Full velocity difference model for a car-following theory

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In this paper, we present a full velocity difference model for a car-following theory based on the previous models in the literature. To our knowledge, the model is an improvement over the previous ones theoretically, because it considers more aspects in car-following process than others. This point is verified by numerical simulation. Then we investigate the property of the model using both analytic and numerical methods, and find that the model can describe the phase transition of traffic flow and estimate the evolution of traffic congestion.

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For these last few decades, the development of various theories concerning traffic phenomena has received considerable attention. An increasing number of investigators with different backgrounds and points of view have considered various aspects of traffic phenomena with very gratifying results. There are essentially two different types of approaches of studying the traffic problem, namely, macroscopic and microscopic ones. Here, we are mainly concerned with the latter ones, which are not only of great importance with regard to an autonomous cruise control system, but that have also emerged as important evaluation tools for intelligent transportation system strategies since the early 1990s. As basic and important components of microscopic approaches, car-following theories have been given much research interest.

Car-following theories were developed to model the motion of vehicles following each other on a single lane without any overtaking. It is based on the assumption each driver reacts in some specific fashion to a stimulus from the vehicle ahead of him. Reuschel [1] and Pipes [2] were pioneers in the development of the theories in the early 1950s. Now the list of contributions to the theories is a long one [3–8]. Among these theories, the classical car-following model was of particular importance because of the accompanying comprehensive field experiments and the discovery of the mathematical bridge between microscopic and macroscopic theories of traffic flow.

The equation of the classical model, which describes the motion of the $(n+1)$th car following the nth car in a single lane of traffic, has been taken as

$$\frac{dv_{n+1}}{dt}(t+\Delta t)=\lambda\Delta v, \tag{1}$$

where $\Delta v=v_n(t)-v_{n+1}(t)$ and $v_n(t)$ is the velocity of the nth car, Δt is the time lag of response, λ is the sensitivity. For the sensitivity, different functions have been assumed, including (1) constant [4]

$$\lambda=a, \tag{2}$$

and (2) step function [5]

$$\lambda=\begin{cases} a: & s\leqslant s_c \\ b: & s>s_c, \end{cases} \tag{3}$$

where s is the headway, i.e., $s=x_n(t)-x_{n+1}(t)$, here x_n is the position of the nth car. a, b, s_c are constants.

Applying the classical model, we can describe the traffic dynamics from the microscopic point of view, i.e., we can track the following vehicle over space and time as a function of the trajectory of the lead vehicle. Moreover, it enables us to establish a bridge between the microscopic and the macroscopic point of views, which is a very important discovery and may be greatly expanded to provide a connection between the matrix of microscopic models and most macroscopic theories of traffic flow as shown by May [9]. However, despite the importance of the classical model, it has the following defects: When the successive vehicles have identical speeds, from Eq. (1), the model allows the distance between the vehicles to be arbitrarily close. Obviously, it is unrealistic. Apart from that, it cannot describe the acceleration of a single vehicle correctly.

Besides the classical car-following model, there are a few others in the literature. In 1995, Bando *et al.* presented a car-following model called the optimal velocity model (OVM) [7]. It was based on the idea that each vehicle has an optimal velocity, which depends on the following distance of the preceding vehicle. The equation of the model is

$$\frac{dv_{n+1}}{dt}(t)=\kappa[V(s)-v_{n+1}(t)], \tag{4}$$

where κ is a sensitivity constant and V is the optimal velocity that the drivers prefer. Applying the OVM, many properties of real traffic flows can be described, such as the instability of traffic flow, the evolution of traffic congestion, and the formation of stop-and-go waves.

Helbing and Tilch [8] carried out a calibration of the OVM with respect to the empirical data. They adopted the optimal velocity function as

$$V(s)=V_1+V_2\tanh[C_1(s-l_c)-C_2], \tag{5}$$

where l_c is the length of the vehicles, which can be taken as 5 m in simulations. The resulting optimal parameter values are $\kappa=0.85\ \mathrm{s}^{-1}$, $V_1=6.75$ m/s, $V_2=7.91$ m/s, $C_1=0.13\ \mathrm{m}^{-1}$, and $C_2=1.57$. The comparison with field data

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shows that OVM encountered the problems of too high acceleration and unrealistic deceleration. (see Figs. 2–4 in [8]).

