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Image charge effects on electron capture by dust grains in dusty plasmas

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Electron-capture processes by negatively charged dust grains from hydrogenic ions in dusty plasmas are investigated in accordance with the classical Bohr-Lindhard model. The attractive interaction between the electron in a hydrogenic ion and its own image charge inside the dust grain is included to obtain the total interaction energy between the electron and the dust grain. The electron-capture radius is determined by the total interaction energy and the kinetic energy of the released electron in the frame of the projectile dust grain. The classical straight-line trajectory approximation is applied to the motion of the ion in order to visualize the electron-capture cross section as a function of the impact parameter, kinetic energy of the projectile ion, and dust charge. It is found that the image charge inside the dust grain plays a significant role in the electron-capture process near the surface of the dust grain. The electron-capture cross section is found to be quite sensitive to the collision energy and dust charge.

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I. INTRODUCTION

The electron-capture process has been of great interest since it is one of the basic processes in atomic collision physics [1]. The capture processes have been investigated widely using various methods [2–10], including classical and quantum methods, depending on the projectile and target states. For intermediate- and high-energy projectiles, a pure classical approach, the so-called Bohr-Lindhard method [2], has been known to be quite reliable [4,7], since the de Broglie wavelength of the high-energy projectile is smaller than the collision range for the electron-capture interaction. Recently, the electron-capture processes in weakly and strongly coupled plasmas were investigated using the Bohr-Lindhard method with the Debye-Hückel [9] and ion-sphere [10] potentials, respectively.

In recent years, there has been considerable interest in the dynamics of gases and plasmas containing dust grains and highly charged aerosol, including collective effects and strong electrostatic interaction between the charged components [11]. These dust-plasma interactions occur not only in space plasmas but also in the laboratory plasmas. Several atomic processes in dusty plasmas have been investigated in order to obtain information on relevant plasma parameters [12]. However, to the best of our knowledge, the electron-capture process by the negatively charged dust grain in dusty plasmas has not been investigated as yet. Thus, in this paper we investigate a new mechanism of the electron-capture processes by negatively charged dust grains from hydrogenic ions in dusty plasmas due to the image charge induced inside the dust grain. We found that the electron-capture process would be impossible without the image charge effects. We assume that there are cases where dust grains are spherical conductors. The interaction potential between the electron released from a hydrogenic ion and the negatively charged dust grain is given by the repulsive part due to the dust charge and by the attractive part due to the image charge inside the dust grain. The classical straight-line approximation is applied to describe the projectile motion since the classical trajectory method has the great advantage of visualizing the atomic transition phenomena as a function of the impact parameter and plasma parameters.

In Sec. II, we derive the classical electron-capture cross section using the Bohr-Lindhard method for hydrogenic ion-dust grain collisions in dusty plasmas. We also obtain the electron-capture radius using the interaction energy and the kinetic energy of the released electron. In Sec. III, we obtain the scaled electron-capture cross section as a function of the impact parameter, kinetic energy of the projectile ion, and dust charge. The results show that the image charge inside the dust grain plays a significant role in the electron-capture process by the negatively charged dust grain. The electron-capture cross section is found to be quite sensitive to the collision energy and dust charge. It is also found that the position of the maximum electron-capture cross section is shifted toward smaller impact parameter as the collision energy increases. Finally, in Sec. IV, discussions are given.

II. ELECTRON-CAPTURE CROSS SECTION

The classical expression of the electron-capture cross section is given by [5]

