frequency including dynamic screening were given for the first time in [23] and [24].

The aim of the present paper is to continue our investigation on inverse bremsstrahlung absorption in dense fully ionized laser plasma based on quantum kinetic theory. *Especially*, the influence of quantum effects on the electron-ion collision frequency is studied in more detail for arbitrary values of v_0/v_{th}. For this purpose a kinetic equation with a new appropriate form of the quantum collision term in dynamically screened Born approximation is derived and applied to high-frequency laser fields.

The paper is organized as follows. In Sec. II, a brief derivation of the quantum kinetic equation for the Wigner distribution function is presented. Here, the non-Markovian and field-dependent scattering rates are given in terms of the screened potential and the two-time polarization functions that are used in *random phase approximation* (RPA). The resulting balance equations for the energy and the electrical current density are given in Sec. III. In particular, a general non-Markovian equation determining the current density in terms of the field-dependent dielectric function is derived. In Sec. IV, the theory is applied to high-frequency laser fields using a perturbation ansatz for the electron distribution function. In the case of harmonic fields, the current density and the electron-ion collision frequency can be expressed then by Fourier expansion in terms of Bessel functions and the field-free quantum (Lindhard) dielectric function. Numerical results for the current density and the cycle-averaged electron-ion collision frequency for dense, fully ionized plasmas as a function of v_0/v_{th} and for different plasma parameters are discussed in Sec. V. Nonlinear field phenomena such as higher harmonics generation and multiphoton processes in electron-ion collisions are considered, and the influence of quantum effects is demonstrated comparing the results with those obtained from classical theories.

II. QUANTUM KINETIC EQUATION FOR DENSE PLASMAS IN ELECTROMAGNETIC FIELDS

We consider a dense plasma under the influence of intense laser radiation. The plasma is assumed to be fully ionized consisting of electrons with number density n_e and ions of charge $e_i = Ze$ with number density n_i. Focusing on spatially homogeneous electric fields and using the vector potential gauge

$$\mathbf{A}(t) = -\int_{-\infty}^{t} d\bar{t}\, \mathbf{E}(\bar{t}); \quad A_0 = \phi = 0, \tag{1}$$

the following kinetic equation for the gauge invariant Wigner distribution function was derived recently [19]

$$\left\{\frac{\partial}{\partial t} + e_a \mathbf{E}(t)\cdot \nabla_{k_a}\right\} f_a(\mathbf{k}_a, t) = -2\,\mathrm{Re}\int_{t_0}^{t} d\bar{t}\{\Sigma_a^{>} g_a^{<} - \Sigma_a^{<} g_a^{>}\} \\ = I_a(\mathbf{k}_a, t), \tag{2}$$

where a labels the particle species. The collision integral is given in terms of the self-energy functions (scattering rates), $\Sigma_a^{\gtrless}$, and the two-time correlation functions $g_a^{\gtrless}$. Here, the arguments are

$$\Sigma_a^{\gtrless} g_a^{\lessgtr} \equiv \Sigma_a^{\gtrless}[\mathbf{k}_a + \mathbf{K}_a^A(t,\bar{t}); t, \bar{t}]\, g_a^{\lessgtr}[\mathbf{k}_a + \mathbf{K}_a^A(t,\bar{t}); \bar{t}, t], \tag{3}$$

where $\mathbf{K}_a^A$ is the field-induced momentum shift $\mathbf{K}_a^A(t,\bar{t}) \equiv e_a \int_{\bar{t}}^{t} dt' [\mathbf{A}(t) - \mathbf{A}(t')]/(t - \bar{t})$. The gauge-invariant modified Fourier transform is given by [25,26]

$$g_a(\mathbf{k}; t, t') = \int d^3 r\, g_a(\mathbf{r}; t, t') \\ \times \exp\left\{-\frac{i}{\hbar}\mathbf{r}\cdot\left[\mathbf{k} + e_a \int_{t'}^{t} d\bar{t} \frac{\mathbf{A}(\bar{t})}{(t - t')}\right]\right\}. \tag{4}$$

The kinetic equation (2) is still very general, and two steps are necessary to find a closed form with an explicit expression for the collision term: (i) The self-energy functions have to be specified in a certain approximation, and (ii) the two-time correlation functions have to be expressed in terms of the Wigner distribution function. The latter point is known as the reconstruction problem [27].

Powerful schemes are available to determine appropriate approximations for the self-energy functions $\Sigma_a^{\gtrless}$ taking into account nonlinear field dependence as well as many-body and quantum effects relevant for high-density plasmas. We will use here the so-called V^s-approximation that corresponds to a dynamically screened Born approximation applicable to weakly coupled plasmas. It reads for the gauge-invariant Fourier transform

$$\Sigma_a^{\gtrless}(\mathbf{k}; t, t') = i\hbar \int \frac{d^3 q}{(2\pi\hbar)^3} g_a^{\gtrless}(\mathbf{k} - \mathbf{q}; t, t')\, V_{aa}^{s\,\gtrless}(\mathbf{q}; t, t'). \tag{5}$$

After insertion of this expression into Eq. (2) the collision integral can be written as

$$I_a(\mathbf{k}_a,t)=2\,\mathrm{Re}\int_{t_0}^{t}d\bar{t}\int\frac{d^3q}{(2\pi\hbar)^3}\{V_{aa}^{s>}(\mathbf{q};t,\bar{t})\times\tilde{\Pi}_{aa}^{<}[\mathbf{k}_a+\mathbf{K}_a^A(t,\bar{t}),\mathbf{q},\bar{t},t]-V_{aa}^{s<}(\mathbf{q};t,\bar{t})\times\tilde{\Pi}_{aa}^{>}[\mathbf{k}_a+\mathbf{K}_a^A(t,\bar{t}),\mathbf{q};\bar{t},t]\}.\qquad(6)$$

Here, we introduced the auxiliary function $\tilde{\Pi}_{aa}^{\lessgtr}(\mathbf{k_a},\mathbf{q};\bar{t},t)=-i\hbar\, g_a^{\gtrless}(\mathbf{k}_a-\mathbf{q};t,\bar{t})g_a^{\lessgtr}(\mathbf{k}_a;\bar{t},t)$. The key quantities in the collision term (6) are the correlation functions of the dynamically screened potential, $V^{s>}$ and $V^{s<}$, which are related to the correlation functions of the longitudinal field fluctuations [28]. Within the RPA we have

$$V_{ab}^{s\gtrless}(\mathbf{q};t_1,t_2)=\sum_c\int_{t_0}^{\infty}d\bar{t}V_{ac}(\mathbf{q})[\Pi_{cc}^{R}(\mathbf{q};t_1,\bar{t})V_{cb}^{s\gtrless}(\mathbf{q};\bar{t},t_2)+\Pi_{cc}^{\gtrless}(\mathbf{q};t_1,\bar{t})V_{cb}^{s\,A}(\mathbf{q};\bar{t},t_2)],\qquad(7)$$

and

$$V_{ab}^{s\,R/A}(\mathbf{q};t_1,t_2)=V_{ab}(\mathbf{q})\,\delta(t_1-t_2)+\sum_c\int_{t_0}^{\infty}d\bar{t}\,V_{ac}(\mathbf{q})\times\Pi_{cc}^{R/A}(\mathbf{q};t_1,\bar{t})V_{cb}^{s\,R/A}(\mathbf{q};\bar{t},t_2),\qquad(8)$$

where $V_{ab}(\mathbf{q})=e_ae_b\hbar^2/(\varepsilon_0q^2)$ is the Coulomb potential, and $V_{ab}^{s\,R/A}$ are the retarded and advanced screened potentials. The polarization functions $\Pi_{aa}^{\gtrless}(\mathbf{q};t_1,t_2)$ are given by

$$\Pi_{aa}^{\gtrless}(\mathbf{q};t_1,t_2)=\int\frac{d^3k_a}{(2\pi\hbar)^3}\tilde{\Pi}_{aa}^{\gtrless}(\mathbf{k_a},\mathbf{q};t_1,t_2),\qquad(9)$$

and for the retarded and advanced polarization functions we have

$$\Pi_{aa}^{R/A}(\mathbf{q};t_1,t_2)=\pm\Theta[\pm(t_1-t_2)]\,[\Pi_{aa}^{>}(\mathbf{q};t_1,t_2)-\Pi_{aa}^{<}(\mathbf{q};t_1,t_2)]$$

The collision integral (6) contains the dependence on the species in an implicit form. However, for our further considerations, especially the discussion of balance equations, it is necessary to identify the particular integrals for collisions between the different species according to $I_a(\mathbf{k}_a,t)=\Sigma_bI_{ab}(\mathbf{k}_a,t)$. Using Eqs. (7) and (8), we get the result

$$I_{ab}(\mathbf{k}_a,t)=2\,\mathrm{Re}\int\frac{d^3q}{(2\pi\hbar)^3}V_{ab}(\mathbf{q})\int_{t_0}^{t}d\bar{t}_1\int_{t_0}^{t}d\bar{t}_2\times\{\Pi_{bb}^{R}(\mathbf{q};t,\bar{t}_2)V_{ba}^{s\,R}(\mathbf{q};\bar{t}_2,\bar{t}_1)\times\tilde{\Pi}_{aa}^{<}(\mathbf{k}_a+\mathbf{K}_a^A,\mathbf{q};\bar{t}_1,t)+[\Pi_{bb}^{R}(\mathbf{q};t,\bar{t}_2)\times V_{ba}^{s<}(\mathbf{q};\bar{t}_2,\bar{t}_1)+\Pi_{bb}^{<}(\mathbf{q};t,\bar{t}_2)V_{ba}^{s\,A}(\mathbf{q};\bar{t}_2,\bar{t}_1)]\times\tilde{\Pi}_{aa}^{A}(\mathbf{k}_a+\mathbf{K}_a^A,\mathbf{q};\bar{t}_1,t)]\},\qquad(10)$$

where $\mathbf{K}_a^A$ is a short hand notation for the field-induced momentum shift (3) with $\mathbf{K}_a^A=\mathbf{K}_a^A(t,\bar{t}_1)$. Here we want to mention, that Eq. (7) could be transformed into another form, $V_{ab}^{\gtrless}=\Sigma_c\int V_{ac}^{s\,R}\Pi_{cc}^{\gtrless}V_{cb}^{s\,A}$, which is often used [cf. Eq. (56) in [23]]. However, this form is unfavorable to derive non-Markovian source terms in the balance equations.