In order to solve the problems, Helbing and Tilch [8] proposed a generalized force model (GFM). One term is increased on the right-hand side (RHS) of Eq. (4). Thus, the formula of GFM reads

$$\frac{dv_{n+1}}{dt}(t)=\kappa[V(s)-v_{n+1}(t)]+\lambda\Theta(-\Delta v)\Delta v, \qquad (6)$$

where Θ is the Heaviside function. In order to reduce the number of parameters, they replaced the previous *V* function (5) by another slightly different optimal velocity, which only causes a negligible effect on the results. Therefore, we still adopt the *V* function (5). The calibration shows that in GFM, $\kappa=0.41\ \mathrm{s}^{-1}$, which is much smaller than that in OVM. And the results show that GFM reaches better agreement with the field data than OVM.

Comparing GFM with OVM, we find out that when $\Delta v \geqslant 0$, GFM has the same form as OVM, the difference lies in that they have different values of sensitivity κ. To find out the effect of the sensitivity on traffic-flow dynamics, we next carry out a numerical simulation of the motion of cars starting from a traffic signal. For this condition, $\Delta v \geqslant 0$ is always guaranteed.

We carry out the simulation as in Ref. [10]. First a traffic signal is red and all cars (11 cars in the simulation) are waiting with a headway of 7.4 m, at which the optimal velocity (5) is zero. Then at time $t=0$, the signal changes to green and cars start.

From the simulation we can obtain the delay time of car motion. Consider a pair of cars, a leader and a follower. Assume the leader changes the velocity according to $v_l=v_0(t)$ and the follower duplicates the leader's velocity but with some delay time , that is, $v_f=v_0(t-\delta t)$. We define the delay time of car motion by δt. Moreover, from the time delay of car motion, we can estimate the kinematic wave speed at jam density c_j, which is equal to the quotient of the headway 7.4 m divided by the delay time of motion.

The simulation results are shown in Fig. 1(a,b) and Table I, which are obtained from the behavior of the velocities of the 7th–10th cars because these cars behave almost in the same manner. From the Table, we learn that κ has an effect on δt and c_j. A smaller sensitivity κ leads to a larger δt and a smaller c_j. As Bando *et al.* [10] pointed out, the observed δt is of the order of $1 s$, and Del Castillo and Benitez [11] indicated that c_j ranges between 17 and 23 km/h. Therefore, we can see that GFM is poor in anticipating the two parameters.

Why does GFM not behave well in the aspect? We believe it may be because the model does not take the effect of positive Δv on traffic dynamics into account. We think the term including Δv is effective not only under the condition that the velocity of the following vehicle is larger than that of the leading vehicle, but also under the opposite condition. Treiber *et al.* [12] also pointed out that if the preceding cars are much faster, then the vehicle will not brake, even if its headway is smaller than the safe distance, and this instance