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$$\sigma_c = 2\pi \int b\, db\, P_c(b), \tag{1}$$

where b is the impact parameter and $P_c(b)$ is the electron-capture probability. For intermediate- and high-energy projectiles, the classical expression of the electron-capture cross section is known to be valid. A first classical treatment to the charge-capture cross section was given by the Bohr-Lindhard (BL) method [2]. In the BL model, the electron-capture probability is given by the ratio of the collision time to the electron orbital time in the target ion:

$$P_c(b) = \frac{1}{\tau}\int_{-t_c}^{t_c} dt, \tag{2}$$

where t_c is the electron-capture time and $t=0$ is arbitrarily chosen as the instant at which the projectile makes its closest approach to the target ion. Here, the electron orbital time τ is given by

$$\tau = a_n/\nu_n = \frac{n^3}{Z_T^2}\frac{a_0}{c\alpha}, \tag{3}$$

where a_n $(=n^2 a_0/Z_T)$ is the nth Bohr radius of a hydrogenic ion with nuclear charge Z_T, a_0 $(=\hbar^2/me^2)$ is the Bohr radius, ν_n $(=Z_T c\alpha/n)$ is the electron velocity in the nth Bohr orbit, and α $(=e^2/\hbar c \approx 1/137)$ is the fine-structure constant.

Using spherical polar coordinates with their center at the dust grain, we can evaluate the electrostatic potential [12] $\phi(r)$ produced by the dust charge Q with the surrounding plasma:

$$\phi(r) = Q\,\frac{\exp[-(r-a)/\lambda_D]}{r}, \tag{4}$$

where a is the radius of the dust grain and λ_D is the Debye length of the background plasma. Here, for the sake of simplicity, the dust particles are assumed spherical. For typical laboratory and astrophysical dusty plasma, it has been known that $\lambda_D \gg a$. Here, dust grains are considered conductors including the image potential [12]. These conducting dust grains can be found in astrophysical dusty plasma such as metal-rich H II regions and metal-rich globular clusters [13,14]. When a very small particle (its radius$\ll a$) of the electric charge q is placed near the dust grain, the interaction potential energy could be evaluated by a sum of two fields. The first would be produced by the dust charge and the second by the image charge $q' = -(a/r)q$ located at point $r' = a^2/r$ inside the dust sphere. Hence, when the hydrogenic ion is placed within the Debye sphere $(a < r \ll \lambda_D)$, the interaction potential energy $V(r)$ between the bound electron $(q=-e)$ in the ground state of a hydrogenic ion and the negatively charged dust grain $(Q=-Ze)$ is given by

$$V(\mathbf{r}) = \frac{qQ'}{r} + \frac{qq'}{|\mathbf{r} - r'\hat{\mathbf{r}}|} + Z_T^2\mathcal{R}, \tag{5}$$

where $\mathbf{r}$ is the position vector of the projectile electron from the dust grain and $\hat{\mathbf{r}}$ $(\equiv \mathbf{r}/r)$ denotes the unit vector, Q' $(=Q-q')$ is the charge of the dust sphere, and $\mathcal{R}$ $(=me^4/2\hbar^2 \approx 13.6\,\mathrm{eV})$ is the Rydberg constant. Since the dust charge is usually much greater than the electron charge $(Z \gg 1)$ and $r<a$, the charge of the dust sphere Q' $(=-Ze-ae/r)$ can be approximated as Q' $(\cong -Ze)$. Since the Debye screening factor $e^{-(r-a)/\lambda_D}$ is not important inside the Debye sphere [15], i.e., $a<r\ll\lambda_D$ $(\approx 50a)$, the interaction energy between the dust grain and the electron placed near the surface of the dust grain can be given by

$$V(\mathbf{r}) \cong \frac{Ze^2}{r} - \frac{ae^2}{r^2-a^2} + Z_T^2\mathcal{R}, \tag{6}$$

where the first term is the repulsive interaction between the electron and the negative dust charge and the second term is the attractive interaction between the electron and its own positive image charge inside the dust grain. Due to this attractive interaction, the released electron can be captured by the negatively charged dust grain. According to the BL model [2], the capture process happens when the distance between the projectile and a released electron is smaller than the electron-capture radius R_c. Since the capture distance is determined by equating the kinetic energy of the released electron in the frame of the dust grain and the interaction provided by the dust grain [Eq. (6)], the electron-capture radius R_c within the Debye sphere $(r\ll\lambda_D)$ by the dust grain can be determined by