So far the collision integral (10) is given as a functional of the two-time correlation functions $g_a^{\gtrless}(t,t')$. In order to get an expression in terms of one-time distribution functions, one has to solve the reconstruction problem mentioned above. Due to the non–Markovian character of the collision integral (10), we introduce the Wigner distribution functions by the generalized Kadanoff-Baym ansatz (GKBA). It reads in gauge-invariant form [19]

$$\pm g_a^{\gtrless}(\mathbf{k};t,t')=g_a^{R}(\mathbf{k};t,t')f_a^{\gtrless}[\mathbf{k}-\mathbf{K}_a^A(t',t),t']-f_a^{\gtrless}[\mathbf{k}-\mathbf{K}_a^A(t,t'),t]g_a^{A}(\mathbf{k};t,t'),\qquad(11)$$

where $f_a^{<}(t)=-i\hbar g_a^{<}(t,t)=f_a(t)$ defines the Wigner function $f_a(t)$. Furthermore, we have $f_a^{>}(t)=i\hbar g_a^{>}(t,t)=1-f_a(t)$. The gauge-invariant retarded and advanced propagators for free particles in an external field are given by

$$g_a^{R/A}(\mathbf{k};t,t')=\pm\frac{1}{i\hbar}\Theta(\pm(t-t'))\exp\left(-\frac{i}{\hbar}\left\{\frac{k^2}{2m_a}(t-t')+\frac{e_a^2}{2m_a}\left[\int_{t'}^{t}d\bar{t}\mathbf{A}^2(\bar{t})+\frac{1}{t-t'}\left(\int_{t'}^{t}d\bar{t'}\,\mathbf{A}(\bar{t})\right)^2\right]\right\}\right).\qquad(12)$$

In fact, the quantities given in the collision integral (10) can now be expressed in terms of the Wigner distribution functions. We get, e.g., the following field-dependent expression for the retarded polarization function in RPA

$$\Pi_{aa}^{R}(\mathbf{q};t,t')=-\frac{i}{\hbar}\Theta(t-t')e^{(i/\hbar)\mathbf{q}\cdot\mathbf{R}_a(t,t')}\int\frac{d^3k}{(2\pi\hbar)^3}\times e^{-(i/\hbar)(\epsilon_{\mathbf{k+q}}^{a}-\epsilon_{\mathbf{k}}^{a})(t-t')}\{f_a[\mathbf{k}+\mathbf{Q}_a(t,t');t']-f_a[\mathbf{k}+\mathbf{q}+\mathbf{Q}_a(t,t');t']\},\qquad(13)$$

with $\epsilon_{\mathbf{k}}^{a}=k^2/2m_a$ being the single particle energy. The advanced polarization function follows from the relation $\Pi_{aa}^{A}(\mathbf{q};t,t')=[\Pi_{aa}^{R}(\mathbf{q};t',t)]^*$. The quantities $\mathbf{Q}_a$ and $\mathbf{R}_a$ are given by

$$\mathbf{Q}_a(t,\bar{t})\equiv-e_a\int_{\bar{t}}^{t}dt'\;\mathbf{E}(t')=e_a[\mathbf{A}(t)-\mathbf{A}(\bar{t})],\qquad(14)$$

$$\mathbf{R}_a(t,\bar{t})\equiv\frac{e_a}{m_a}\int_{\bar{t}}^{t}dt'\int_{t'}^{t}dt''\mathbf{E}(t'')=\frac{e_a}{m_a}\int_{\bar{t}}^{t}dt'[\mathbf{A}(t')-\mathbf{A}(t)].\qquad(15)$$

The physical meaning of the two latter quantities becomes obvious from the equation of motion of charged particles in an electric field $\mathbf{E}$. $\mathbf{Q}_a(t,t')$ is just the momentum gain of a particle in the E field during the time $t-t'$ that is of the order of the duration of the collision. $\mathbf{R}_a(t,t')$ is the field-induced replacement of the particle during this time.

Finally, the collision integral can be written as

$$I_{ab}(\mathbf{k}_a,t)=-\frac{2}{\hbar^2}\mathrm{Re}\int_{t_0}^{t}d\overline{t_1}\int_{t_0}^{t}d\overline{t_2}\int\frac{d^3q}{(2\pi\hbar)^3}\int\frac{d^3k_b}{(2\pi\hbar)^3}V_{ab}(\mathbf{q})\exp(-i\{[\epsilon^a_{\mathbf{k}_a-\mathbf{q}}-\epsilon^a_{\mathbf{k}_a}](t-\overline{t_1})-\mathbf{q}\cdot\mathbf{R}_a(t,\overline{t_1})\}/\hbar)$$
$$\times\exp(-i\{[\epsilon^b_{\mathbf{k}_b+\mathbf{q}}-\epsilon^b_{\mathbf{k}_b}](t-\overline{t_2})+\mathbf{q}\cdot\mathbf{R}_b(t,\overline{t_2})\}/\hbar)\{[f_b(\overline{t_2})-\overline{f_b}(\overline{t_2})]V^{s\,R}_{ba}(\mathbf{q};\overline{t_2},\overline{t_1})f_a(\overline{t_1})[1-\overline{f_a}(\overline{t_1})]$$
$$+[f_b(\overline{t_2})-\overline{f_b}(\overline{t_2})]V^{s\,<}_{ba}(\mathbf{q};\overline{t_2},\overline{t_1})[f_a(\overline{t_1})-\overline{f_a}(\overline{t_1})]+\overline{f_b}(\overline{t_2})[1-f_b(\overline{t_2})]V^{s\,A}_{ba}(\mathbf{q};\overline{t_2},\overline{t_1})[f_a(\overline{t_1})-\overline{f_a}(\overline{t_1})]\}. \tag{16}$$

For a shorter notation, the momentum arguments of the distribution functions were dropped, i.e., one has $f_c(\overline{t})=f_c(\mathbf{k}_c+\mathbf{Q}_c(t,\overline{t});\overline{t})$ with $c=a,b$. For the functions $\overline{f_a}$ and $\overline{f_b}$ one has to replace $\mathbf{k}_c$ by $\mathbf{k}_a-\mathbf{q}$ and $\mathbf{k}_b+\mathbf{q}$, respectively.

The above quantum collision term is valid for arbitrary field strengths. The comparison with the field-free case shows that the applied time-dependent field modifies the collision integral in several ways: (i) The momentum arguments of the distribution functions are shifted by $\mathbf{Q}_c$, i.e., they contain an additional retardation. This is called intracollisional field effect. (ii) A further modification occurs in the exponential functions that essentially govern the energy balance. There appear additional terms leading to a field-dependent broadening determined by the function $\mathbf{R}_c$ that describes the field-induced displacement of a free particle c. (iii) The collision integral is dependent on the field in a nonlinear way. This will lead to typical nonlinear effects like the occurrence of higher harmonics and multiphoton absorption that will be, together with quantum effects, considered below in detail.

III. BALANCE EQUATIONS, COLLISIONAL ABSORPTION

As we are interested here in the collisional absorption by the dense plasma, it is obvious to start from the balance equation for the energy resulting from the second moment of the kinetic equation (2) with (10)

$$\frac{dW^{\mathrm{kin}}}{dt}-\mathbf{j}\cdot\mathbf{E}=\sum_{a,b}\int\frac{d^3k_a}{(2\pi\hbar)^3}\frac{k_a^2}{2m_a}I_{ab}(\mathbf{k}_a). \tag{17}$$

Here W^{kin} and $\mathbf{j}$ are the mean kinetic energy and the total electrical current density, respectively, according to

$$W^{\mathrm{kin}}(t)=\sum_a\int\frac{d^3k_a}{(2\pi\hbar)^3}\frac{k_a^2}{2m_a}f_a(k_a,t), \tag{18}$$

$$\mathbf{j}(t)=\sum_a e_a\int\frac{d^3k_a}{(2\pi\hbar)^3}\frac{\mathbf{k}_a}{2m_a}f_a(k_a,t). \tag{19}$$

In the Appendix, we show that the right hand side of Eq. (17) is just $-(d/dt)W^{\mathrm{pot}}$ with the mean potential energy density given by

$$W^{\mathrm{pot}}=\frac{1}{2}(i\hbar)\sum_{a,b}\int_{t_0}^{\infty}d\overline{t_1}\int_{t_0}^{\infty}d\overline{t_2}\int\frac{d^3q}{(2\pi\hbar)^3}V_{ab}(q)$$
$$\times[\Pi^R_{bb}(\mathbf{q};t,\overline{t_2})V^{s\,R}_{ba}(\mathbf{q};\overline{t_2},\overline{t_1})\Pi^<_{aa}(\mathbf{q};\overline{t_1},t)$$
$$+\Pi^R_{bb}(\mathbf{q};t,\overline{t_2})V^{s\,<}_{ba}(\mathbf{q};\overline{t_2},\overline{t_1})\Pi^A_{aa}(\mathbf{q};\overline{t_1},t)$$
$$+\Pi^<_{bb}(\mathbf{q};t,\overline{t_2})V^{s\,A}_{ba}(\mathbf{q};\overline{t_2},\overline{t_1})\Pi^A_{aa}(\mathbf{q};\overline{t_1},t)]. \tag{20}$$

Thus the energy balance (17) reads now

$$\frac{dW^{\mathrm{kin}}}{dt}+\frac{dW^{\mathrm{pot}}}{dt}=\mathbf{j}\cdot\mathbf{E}, \tag{21}$$

i.e., the change of the total energy of the system of particles is equal to $\mathbf{j}\cdot\mathbf{E}$ that is in turn the energy loss of the electromagnetic field due to Poynting's theorem. Both the mean kinetic energy and the potential energy are functionals of the actual distribution functions that follow from the kinetic equation. It is an important feature of Eq. (21) that there occurs the total energy on the left hand side. This means that a nonideal system is described by the underlying non-Markovian kinetic equation.