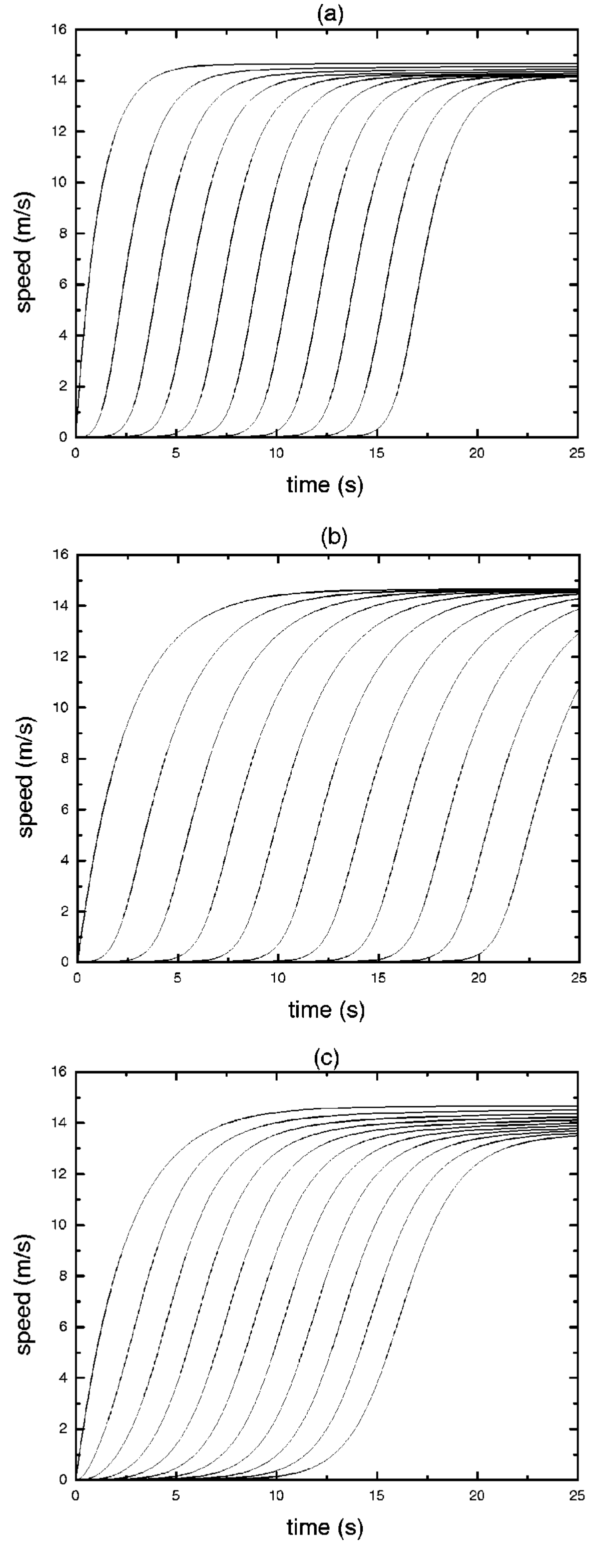

FIG. 1. Motions of cars 1–11 starting from a traffic signal. (a) for OVM; (b) for GFM; (c) for FVDM.

cannot be explained by either OVM or GFM. According to our observation of real traffic, this instance does exist.

In accordance with the above concept, on the basis of GFM, taking the positive Δv factor into account, we obtain a more systematic model, one whose dynamics equation is as

TABLE I. Delay times of car motions from a traffic signal and disturbance propagation speed at jam density in different models.

Model	δt (s)	c_j (km/h)
OVM ($\kappa = 0.85\ \mathrm{s}^{-1}$)	1.6	16.65
GFM ($\kappa = 0.41\ \mathrm{s}^{-1}$)	2.2	12.11
FVDM ($\kappa = 0.41\ \mathrm{s}^{-1}$)	1.4	19.03

follows:

$$\frac{dv_{n+1}}{dt}(t) = \kappa[V(s) - v_{n+1}(t)] + \lambda \Delta v. \tag{7}$$

Since the model takes both positive and negative velocity differences into account, we call it a full velocity difference model (FVDM). Note that in GFM, Eq. (6) may be rewritten as

$$\frac{dv_{n+1}}{dt}(t) = \kappa[v_m - v_{n+1}(t)] + \kappa[V(s) - v_m] + \lambda \times \Theta(-\Delta v)\Delta v, \tag{8}$$

where v_m is the maximum speed. The first term on the RHS is the acceleration force, and the last two terms represent the interaction force. Our model Eq. (7) may be reformulated into a similar form:

$$\frac{dv_{n+1}}{dt}(t) = \kappa[v_m - v_{n+1}(t)] + \kappa[V(s) - v_m] + \lambda \times \Theta(-\Delta v)\Delta v + \lambda \Theta(\Delta v)\Delta v. \tag{9}$$

Comparing with Eq. (8), FVDM differs in the expression of interaction term, where GFM assumes the positive Δv does not contribute to the vehicle interaction, while FVDM suggests it does contribute to vehicle interaction by reducing interaction force because $\kappa[V(s) - v_m]$ is always negative and $\lambda \Theta(\Delta v)\Delta v$ is always positive.