$$\frac{ae^2}{R_c^2-a^2} - \frac{ze^2}{R_c} - Z_T^2\mathcal{R} = \tfrac{1}{2}m\nu^2, \tag{7}$$

where ν is the collision velocity and m is the electron rest mass since the reduced mass of the dust grain and the released electron is almost equal to the electron mass. Near the surface of the dust grain $(R_c \approx a)$, the negatively charged dust grain can capture the released electron since the first term in Eq. (7), i.e., attractive interaction, is stronger than the second term, i.e., repulsive interaction. Since the image charge effect is only important near the surface of the dust grain, the Debye screening factor $e^{-(R_c-a)/\lambda_D}$ can be replaced by unity so that Eq. (7) is quite reasonable to obtain the solution of the scaled capture radius near the dust surface. Equation (7) can be rewritten in the scaled form,

$$\bar{R}_c^3 + \xi Z\bar{R}_c^2 - (\xi+1)\bar{R}_c - \xi Z = 0, \tag{8}$$

where $\bar{R}_c$ $(\equiv R_c/a)$ is the scaled capture radius and $\xi \equiv (2e^2/ma)/(\nu^2 + Z_T^2 e^4/\hbar^2)$. Even though Eq. (8) has three solutions, we need a real physical solution because of the physical constraint $1<\bar{R}_c<\bar{\lambda}_D$, where $\bar{\lambda}_D$ $(\equiv\lambda_D/a)$ is the scaled Debye length. After some algebraic manipulations, the physical solution of the scaled electron-capture radius is found to be

$$\bar{R}_c(\xi,Z) = \tfrac{1}{3}\{-\xi Z + (3+3\xi+\xi^2Z^2)[2/F(\xi,Z)]^{1/3} + [F(\xi,Z)/2]^{1/3}\}, \tag{9}$$

where

$$F(\xi,Z)=18\xi Z-9\xi^2 Z-2\xi^3 Z^3+\sqrt{-4(3+3\xi+\xi^2 Z^2)^3+[27\xi Z-9(1+\xi)\xi Z-2\xi^3 Z^3]^2}. \tag{10}$$

In the following section, we shall discuss the differential electron-capture cross section using the electron-capture radius given by Eq. (9).

III. DIFFERENTIAL ELECTRON-CAPTURE CROSS SECTION

For heavy dust grains, the projectile path can be described by the classical straight-line trajectory, i.e., $\mathbf{r}(t)=\mathbf{b}+\mathbf{v}t$ with $\mathbf{b}\cdot\mathbf{v}=0$. Then, the collision time t_c in Eq. (2) is given by $(R_c^2-b^2)^{1/2}/v$. Using the straight-line trajectory, $P_c(b)$ becomes

$$P_c(b)=\frac{2}{\tau v}(R_c^2-b^2)^{1/2}. \tag{11}$$

The total electron-capture cross section, Eq. (1), can be represented as

$$\sigma_c=2\pi a^2\int_1^{\bar{\lambda}_D}\bar{b}\,d\bar{b}\,P_c(\bar{b}), \tag{12}$$

where $\bar{b}$ $(\equiv b/a)$ is the scaled impact parameter. Then, Eq. (12) can be rewritten as

$$\frac{d\sigma_c/d\bar{b}}{\pi a^2}=2\bar{b}P_c(\bar{b}) \tag{13a}$$

$$=\frac{4a}{\tau v}\overline{bP_c}(\bar{b}), \tag{13b}$$

where $\overline{bP_c}(\bar{b})$ is the scaled differential electron-capture cross section:

$$\overline{bP_c}(\bar{b},\xi,Z)=\bar{b}[\bar{R}_c^2(\xi,Z)-\bar{b}^2]^{1/2}, \tag{14}$$

where the scaled electron-capture radius $\bar{R}_c(\xi,Z)$ is given in Eq. (9). The parameter ξ can be rewritten as $\xi=2(\mu/m)(a_0/a)/(\bar{E}+\mu Z_T^2/m)$, $\mu[=Mm_i/(M+m_i)]$ is the reduced mass of the dust grain (M) and the ion (m_i), and $\bar{E}$ $(\equiv\mu v^2/2\mathcal{R})$ is the scaled kinetic energy of the projectile ion.