The balance equation for the electrical current reads

$$\frac{d\mathbf{j}}{dt}-\sum_a\frac{n_ae_a^2}{m_a}\mathbf{E}=\sum_{ab}\int\frac{d^3k_a}{(2\pi\hbar)^3}\frac{e_a\mathbf{k}_a}{m_a}I_{ab}(\mathbf{k}_a). \tag{22}$$

If collision could be neglected completely, we would find $\mathbf{j}^{(0)}=\Sigma_a(e_a^2/m_a)n_a\int_{t_0}^{t}dt'\,\mathbf{E}(t')$. Now, defining $\mathbf{j}\equiv\mathbf{j}^{(0)}+\mathbf{j}^{(1)}$, we have

$$\frac{d\mathbf{j}^{(1)}}{dt}=\sum_{ab}\int\frac{d^3k_a}{(2\pi\hbar)^3}\frac{e_a\mathbf{k}_a}{m_a}I_{ab}(\mathbf{k}_a). \tag{23}$$

With the collision integral (10), the equation for the current $\mathbf{j}^{(1)}$ is therefore given by

$$\frac{d\mathbf{j}^{(1)}}{dt}=\mathrm{Re}\int_{t_0}^{\infty}d\overline{t}_1\int_{t_0}^{\infty}d\overline{t}_2\int\frac{d^3q}{(2\pi\hbar)^3}\times\frac{1}{2}\sum_{ab}\left(\frac{e_a}{m_a}-\frac{e_b}{m_b}\right)\mathbf{q}\,V_{ab}(q)\times\{\Pi^R_{bb}(\mathbf{q};t,\overline{t}_2)V^{s\,R}_{ba}(\mathbf{q};\overline{t}_2,\overline{t}_1)\Pi^<_{aa}(\mathbf{q};\overline{t}_1,t)+\Pi^R_{bb}(\mathbf{q};t,\overline{t}_2)V^{s\,<}_{ba}(\mathbf{q};\overline{t}_2,\overline{t}_1)\Pi^A_{aa}(\mathbf{q};\overline{t}_1,t)+\Pi^<_{bb}(\mathbf{q};t,\overline{t}_2)V^{s\,A}_{ba}(\mathbf{q};\overline{t}_2,\overline{t}_1)\Pi^A_{aa}(\mathbf{q};\overline{t}_1,t)\},\quad(24)$$

where the upper limits of the time integration are determined by the heaviside functions contained in the advanced and retarded functions, respectively. It is easy to see that only collision terms I_{ab} with $a\neq b$ contribute to the current balance.

In the following, we will consider a two-component plasma consisting of electrons and ions ($m_i\gg m_e$). The ions will be treated in adiabatic approximation that leads to $\Pi^{R/A}_{ii}(\mathbf{q};t,t')\approx 0$ and $i\hbar\Pi^<_{ii}(\mathbf{q};t,t')\approx n_i$ with n_i being the ion density. Further, $V_{ab}(q)=e_ae_bV(q)$ is the Coulomb potential with $V(q)=1/(\varepsilon_0q^2/\hbar^2)$. With these assumptions, expression (24) is simplified considerably

$$\frac{d\mathbf{j}^{(1)}}{dt}=n_i\,\mathrm{Re}\int_{t_0}^{t}d\overline{t}_1\int_{\overline{t}_1}^{t}d\overline{t}_2\int\frac{d^3q}{(2\pi\hbar)^3}\frac{e_ee_i^2}{m_ei\hbar}\mathbf{q}\times V(q)e_e^2\Pi^R_{ee}(\mathbf{q};t,\overline{t}_2)V^{s\,R}(\mathbf{q};\overline{t}_2,\overline{t}_1),\quad(25)$$

where only the electrons contribute to the dynamically screened potential $V^{s\,R}=V+V\,e^2\Pi^R_{ee}\,V^{s\,R}$. If, finally, the following quantity is introduced

$$\varepsilon_R^{-1}(\mathbf{q};t,t')=\delta(t-t')+\int_{t'}^{t}d\overline{t}\,e_e^2\,\Pi^R_{ee}(\mathbf{q};t,\overline{t})V^{s\,R}(\mathbf{q};\overline{t},t'),\quad(26)$$

we have

$$\frac{d\mathbf{j}^{(1)}}{dt}=\frac{e_en_ie_i^2}{m_e\hbar}\mathrm{Re}\int_{t_0}^{t}d\overline{t}_1\int\frac{d^3q}{(2\pi\hbar)^3}\mathbf{q}V(q)\times\frac{1}{i}[\varepsilon_R^{-1}(\mathbf{q};t,\overline{t}_1)-\delta(t-\overline{t}_1)].\quad(27)$$

This expression for the current is a very important result. The quantity ε^{-1} represents a generalization of the dielectric function. It is a functional of the electron Green's functions, and it includes the full memory. The Wigner distribution functions can be introduced in an approximation by the GKBA in its gauge-invariant form, c.f. Sec. II.

If one is able to solve equation (27) for $\mathbf{j}^{(1)}$, it is simple to calculate $\mathbf{j}\cdot\mathbf{E}$ and the respective energy absorption rate. The derivation of analytical expressions will, of course, require further approximations that will be considered in the next section.

IV. HIGH-FREQUENCY FIELDS

So far the calculation of the inverse dielectric function includes all the memory. Now we consider the collision frequency as low compared to the oscillation frequency of the field, i.e., the collisions can be treated as a small perturbation. This enables us to use a perturbation ansatz for the electron distribution function introduced by Silin [4], $f_e=f_e^0+f_e^1$, in which the distribution function f_e^0 obeys the collisionless equation

$$\left\{\frac{\partial}{\partial t}-e_e\frac{\partial\mathbf{A}}{\partial t}\cdot\nabla_{k_e}\right\}f_e^0(\mathbf{k}_e,t)=0.\quad(28)$$

We have [23]

$$f_e^0(\mathbf{k},t)=f_{e\,0}[\mathbf{k}+e_e\mathbf{A}(t)],\quad(29)$$

with $f_{e\,0}$ being an arbitrary function depending on the initial conditions. In this section we will adopt equilibrium distributions.

For the dielectric function follows

$$\varepsilon_R^{-1}(\mathbf{q};t,t')=\exp\left[\frac{i}{\hbar}\frac{e_e\mathbf{q}}{m_e}\cdot\int_{t'}^{t}d\overline{t}\mathbf{A}(\overline{t})\right]\varepsilon^{-1}_{\mathrm{RPA}}\times(\mathbf{q};t-t'),\quad(30)$$

where $\varepsilon_{\mathrm{RPA}}$ is the RPA (Lindhard) dielectric function being a functional of $f_{e\,0}$.

This simplified expression for the dielectric function can be inserted in Eq. (27) for the current that reads now

$$\frac{d\mathbf{j}^{(1)}}{dt}=\frac{e_en_ie_i^2}{m_e\hbar}\mathrm{Re}\frac{1}{i}\int_{t_0}^{t}d\overline{t}_1\int\frac{d^3q}{(2\pi\hbar)^3}\mathbf{q}\,V(q)\times\exp\left[\frac{i}{\hbar}\frac{e_e\mathbf{q}}{m_e}\cdot\int_{\overline{t}_1}^{t}d\overline{t}\mathbf{A}(\overline{t})\right]\varepsilon^{-1}_{\mathrm{RPA}}(\mathbf{q};t-\overline{t}_1),\quad(31)$$

and which will serve as the basis for the investigation of higher harmonics in the current, and for the calculation of the inverse Bremsstrahlung absorption in a dense quantum plasma.

A. Higher harmonics in the electrical current

For a harmonic field, $\mathbf{E}=\mathbf{E}_0\cos\omega t$, one can expand Eq. (31) into a Fourier series using $e^{-iz\cos\omega t}=\sum_{l=-\infty}^{\infty}(-i)^lJ_l(z)e^{il\omega t}$, where J_l is the Bessel function of lth order. This leads to

$$\frac{d\mathbf{j}^{(1)}}{dt}=\frac{e_en_ie_i^2}{m_e\hbar}\mathrm{Re}\int\frac{d^3q}{(2\pi\hbar)^3}\mathbf{q}\,V(q)\times\sum_m\sum_n(-i)^{m+1}J_n\left(\frac{\mathbf{q}\cdot\mathbf{v}_e^0}{\hbar\omega}\right)J_{n-m}\left(\frac{\mathbf{q}\cdot\mathbf{v}_e^0}{\hbar\omega}\right)e^{im\omega t}\times\int_0^{t-t_0}d\tau\,e^{-(i/\hbar)n\omega\tau}[\varepsilon^{-1}_{\mathrm{RPA}}(\mathbf{q};\tau)-\delta(\tau)],\quad(32)$$

with $\mathbf{v}_e^0 = e_e \mathbf{E}_0 / m_e \omega$. For times $t \gg t_0$, which will be considered in the following, the expression in the second line of the above equation is just the Fourier transform of the inverse dielectric function.