Now we apply FVDM to simulate the car motion under a traffic signal. Without loss of generality, here we take step-function (3) for λ, where parameters a, b, s_c are taken as $a = 0.5\ \mathrm{s}^{-1}$, $b = 0$ and $s_c = 100$ m. The results are shown in Fig. 1(c) and also in Table I. From the Table, we can see that δt of FVDM is quite smaller than that of GFM, which is the most exact in the three models. And c_j fall into the desired range. From this point of view, FVDM describes the traffic dynamics most exactly, which verifies that the improvement in FVDM is reasonable and realistic.

Next we examine some properties of FVDM. First, by simulation, we explore whether the model causes unrealistically high acceleration just as OVM. Considering two cars initially at rest, the leader car is unobstructed. At $t = 0$, the two cars start up according to GFM and FVDM respectively. We obtain the acceleration in Fig. 2. Parameters are the same as those in the previous simulation. From the figure, we can see that the maximum value of acceleration in FVDM is not greater than that in GFM. For the leading car in FVDM, since it is unobstructed, the headway can be assumed $s \to +\infty$, therefore $\lambda = 0$, thus it has the same acceleration as that in GFM. As for the following car, the main cause of the difference is that the car in FVDM accelerates more quickly than the car in GFM. Therefore, the delay time in FVDM is smaller than that in GFM.

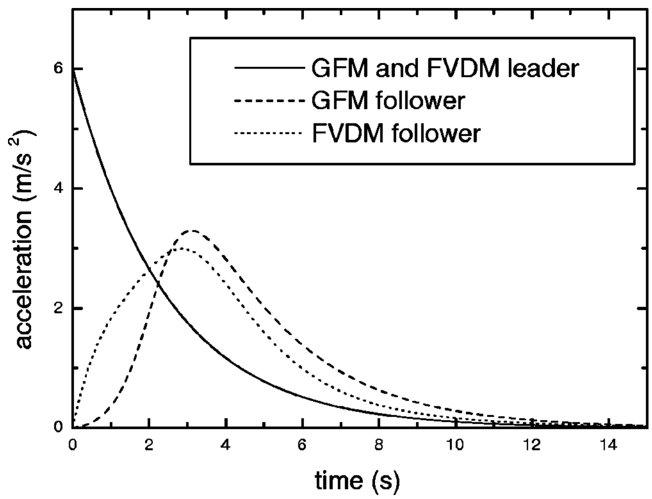

FIG. 2. Acceleration of unobstructed leading car and its following car both initially at rest according to GFM and FVDM.

Making a linear stability analysis of FVDM similar to Ref. [7], one obtains that only when the condition

$$f = V'(b) < \frac{\kappa}{2} + \lambda \tag{10}$$

is met, the traffic is stable. For OVM, the criteria for stability is

$$f < \frac{\kappa}{2}. \tag{11}$$

Comparing the criterion (10) with (11), we find out that they are consistent because if we assume $\lambda = 0$ in FVDM, it reduces to OVM and then Eq. (10) is the same as Eq. (11).