In order to explicitly investigate the scaled differential electron-capture cross section, specifically, we chose $a=0.01\,\mu\text{m}$, $Z_T=1$ (hydrogen atom), and the density of the dust grain is $\rho\cong 2\text{ g cm}^{-3}$. Figure 1 shows the scaled differential electron-capture cross section as a function of the scaled impact parameter in hydrogen-atom–dust-grain collisions in dusty plasmas. As we can see in this figure, the electron-capture cross section is decreased with increasing the scaled kinetic energy of the projectile ion. It is also found that the ion must pass very close to the dust grain to be captured. The dust grains are principally charged by the thermal fluxes reaching the grain surface [16]. Even though the charge capture by the image charge produced inside the dust grain is not the main charging process, it is important to note that the induced positive image charge inside the dust grain plays an important role in the electron capture by the negatively charged dust grain near the dust surface.

IV. DISCUSSIONS

We investigate the electron-capture processes by negatively charged dust grains from hydrogenic ions in dusty plasmas using the classical Bohr-Lindhard model. The interaction potential between the released electron and the negatively charged dust grain is given by the repulsive part due to the dust charge and by the attractive part due to the positive

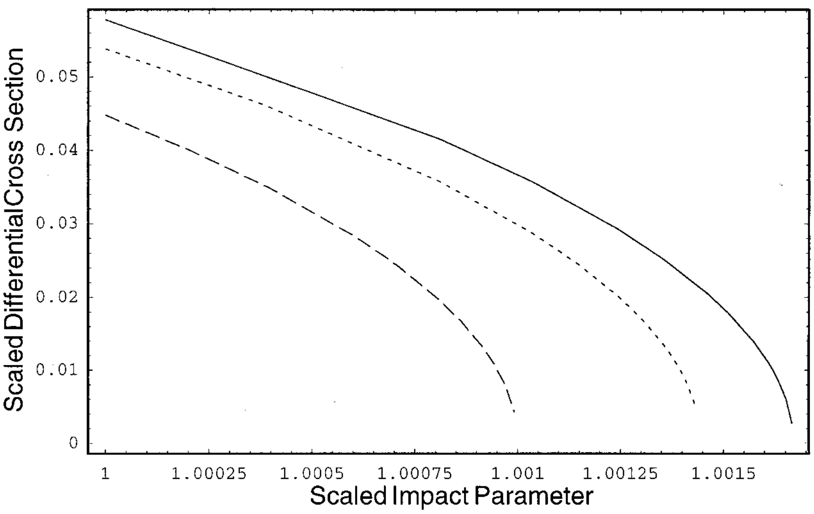

FIG. 1. The scaled differential electron-capture cross section $\overline{bP_c}(\bar{b})$ [Eq. (14)] from a hydrogen atom by the negatively charged dust grain as a function of the scaled impact parameter $\bar{b}$ $(=b/a$, a is the grain radius). The solid line represents the differential electron-capture cross section for $Z=200$ and $\bar{E}=100$. The dashed line represents the differential electron-capture cross section for $Z=400$ and $\bar{E}=100$. The dotted line represents the differential electron-capture cross section for $Z=200$ and $\bar{E}=1000$.

image charge inside the dust grain. The electron-capture radius is obtained by the total interaction potential and the kinetic energy of the released electron in the frame of the projectile dust grain. The classical straight-line approximation is applied to describe the projectile motion since the classical trajectory method has the great advantage of visualizing the electron-capture cross section as a function of the impact parameter, projectile energy, and dust charge. Without the image charge effects, the released electron cannot be captured by the negatively charged dust grain. The electron-capture cross section is quite sensitive to the collision velocity and dust charge. It should be noted that the image charge inside the dust grain plays a significant role in the electron-capture process near the surface of the dust grain. These results provide a general description of the electron-capture phenomena by the image charge produced inside the negatively charged dust grains in dusty plasmas.

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