After integration in time one has

$$\mathbf{j}^{(1)}(t) = \frac{e_e n_i e_i^2}{m_e \hbar} \mathrm{Im} \int \frac{d^3q}{(2\pi\hbar)^3} \mathbf{q} V(q) \times \sum_m \sum_n (-i)^{m+2} J_n\left(\frac{\mathbf{q}\cdot\mathbf{v}_e^0}{\hbar\omega}\right) J_{n-m}\left(\frac{\mathbf{q}\cdot\mathbf{v}_e^0}{\hbar\omega}\right) \times \frac{e^{im\omega t}}{m\omega}\left[\frac{1}{\varepsilon_{\mathrm{RPA}}(\mathbf{q};-n\omega)} - 1\right], \tag{33}$$

with the Lindhard dielectric function

$$\varepsilon_{\mathrm{RPA}}(\mathbf{q}\omega) = 1 + e^2 V(q) \int \frac{d^3p}{(2\pi\hbar)^3} \frac{f_{e0}(\mathbf{p}+\mathbf{q}) - f_{e0}(\mathbf{p})}{\hbar\omega - \epsilon_e(\mathbf{p}+\mathbf{q}) + \epsilon_e(\mathbf{p}) + i0}. \tag{34}$$

Eq. (33) is clearly a Fourier expansion of the current in terms of all harmonics. The Fourier coefficients of the current, $\mathbf{j}(t) = \Sigma_{m=-\infty}^{\infty} \mathbf{j}_m(\omega) e^{-im\omega t}$ with $j_m = j_{-m}^*$, can be identified easily from Eq. (33). One can show that only the odd harmonics are allowed due to the symmetry of the interaction that is characterized by $\mathrm{Re}\,\varepsilon_0^{-1}(\mathbf{q},\omega) = \mathrm{Re}\,\varepsilon_0^{-1}(-\mathbf{q},-\omega)$ and $\mathrm{Im}\,\varepsilon_0^{-1}(\mathbf{q},\omega) = -\mathrm{Im}\,\varepsilon_0^{-1}(-\mathbf{q},-\omega)$. In particular, we get for the real parts ($l = 0,1,2,\ldots$)

$$\mathrm{Re}\,\mathbf{j}_{2l+1}(\omega) = \frac{(-1)^l}{(2l+1)} n_i e_i^2 \int \frac{d^3q}{(2\pi\hbar)^3} \frac{e_e}{m_e \hbar\omega} \mathbf{q} \times V(q) \sum_{n=0}^{\infty} J_n\left(\frac{\mathbf{q}\cdot\mathbf{v}_e^0}{\hbar\omega}\right)\left[J_{n-(2l+1)}\left(\frac{\mathbf{q}\cdot\mathbf{v}_e^0}{\hbar\omega}\right) + J_{n+(2l+1)}\left(\frac{\mathbf{q}\cdot\mathbf{v}_e^0}{\hbar\omega}\right)\right] \mathrm{Im}\,\varepsilon_{\mathrm{RPA}}^{-1}(\mathbf{q};-n\omega), \tag{35}$$

whereas for the imaginary parts follows

$$\mathrm{Im}\,\mathbf{j}_{2l+1}(\omega) = \delta_{l,0} \frac{n_e e^2}{m_e} \frac{\mathbf{E}_0}{2} + \frac{(-1)^l}{(2l+1)} n_i e_i^2 \int \frac{d^3q}{(2\pi\hbar)^3} \frac{e_e}{m_e\hbar\omega} \mathbf{q} \times V(q) \sum_{n=0}^{\infty} J_n\left(\frac{\mathbf{q}\cdot\mathbf{v}_e^0}{\hbar\omega}\right)\left[J_{n-(2l+1)}\left(\frac{\mathbf{q}\cdot\mathbf{v}_e^0}{\hbar\omega}\right) - J_{n+(2l+1)}\left(\frac{\mathbf{q}\cdot\mathbf{v}_e^0}{\hbar\omega}\right)\right][\mathrm{Re}\,\varepsilon_{\mathrm{RPA}}^{-1}(\mathbf{q};-n\omega) - 1]. \tag{36}$$

Evaluation of these expressions allows to investigate the spectrum of the time-dependent electrical current as a function of the electrical field strength, of the frequency, and of the plasma temperature and density, respectively.

B. Collisional absorption rate

The energy dissipation in an electrical field $\mathbf{E} = \mathbf{E}_0 \cos \omega t$ is then given by

$$\mathbf{j}(t)\cdot\mathbf{E}(t) = \mathbf{E}_0 \cdot \Big\{ \mathrm{Re}\,\mathbf{j}_1(\omega) + \sum_{l=1}^{\infty} [\mathrm{Re}\{\mathbf{j}_{2l+1}(\omega) + \mathbf{j}_{2l-1}(\omega)\}\cos(2l\omega t) + \mathrm{Im}\{\mathbf{j}_{2l+1}(\omega) + \mathbf{j}_{2l-1}(\omega)\}\sin(2l\omega t)] \Big\}, \tag{37}$$

containing beside the constant term even harmonics only.

Now the dissipation of energy is calculated averaged over one oscillation cycle

$$\langle \mathbf{j}\cdot\mathbf{E}\rangle \equiv \frac{1}{T}\int_{t-T}^{t} dt'\, \mathbf{j}(t')\cdot\mathbf{E}(t') = \mathbf{E}_0 \cdot \mathrm{Re}\,\mathbf{j}_1(\omega). \tag{38}$$

The current $\mathbf{j}^{(0)}$ gives no contribution here. We want to mention that with the Silin ansatz (29), there holds $\langle \mathbf{j}\cdot\mathbf{E}\rangle = \langle dW^{\mathrm{kin}}/dt\rangle$. That means that the potential energy averaged over an oscillation cycle is constant in this approximation.

In the expression for $\mathrm{Re}\,\mathbf{j}_1$ one can use the recursion formula for the Bessel functions

$$J_{n-1}(z) + J_{n+1}(z) = \frac{2n}{z} J_n(z), \tag{39}$$

which leads to

$$\langle \mathbf{j}\cdot\mathbf{E}\rangle = n_i e_i^2 \int \frac{d^3q}{(2\pi\hbar)^3} V(q) \sum_{n=1}^{\infty} n\omega\, J_n^2\left(\frac{\mathbf{q}\cdot\mathbf{v}_e^0}{\hbar\omega}\right) \times \mathrm{Im}\frac{1}{\varepsilon_{\mathrm{RPA}}(\mathbf{q};-n\omega)}. \tag{40}$$

This result has a similar form as that of the nonlinear Dawson-Oberman model [3]. We want to stress, however, that in the present paper, the dielectric function is given by the quantum Lindhard form, whereas the dielectric theory of Decker *et al.* leads to the classical Vlasov dielectric function.

Finally, using $\mathrm{Im}\,\varepsilon^{-1} = -\mathrm{Im}\,\varepsilon/|\varepsilon|^2$, we get

$$\langle \mathbf{j}\cdot\mathbf{E}\rangle = n_i e_i^2 \int \frac{d^3q}{(2\pi\hbar)^3} \times V(q) \sum_{n=1}^{\infty} n\omega\, J_n^2\left(\frac{\mathbf{q}\cdot\mathbf{v}_e^0}{\hbar\omega}\right) \frac{\mathrm{Im}\,\varepsilon_{\mathrm{RPA}}(\mathbf{q};n\omega)}{|\varepsilon_{\mathrm{RPA}}(\mathbf{q};n\omega)|^2}. \tag{41}$$

The Lindhard dielectric function has to be calculated numerically, that can be done for arbitrary degeneracy. For a transparent discussion of the different quantum effects, however, it is advantageous to consider especially the nondegenerate case, in which some necessary integrations can be done analytically. For this case of a Maxwellian electron distribution function, we get [24]

$$\langle \mathbf{j}\cdot\mathbf{E}\rangle = \frac{8\sqrt{2\pi}Z^2e^4n_en_i\sqrt{m_e}}{(4\pi\varepsilon_0)^2(k_BT)^{3/2}}\,\omega^2\sum_{n=1}^{\infty} n^2\int_0^{\infty}\frac{dk}{k^3}\,\frac{1}{|\varepsilon_{\mathrm{RPA}}(k,n\omega)|^2}\times e^{-(n^2m_e\omega^2/2k_BT\,k^2)}e^{-(\hbar^2k^2/8m_ek_BT)}\times\frac{\sinh(n\hbar\omega/2k_BT)}{(n\hbar\omega/2k_BT)}\int_0^1 dz\,J_n^2\left(\frac{eE_0k}{m_e\omega^2}z\right). \tag{42}$$

In the classical limit, $\hbar\to 0$, this expression is well known. Within the classical kinetic theory it was derived first by Klimontovich [5]. Later Decker *et al.* [3] got such an expression in the framework of the nonlinear Dawson-Oberman model. The classical formulas have the well-known problem of a divergency at large k that is solved by some cutoff procedures. In contrast, in our quantum approach no divergencies exist.

Quantum effects, indicated by $\hbar$, occur here at different places. The first place is one of the exponential functions in Eq. (42) describing the quantum diffraction effect at large momenta k. This exponential function ensures the convergence of the integral. The second place is the term with the sinh function that is connected with the Bose statistics of multiple photon emission and absorption. Finally, quantum effects enter also the calculation of $|\varepsilon(q,n\omega)|^2$ itself. These effects will be discussed in detail in the next section.