Now we carry out a numerical simulation to check the analysis, still taking Eq. (5) for the function of V and the parameters are the same as before. Since the headway s in the following simulation never exceeds $s_c = 100$ m, thus constant $\lambda = 0.5\ \mathrm{s}^{-1}$ can be adopted instead of step-function (3). We take car number $N = 100$, the circuit length $L = 1500$ m. We set an initial disturbance as

$$x_1(0) = 1\ \mathrm{m}; \quad x_n(0) = (n-1)L/N \ \text{ for } n \neq 1, \tag{12}$$

$$v_n(0) = V(L/N). \tag{13}$$

Substituting the values of the parameters into criterion (10), we learn the initial disturbance is unstable. Figure 3 shows the snapshots at $t = 300$ s and $t = 2000$ s. The homogeneous flow eventually develops into congestion, which corresponds to stop-and-go traffic. In the phase space ($s - v$ space), we can see that after enough time, when the congestion becomes stationary, the motion of vehicles organizes a ‘‘hysteresis loop’’ as shown in Fig. 4.

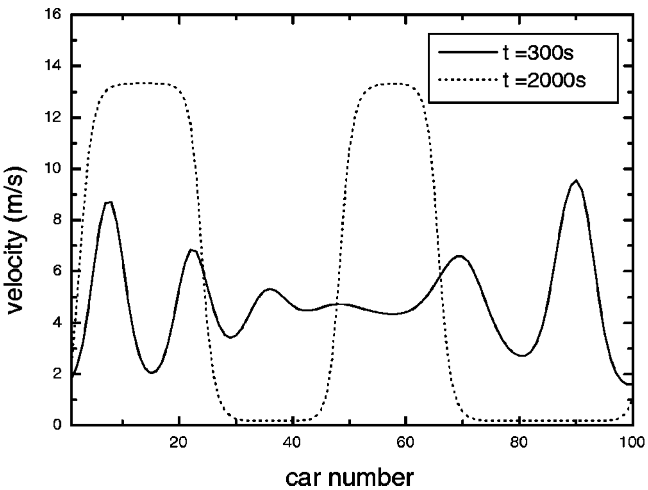

FIG. 3. The snapshots of velocity of all vehicles at different times.

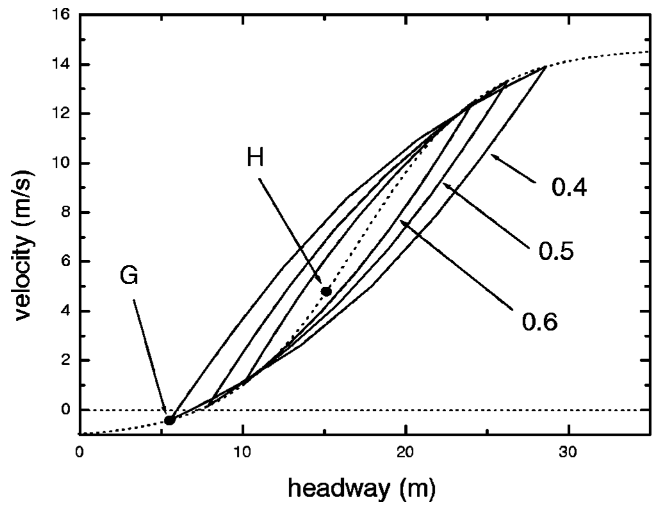

FIG. 4. Hysteresis loops for FVDM at different values of λ.

Changing the value of λ, loops of different size can be obtained [cf. Fig. 4]. Here, two points should be noted. First, for $\lambda=0.4$, part of the loop (for example, point G) lies in the region where $v<0$ and s is smaller than the minimum headway 7.4 m. Bando *et al.* [10] suggested two possibilities: (1) There may be an existence of a new phase; (2) It is artificial due to finite-size effects. It is our intent that our future work will determine which one is right. Second, when $\lambda=0.8$, criterion (10) is held, the traffic flow is stable, the hysteresis loop will not be generated, and in phase space, there will be only a point H on the optimal velocity curve instead.

In summary, we develop a full velocity difference model (FVDM) for a car-following theory based on the previous OVM and GFM. Then we apply FVDM to several simulations. The results reveal that FVDM predicts correct delay time of car motion and kinematic wave speed at jam density. Moreover, unrealistically high acceleration will not appear. Linear stability analysis has been done and the stable criterion is given, simulation indicates that FVDM can produce the desired results such as the formation of congestion from an initially homogenous condition, etc.

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