Often the energy absorption rate ν_E is discussed that is defined by $\nu_{\mathrm{E}}=\langle \mathbf{j}\cdot\mathbf{E}\rangle/\langle \epsilon_0\mathbf{E}^2\rangle$, or the electron-ion collision frequency ν_{ei} that is introduced according to a high-frequency Drude model with

$$\nu_{\mathrm{ei}}=\frac{\omega^2}{\omega_{\mathrm{pl}}^2}\,\frac{\langle \mathbf{j}\cdot\mathbf{E}\rangle}{\langle \varepsilon_0\mathbf{E}^2\rangle}. \tag{43}$$

V. NUMERICAL RESULTS

In this section we will present numerical results for the collision frequency and the higher harmonics in the current density based on the formulas of the foregoing chapter. Our emphasis will be to show the importance of a quantum approach. In the following a hydrogen plasma is considered that is assumed to be fully ionized.

In Fig. 1 the collision frequency in such a plasma is shown as a function of the quiver velocity. For comparison there are given curves (dashed line) following from the asymptotic formulas of Silin for the cases of small and of big quiver velocities, respectively. These formulas read [4]

$$\nu_{ei}=\frac{4}{3}\,\frac{\sqrt{2\pi}e^4Z^2n_i}{(4\pi\varepsilon_0)^2m_e^2v_{\mathrm{th}}^3}\ln\frac{k_{\max}}{k_{\min}},\quad v_0\ll v_{\mathrm{th}}, \tag{44}$$

$$\frac{\nu_{ei}}{\omega_p}=\frac{Z}{\pi^2}\,\frac{1}{n\lambda_D^3}\left(\frac{v_{\mathrm{th}}}{v_0}\right)^3\left[\ln\left(\frac{v_0}{2v_{\mathrm{th}}}\right)+1\right]\ln\frac{k_{\max}}{k_{\min}},\quad v_0\gg v_{\mathrm{th}}, \tag{45}$$

with λ_D being the Debye length. In the high-frequency field under consideration, $k_{\min}$ is given as usual by $k_{\min}=\omega_0/v_{\mathrm{th}}$.

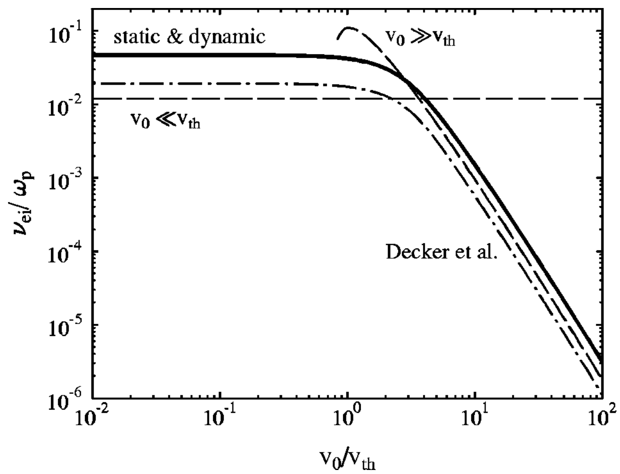

FIG. 1. Electron-ion collision frequency as a function of the quiver velocity $v_0=eE/\omega m_e$ for a hydrogen plasma in a laser field ($Z=1; n_e=10^{22}\ cm^{-3}; T=3\cdot 10^5$ K; $\omega/\omega_p=5$). For comparison, results of Decker *et al.* (dash-dotted line) and of the asymptotic formulas (44) and (45) of Silin (dashed line) are given.

For the upper cutoff we take here for the comparison of the classical one, $k_{\max}=4\pi\varepsilon_0k_BT/(Ze^2)$.

Further, the classical expression of Decker *et al.* [unfortunately, in their Eq. (22) in [3], a factor $2/(n_e\lambda_D^3)$ is missing] is evaluated (dash-dotted line). Our quantum expression, Eq. (43) with (41), was evaluated with the quantum Lindhard dielectric function fully dynamical and, for comparison, also with static screening in the denominator in Eq. (41). These results are given in Fig. 1 with solid lines. In the given logarithmic scale, there is almost no difference between these two cases to be observed. The qualitative behavior of the results from the classical dielectric theory of Decker *et al.* and from our quantum approach is very similar. The collision frequency is nearly constant for small field strengths up to $v_0/v_{\mathrm{th}}=1$ and then decreases rapidly for higher fields. This is in agreement with the asymptotic formulas of Silin, too. There are, however, quantitative differences. These can be attributed to the use of Coulomb logarithms in the classical approaches that correspond to cutting procedures in the integral over momentum.

In order to make the differences between the different approximations in the range of weak and moderate values of v_0 more prominent, in Fig. 2 the collision frequency is given in a linear scale for the same parameters as in the foregoing figure, and in Fig. 3 for a higher temperature.

In this linear scale, the small differences between the cases of dynamical screening and statical screening are to be seen. The static approximation for $1/|\varepsilon|^2$ slightly overestimates the effect of screening. The difference to the classical results of Decker *et al.* is much bigger in Fig. 2 whereas for the case of a higher temperature [lower coupling parameter $\Gamma=(e^2/4\pi\varepsilon_0)/dk_BT$ with $d=(4\pi n_i/3)^{-1/3}$] the agreement is much better, cf. Fig. 3.

The dependence of the collision frequency on the coupling parameter will be considered now more in detail. First, in Fig. 4, the collision frequency is shown as a function of coupling parameter Γ for a small quiver velocity, i.e., a small

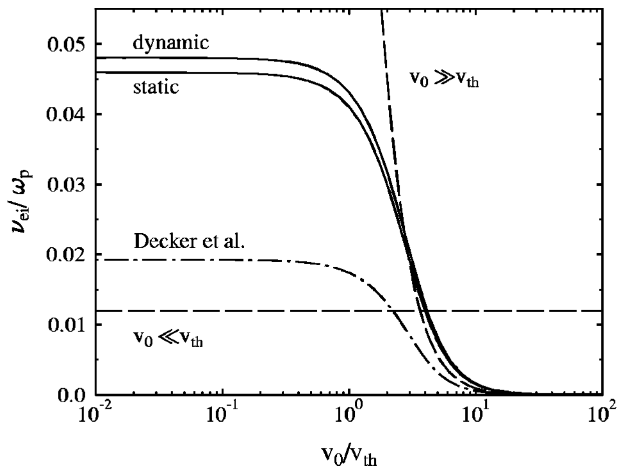

FIG. 2. Electron-ion collision frequency vs quiver velocity for the same parameters as in Fig. 1.

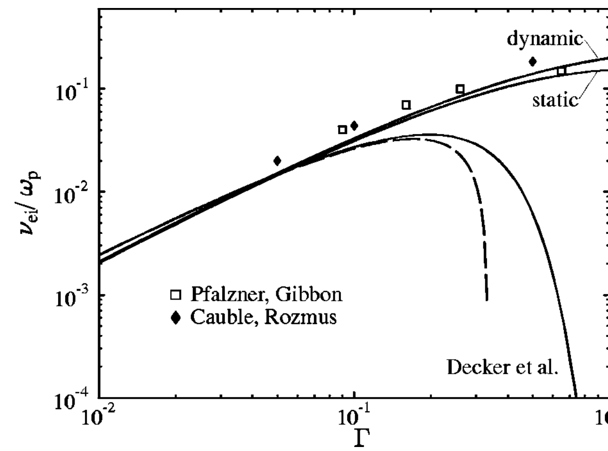

FIG. 4. Electron-ion collision frequency as a function of the coupling parameter Γ for a hydrogen plasma in a laser field ($Z=1;v_0/v_{th}=0.2;n_e=10^{22}$ cm^{-3};$\omega/\omega_p=3$). Comparison is given with the theory of Decker *et al.* and with the asymptotic formula (44) of Silin (dashed line.)

field strength. Results of the evaluation of Eqs. (43) and (40) are given by the upper solid line. The static screening results are the lower solid curve. Again the asymptotic formula of Silin for small quiver velocities is given, and the classical expression of Decker *et al.* was evaluated (Fig. 5). Furthermore, numerical results of Cauble and Rozmus [15] are plotted. They considered small field strengths and used a memory function kinetic approach that allows to consider plasmas up to strong coupling. The points in Fig. 4 correspond to their so-called Debye-Hückel mean field approximation, cf. Ref. [15]. Finally, we compare in Fig. 4 also with numerical simulation results of Pfalzner and Gibbon [11]. They applied a tree code method to classical molecular dynamics simulations using a soft Coulomb potential.

According to Fig. 4 the collision frequency increases with increasing coupling Γ. For small Γ, the dielectric theory and our theory give almost the same results. The values of the asymptotic formula of Silin are slightly bigger. With increasing Γ this asymptotic formula as well as the dielectric approach reach a maximum around $\Gamma \sim 0.2$ and sharply drop down afterwards. This behavior is governed by the Coulomb logarithm used in these approaches. It results from a cutoff procedure at large momenta *k*. Such a cutoff, inherent in many classical approaches, is avoided in our approach because the *k* integration is automatically convergent, cf. the second exponential function in Eq. (42). Therefore the range of applicability of our approach is extended to higher values of Γ.

The agreement with the results of Cauble *et al.* and of Pfalzner *et al.* is rather good with the values of the present theory being slightly smaller. One has to take into account, however, that our approximation is a weak coupling theory whereas the approaches we compare with include the correlations in higher approximations. We want to mention that

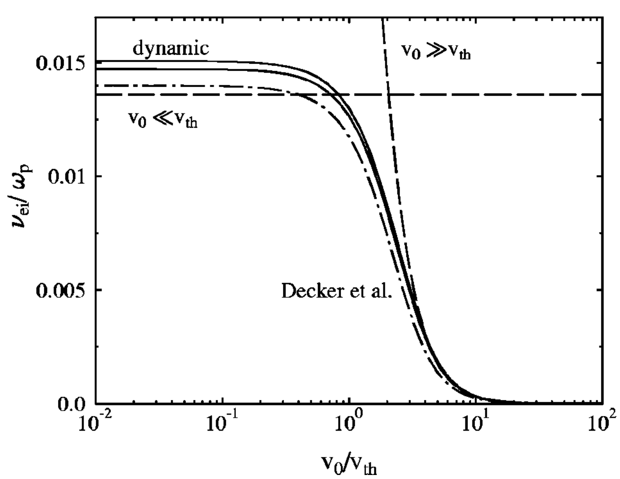

FIG. 3. Electron-ion collision frequency as a function of the quiver velocity $v_0=eE/\omega m_e$ for a hydrogen plasma in a laser field ($Z=1;n_e=10^{22}$ cm^{-3};$T=10^6$ K;$\omega/\omega_p=5$). For comparison, results of Decker *et al.* (dash-dotted line) and of the asymptotic formulas (44) and (45) of Silin (dashed line) are given.

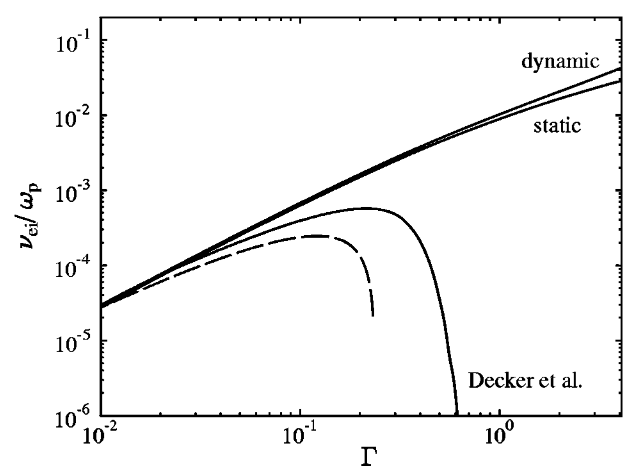

FIG. 5. Electron-ion collision frequency as a function of the coupling parameter Γ for a hydrogen plasma in a laser field ($Z=1;v_0/v_{th}=10;n_e=10^{22}$ cm^{-3};$\omega/\omega_p=5$). Comparison is given with the theory of Decker *et al.* and with the asymptotic formula (45) of Silin (dashed line.)

TABLE I. Electron-ion collision frequency, $\nu_{ei}/\omega_{\mathrm{p}}$, calculated from Eq. (41) for different values of Γ and v_0/v_{th} with $n_e = 10^{22}\ \mathrm{cm}^{-3}$ and $\omega/\omega_{\mathrm{p}}=5$.

	v_0/v_{th}			
Γ	0.1	1.0	5.0	10
0.01	1.8767×10^{-3}	1.5766×10^{-3}	1.9410×10^{-4}	3.5183×10^{-5}
0.10	2.6476×10^{-2}	2.3191×10^{-2}	3.6981×10^{-3}	7.2497×10^{-4}
0.50	8.7018×10^{-2}	8.1765×10^{-3}	2.2703×10^{-2}	4.8870×10^{-3}
1.00	1.0550×10^{-1}	1.0221×10^{-1}	4.3577×10^{-2}	1.0254×10^{-2}

our results do not depend solely on Γ, but depend on temperature and on density (as well as the results of Cauble *et al.* do because of the usage of a modified potential taking into account short-range quantum effects).

Before we are going to elucidate the quantum effects, we want to present in Fig. 5 the behavior of the collision frequency vs Γ for a higher value of the electrical field strength ($v_0/v_{\mathrm{th}}=10$). As the theory of Cauble and Rozmus is limited to small field strengths, and because there are no results for this high fields from molecular dynamics calculations available up to now, we can compare our results only with those of the classical dielectric theory and with the asymptotic formula of Silin for big quiver velocities. The qualitative behavior is similar to that for $v_0/v_{\mathrm{th}}=0.2$. Numerical values of the collision frequency for different Γ and various ratios v_0/v_{th} are given in Table I.

Now the consequences of the quantum approach in contrast to the classical dielectric theory will be investigated. We consider such parameters that the plasma can be assumed to be nondegenerate, i.e., Eq. (42) can be used. In this case a direct comparison with the classical dielectric theory of Decker *et al.* is possible. Quantum effects indicated by $\hbar$ occur in Eq. (42) at two places. One is the quantum diffraction effect ensuring the convergence of the integral at big k (cf. the second exponential function). The other is the factor $\sinh x/x$ with $x=n\hbar\omega/(2k_BT)$. The classical theory uses, instead, a cutoff $k_{\mathrm{max}}=mv_{\mathrm{th}}^2 4\pi\varepsilon_0/Ze^2$, and the sinh factor is missing.

An important feature of the expressions (40)–(42) is the sum over n that can be interpreted as a sum over the different multiphoton processes, i.e., the emission and absorption of energies $n\hbar\omega$. The different contributions ν_n in the sum $\nu_{ei}=\Sigma_n\nu_n$ are dependent on the field strength. It is obvious that with increasing field the number of terms contributing essentially to the sum is also increasing. The following two figures, Fig. 6 and Fig. 7, showing ν_n vs n (full solution of Eq. (42)—solid line) for two different field strengths illustrate this issue. Moreover, we compare with the classical dielectric theory (dotted line) and the case in which the factor $\sinh x/x$ in Eq. (42) is set to unity (dashed line). One observes that the differences between these three cases grow with increasing photon number n. The reason for the faster decreasing contributions in the classical approach is the hard cutoff in the k integration while the maximum of the integrand is shifted to higher k due to the exponential factor $\exp[-(n^2\omega^2m_e)/(2k_BTk^2)]$. Thus the relative error of the ν_n increases with n.

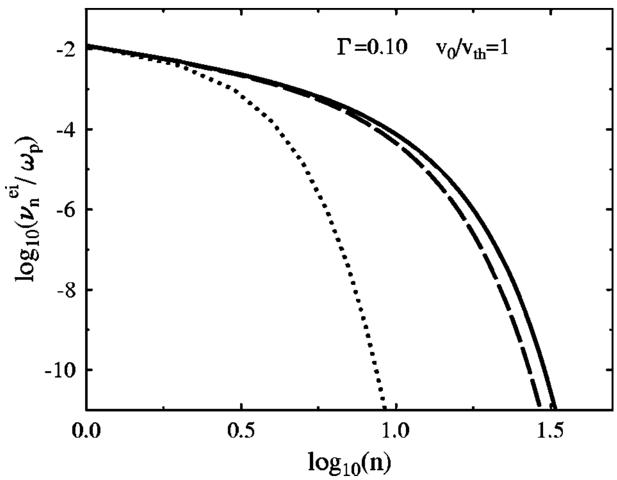

FIG. 6. Contributions ν_n vs photon number n in a hydrogen plasma ($n_e=10^{22}\ \mathrm{cm}^{-3}$; $\omega/\omega_p=5$; $\Gamma=0.1$) for $v_0/v_{th}=1.0$. Present approach (solid line), sinh term neglected (dashed line), classical dielectric theory (dotted line).

The other quantum effect is connected with the $\sinh x$ factor that behaves for large $x(n)$ as $e^x/2x$. Therefore this factor becomes more important for large n, the processes involving large numbers of photons are enhanced. This can be seen in Fig. 7 that considers the case $v_0/v_{\mathrm{th}}=10$. The solid curve corresponding to the full solution extends to much higher n values than that curve that results from a neglection of the sinh term. An interesting feature is the plateau like behavior up to $n\sim350$ with the subsequent sharp drop down. We can conclude at this point, that especially in the strong field case where multiphoton processes play an increasing role, it is important to treat the problem on the basis of quantum mechanics. This problem was also discussed in [18] based on an asymptotic solution of expression (42) for strong fields.

In order to complete the discussion of the collision frequency, the dependence on the laser frequency is considered. This is shown in Fig. 8 for two different field strengths. The

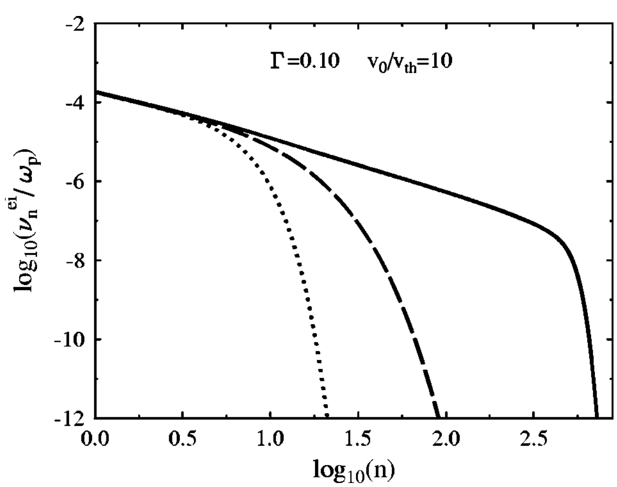

FIG. 7. Contributions ν_n vs photon number n in a hydrogen plasma ($n_e=10^{22}\ \mathrm{cm}^{-3}$; $\omega/\omega_p=5$; $\Gamma=0.1$) for $v_0/v_{th}=10$. Present approach (solid line), sinh term neglected (dashed line); classical dielectric theory (dotted line).

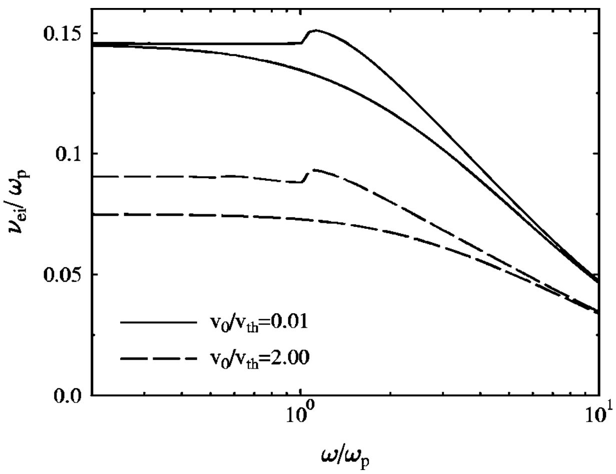

FIG. 8. Electron-ion collision frequency as a function of the laser frequency for a hydrogen plasma ($n_e=10^{21}\ \mathrm{cm}^{-3}$; $T=10^5$ K) for two different field strengths. The upper curve of each pair corresponds to the full dynamical screening, the lower one to the static screening approximation.

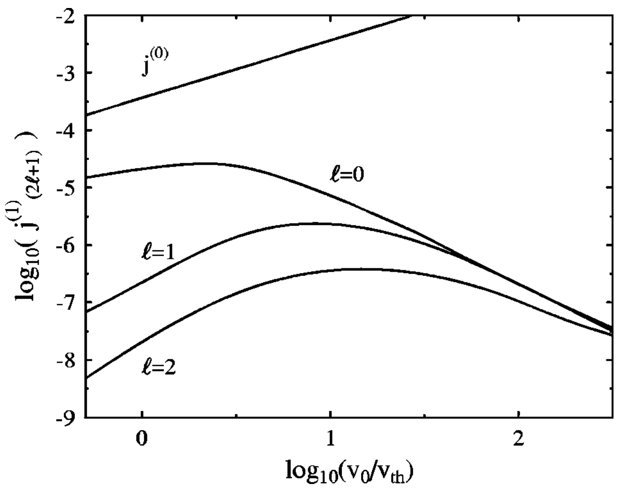

FIG. 9. Amplitudes $2\sqrt{(\mathrm{Re}j_m)^2+(\mathrm{Im}j_m)^2}$ of the $m=(2\ell+1)$th harmonics of the current vs field strength denoted by v_0/v_{th}. The parametes are $n_e=4\times10^{21}\ \mathrm{cm}^{-3}$ and $T=1.2\times10^5$ K.

full dynamic solution is compared with the static screening approximation for $1/|\varepsilon|^2$. For large frequencies the differences between the two approximation decrease. Collective effects in the dielectric function play a role only in the vicinity of the plasma frequency. This behavior is to be seen also for the higher field strength. In the high-field case, the static screening approximation deviates from the dynamical one also for the lower frequencies. This is caused by collective effects in the terms with higher n in the respective sum in Eq. (42) for arguments of the dielectric function $n\omega\approx\omega_{\mathrm{p}}$.

The discussion of the behavior of $\nu_{ei}(\omega)$ for laser frequencies around and below the plasma frequency has to be treated, of course, with some care because the underlying theory is a high-frequency approximation.

A further issue that is very interesting is the possibility of higher harmonics [29] in a strong laser field. The higher harmonics in the current density can be calculated according to Eqs. (35) and (36). In Fig. 9, the amplitudes of the different harmonics, $2\sqrt{(\mathrm{Re}j_m)^2+(\mathrm{Im}j_m)^2}$, are given as a function of the field strength. The harmonics have amplitudes increasing with the field strength up to maxima at certain values and decreasing afterwards. For high fields the differences between the higher harmonics decrease. The ratio to $j^{(0)}$, however, becomes very small.

VI. CONCLUSION

Starting from a generalized quantum kinetic equation for dense, laser plasmas in the framework of gauge-invariant Green's functions, we derived balance equations for the energy and the electrical current of a plasma in strong laser fields. The non-Markovian collision integral was considered in dynamically screened Born approximation (RPA). In the case of high frequencies the so-called Silin ansatz could be used leading to explicit expressions for the current and the electron-ion collision frequency in terms of the quantum Lindhard dielectric function. This generalizes results of earlier papers using classical kinetic theory and the classical dielectric theory, respectively. An important feature of the quantum approaches is that no cutting procedures for large momenta have to be introduced in the respective integrals. Moreover, it turned out that quantum effects become especially important for strong fields when the energy absorbed in the collisions is much greater than k_BT and multiphoton processes play an essential role. This was found also recently in the framework of the quantum Vlasov theory of Kull and Plagne [18] who could reproduce our equation (42). It should be mentioned that the Green's functions method allows to generalize the approach rather straightforwardly, e.g., to include strong coupling effects as ion correlations and bound states.

ACKNOWLEDGMENTS

Valuable discussions with V.P. Silin, P. Mulser, M. Bonitz, and H.-J. Kull are gratefully acknowledged. Further we want to thank S. Pfalzner and P. Gibbon for providing us with their numerical data. This work was supported by the Deutsche Forschungsgemeinschaft (Schwerpunkt "Wechselwirkung intensiver Laserfelder mit Materie").

APPENDIX: MEAN POTENTIAL ENERGY

In general the mean potential energy is given by

$$\langle V\rangle=\frac{1}{2}\sum_{a,b}\int d^3r_1d^3r_2V_{ab}(r_1-r_2)\,(i\hbar)^2g_{ab}^{<}(\mathbf{r}_1\mathbf{r}_2t,\mathbf{r}_1\mathbf{r}_2t), \tag{A1}$$

with $g_{ab}^{<}(t,t')$ being the two-time two-particle correlation function

$$g_{ab}^{<}(r_1r_2t,r_1'r_2't')=\frac{1}{(i\hbar)^2}\langle\Psi_a^\dagger(r_1't')\Psi_b^\dagger(r_2't')\times\Psi_b(r_2t)\Psi_a(r_1t)\rangle. \tag{A2}$$

We will consider here the dynamically screened ladder approximation [30] up to first order

$$g_{ab}=g_a g_b+i\hbar\, g_a g_b V^s_{ab} g_a g_b+\cdots, \tag{A3}$$

which has to be evaluated on the Keldysh time contour. When this expression is inserted into Eq. (A1) the zeroth-order term (Hartree term) gives no contribution for a neutral plasma, and therefore we get (neglecting exchange terms and initial correlation terms)

$$\begin{aligned}\langle V\rangle(t)=&\frac{1}{2}(i\hbar)\sum_{a,b}\int_{t_0}^{\infty}d\bar{t}_1\int_{t_0}^{\infty}d\bar{t}_2\int d^3r_1 d^3r_2\int d^2\bar{r}_1 d^3\bar{r}_2\\&\times V_{ab}(|\mathbf{r}_1-\mathbf{r}_2|)[\Pi^R_{bb}(\mathbf{r}_2 t,\bar{\mathbf{r}}_2\bar{t}_2)V^{s\,R}_{ba}(\bar{\mathbf{r}}_2\bar{t}_2,\bar{\mathbf{r}}_1\bar{t}_1)\\&\times\Pi^<_{aa}(\bar{\mathbf{r}}_1\bar{t}_1,\mathbf{r}_1 t)+\Pi^R_{bb}(\mathbf{r}_2 t,\bar{\mathbf{r}}_2\bar{t}_2)V^{s\,<}_{ba}(\bar{\mathbf{r}}_2\bar{t}_2,\bar{\mathbf{r}}_1\bar{t}_1)\\&\times\Pi^A_{aa}(\bar{\mathbf{r}}_1\bar{t}_1,\mathbf{r}_1 t)+\Pi^<_{bb}(\mathbf{r}_2 t,\bar{\mathbf{r}}_2\bar{t}_2)V^{s\,A}_{ba}(\bar{\mathbf{r}}_2\bar{t}_2,\bar{\mathbf{r}}_1\bar{t}_1)\\&\times\Pi^A_{aa}(\bar{\mathbf{r}}_1\bar{t}_1,\mathbf{r}_1 t)].\end{aligned} \tag{A4}$$

Now we use the gauge-invariant Fourier (back) transformation (in the following a homogeneous system is assumed)

$$g_a(\mathbf{r},tt')=\int\frac{d^3k}{(2\pi\hbar)^3}\times\exp\left\{\frac{i}{\hbar}\mathbf{r}\cdot\left[\mathbf{k}+e_a\int_{t'}^{t}d\bar{t}\frac{\mathbf{A}(\bar{t})}{(t-t')}\right]\right\}g_a(\mathbf{k},tt'), \tag{A5}$$

which leads to

$$\begin{aligned}\langle V\rangle(t)=&\frac{1}{2}(i\hbar)\sum_{a,b}\Omega\int_{t_0}^{\infty}d\bar{t}_1\int_{t_0}^{\infty}d\bar{t}_2\int\frac{d^3q}{(2\pi\hbar)^3}V_{ab}(q)\\&\times[\Pi^R_{bb}(\mathbf{q};t,\bar{t}_2)V^{s\,R}_{ba}(\mathbf{q};\bar{t}_2,\bar{t}_1)\Pi^<_{aa}(\mathbf{q};\bar{t}_1,t)\\&+\Pi^R_{bb}(\mathbf{q};t,\bar{t}_2)V^{s\,<}_{ba}(\mathbf{q};\bar{t}_2,\bar{t}_1)\Pi^A_{aa}(\mathbf{q};\bar{t}_1,t)\\&+\Pi^<_{bb}(\mathbf{q};t,\bar{t}_2)V^{s\,A}_{ba}(\mathbf{q};\bar{t}_2,\bar{t}_1)\Pi^A_{aa}(\mathbf{q};\bar{t}_1,t)]\end{aligned} \tag{A6}$$

with Ω being the volume. The potential energy density is therefore $W^{\rm pot}(t)=\langle V\rangle/\Omega$.

The dependence on the time t can be made explicit using the GKBA. Using expression (13) for Π^R and similar ones for $\Pi^<$ and Π^A, we get the following expression

$$\begin{aligned}W^{\rm pot}(t)=&\frac{1}{2i\hbar}\sum_{a,b}\int_{t_0}^{t}d\bar{t}_1\int_{t_0}^{t}d\bar{t}_2\int\frac{d^3k_a}{(2\pi\hbar)^3}\frac{d^3k_b}{(2\pi\hbar)^3}\frac{d^3q}{(2\pi\hbar)^3}V_{ab}(q)\exp\left(-\frac{i}{\hbar}\left[(\epsilon^b_{\mathbf{k}_b+\mathbf{q}}-\epsilon^b_{\mathbf{k}_b})(t-\bar{t}_2)-\frac{e_b}{m_b}\mathbf{q}\cdot\int_{\bar{t}_2}^{t}dt'\,\mathbf{A}(t')\right]\right)\\&\times\exp\left(-\frac{i}{\hbar}\left[(\epsilon^a_{\mathbf{k}_a-\mathbf{q}}-\epsilon^a_{\mathbf{k}_a})(t-\bar{t}_1)+\frac{e_a}{m_a}\mathbf{q}\cdot\int_{\bar{t}_1}^{t}dt'\,\mathbf{A}(t')\right]\right)\{[F_b(\mathbf{k}_b,\bar{t}_2)-F_b(\mathbf{k}_b+\mathbf{q},\bar{t}_2)]V^{s\,R}_{ba}(\mathbf{q};\bar{t}_2,\bar{t}_1)\\&\times F_a(\mathbf{k}_a,\bar{t}_1)[1-F_a(\mathbf{k}_a-\mathbf{q},\bar{t}_1)]+[F_b(\mathbf{k}_b,\bar{t}_2)-F_b(\mathbf{k}_b+\mathbf{q},\bar{t}_2)]V^{s\,<}_{ba}(\mathbf{q};\bar{t}_2,\bar{t}_1)[F_a(\mathbf{k}_a,\bar{t}_1)-F_a(\mathbf{k}_a-\mathbf{q},\bar{t}_1)]\\&+F_b(\mathbf{k}_b+\mathbf{q},\bar{t}_2)[1-F_b(\mathbf{k}_b,\bar{t}_2)]V^{s\,A}_{ba}(\mathbf{q};\bar{t}_2,\bar{t}_1)[F_a(\mathbf{k}_a,\bar{t}_1)-F_a(\mathbf{k}_a-\mathbf{q},\bar{t}_1)]\},\end{aligned} \tag{A7}$$

where we have defined $F_a(\mathbf{k},\bar{t})\equiv f_a(\mathbf{k}-(e_a/c)\mathbf{A}(\bar{t}),\bar{t})$. Now it is easy to calculate also the temporal derivation of this expression

$$\begin{aligned}\frac{d}{dt}W^{\rm pot}(t)=&-\frac{1}{\hbar^2}\sum_{a,b}\int_{t_0}^{t}d\bar{t}_1\int_{t_0}^{t}d\bar{t}_2\int\frac{d^3k_a}{(2\pi\hbar)^3}\frac{d^3k_b}{(2\pi\hbar)^3}\frac{d^3q}{(2\pi\hbar)^3}V_{ab}(q)\left[\epsilon^a_{\mathbf{k}_a-\mathbf{q}}+\epsilon^b_{\mathbf{k}_b+\mathbf{q}}-\epsilon^a_{\mathbf{k}_a}-\epsilon^b_{\mathbf{k}_b}+\left(\frac{e_a}{m_a}-\frac{e_b}{m_b}\right)\mathbf{q}\cdot\mathbf{A}(t)\right]\\&\times\exp\left(-\frac{i}{\hbar}\left[(\epsilon^b_{\mathbf{k}_b+\mathbf{q}}-\epsilon^b_{\mathbf{k}_b})(t-\bar{t}_2)-\frac{e_b}{m_b}\mathbf{q}\cdot\int_{\bar{t}_2}^{t}dt'\,\mathbf{A}(t')\right]\right)\exp\left(-\frac{i}{\hbar}\left[(\epsilon^a_{\mathbf{k}_a-\mathbf{q}}-\epsilon^a_{\mathbf{k}_a})(t-\bar{t}_1)\right.\right.\\&\left.\left.+\frac{e_a}{m_a}\mathbf{q}\cdot\int_{\bar{t}_1}^{t}dt'\,\mathbf{A}(t')\right]\right)\{[F_b(\mathbf{k}_b,\bar{t}_2)-F_b(\mathbf{k}_b+\mathbf{q},\bar{t}_2)]V^{s\,R}_{ba}(\mathbf{q};\bar{t}_2,\bar{t}_1)F_a(\mathbf{k}_a,\bar{t}_1)[1-F_a(\mathbf{k}_a-\mathbf{q},\bar{t}_1)]\\&+[F_b(\mathbf{k}_b,\bar{t}_2)-F_b(\mathbf{k}_b+\mathbf{q},\bar{t}_2)]V^{s\,<}_{ba}(\mathbf{q};\bar{t}_2,\bar{t}_1)[F_a(\mathbf{k}_a,\bar{t}_1)-F_a(\mathbf{k}_a-\mathbf{q},\bar{t}_1)]\\&+F_b(\mathbf{k}_b+\mathbf{q},\bar{t}_2)[1-F_b(\mathbf{k}_b,\bar{t}_2)]V^{s\,A}_{ba}(\mathbf{q};\bar{t}_2,\bar{t}_1)[F_a(\mathbf{k}_a,\bar{t}_1)-F_a(\mathbf{k}_a-\mathbf{q},\bar{t}_1)]\}.\end{aligned} \tag{A8}$$

This expression can be compared with that resulting from the right hand side of the energy balance equation (17). First, we summarize important symmetry relations of the non-Markovian collision integral

$$I_{ab}(\mathbf{k}_a,t)=\int \frac{d^3\overline{k}_a d^3k_b d^3\overline{k}_b}{(2\pi\hbar)^6}\tilde{I}_{ab}(\mathbf{k}_a,\mathbf{k}_b,\overline{\mathbf{k}}_a,\overline{\mathbf{k}}_b)\times\delta(\mathbf{k}_a+\mathbf{k}_b-\overline{\mathbf{k}}_a-\overline{\mathbf{k}}_b). \tag{A9}$$

Using that $V^{s\gtrless}_{ab}(\mathbf{q};t_1,t_2)=V^{s\lessgtr}_{ba}(-\mathbf{q};t_2,t_1)$ and $V^{s\,R}_{ab}(\mathbf{q};t_1,t_2)=V^{s\,A}_{ba}(-\mathbf{q};t_2,t_1)$, it is easy to show that the quantity $\tilde{I}_{ab}$ has the following properties

$$\tilde{I}_{ab}(\mathbf{k}_a,\mathbf{k}_b;\overline{\mathbf{k}}_a,\overline{\mathbf{k}}_b)=-\tilde{I}_{ab}(\overline{\mathbf{k}}_a,\overline{\mathbf{k}}_b;\mathbf{k}_a,\mathbf{k}_b), \tag{A10}$$

$$\tilde{I}_{ab}(\mathbf{k}_a,\mathbf{k}_b;\overline{\mathbf{k}}_a,\overline{\mathbf{k}}_b)=\tilde{I}_{ba}(\mathbf{k}_b,\mathbf{k}_a;\overline{\mathbf{k}}_b,\overline{\mathbf{k}}_a).$$

Therefore, for any function $\mathbf{h}_a(\mathbf{k}_a)$, we have

$$\sum_{a,b}\int\frac{d^3k_a}{(2\pi\hbar)^3}\mathbf{h}_a(\mathbf{k}_a)I_{ab}(\mathbf{k}_a) =\sum_{a,b}\int\frac{d^3k_a d^3\overline{k}_a d^3k_b d^3\overline{k}_b}{(2\pi\hbar)^9}\delta(\mathbf{k}_a+\mathbf{k}_b-\overline{\mathbf{k}}_a-\overline{\mathbf{k}}_b)\times\frac{1}{4}[\mathbf{h}_a(\mathbf{k}_a)+\mathbf{h}_b(\mathbf{k}_b)-\mathbf{h}_a(\overline{\mathbf{k}}_a)-\mathbf{h}_b(\overline{\mathbf{k}}_b)]\times\tilde{I}_{ab}(\mathbf{k}_a,\mathbf{k}_b,\overline{\mathbf{k}}_a,\overline{\mathbf{k}}_b). \tag{A11}$$

Inserting the explicit form of the collision integral $\tilde{I}_{ab}$ that follows from a comparison with Eq. (16) and performing the necessary substitution of integration variables, there follows

$$\sum_{a,b}\int\frac{d^3k_1}{(2\pi\hbar)^3}\frac{k_1^2}{2m_a}I_{ab}(\mathbf{k}_1)=-\frac{d}{dt}W^{\mathrm{pot}}. \tag{A12}$$

Thus the energy balance (17) reads now

$$\frac{dW^{\mathrm{kin}}}{dt}+\frac{dW^{\mathrm{pot}}}{dt}=\mathbf{j}\cdot\mathbf{E}, \tag{A13}$$

i.e., the change of the total energy of the system of particles is equal to $\mathbf{j}\cdot\mathbf{E}$. In the case of a vanishing external field, the total energy is conserved.

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