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Soliton interaction in a nonlinear waveguide in the presence of resonances

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The simultaneous propagation of two optical pulses through a nonlinear dispersive medium composed of a resonant three-level system is investigated. By choosing a soliton of area 4π and order $N=2$ at the pump frequency, together with a weaker pulse with a sech profile at the signal frequency, we show that the pump soliton breaks up into a pair of solitary waves which are cloned to the signal frequency. Due to a combination of coherent population trapping and nonlinear dispersive effects, the pair interacts in a repulsive fashion so that the taller wave travels faster than the shorter one.

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I. INTRODUCTION

One of the most interesting phenomena in the physics of nonlinear waves is the formation of a stable wave packet propagating over considerable distances with a permanent profile, the so-called soliton. Solitons appear in many diverse fields such as particle physics, solid state physics, plasma physics, biophysics, acoustics, and nonlinear optics. Particularly in the field of nonlinear optics, there are two soliton types with which we shall be concerned in this work. One is the soliton solution of the nonlinear Schrödinger equation (the NLS soliton) whose mechanism is based on the balance of anomalous dispersion and nonlinear effects in a transparent medium. Another kind of soliton emerges from a complete different reason near atomic transitions as a result of self induced transparency (SIT), a coupling between the electric field and the quantum states describing matter whereby the pulse propagates undistorted and unattenuated inside an absorbing medium. The latter is usually called a SIT soliton, and is shown to obey the area theorem which establishes that initial pulse areas between π and 3π evolve into a steady-state 2π pulse, the pulse area being defined by the integral of the field envelope over time [1]. Furthermore a 4π pulse exhibits the pulse breakup phenomenon which has been observed both experimentally and numerically [2–6]. Many phenomena have been reported in recent years, involving the coherent propagation of light through a three level system, such as subluminal propagation due to electromagnetic induced transparency [7], and parametric amplification and cloning of SIT solitons in a Λ-scheme of a three level model of atoms [8]. The simultaneous propagation of two pulses through an otherwise opaque three level medium without changing their initial temporal shapes has been described by analytical solutions in some special cases such as simultons [9]. The essence of these phenomena is attributed to the coherent population trapping effect [10], under which the otherwise absorbing medium becomes transparent and the pulse propagates freely.

Even more interesting than each type of soliton solution mentioned above is the theoretical evidence of the existence of a type of a mixed state, to which we shall henceforth refer as a SIT-NLS-soliton. As demonstrated in Ref. [11], the condition for this coexistence is that the amplitude and duration of a 2π-SIT pulse should be the exact ones that allow the corresponding self phase modulation to balance the dispersion spread of the pulse. Numerical work on a two-level model confirmed the existence of a stable $2\pi-(N=1)$ SIT-NLS soliton, that is, a soliton of area 2π and order $N=1$, and also confirmed that high order SIT-NLS solitons split into multiple solitons [12]. The cloning of a SIT-NLS soliton was recently demonstrated within the framework of Maxwell-Bloch equations describing a three-level system together with a pair of coupled nonlinear Schrödinger equations, including the cross phase modulation term [13]. It should be noted here that the coherent pulse propagation in resonant fiber waveguides opens new possibilities for optical signal processing such as pulse shaping, control, and cloning. Experimental work dealing with coherent pulse propagation through an erbium doped fiber was carried out, and pulse breakup was shown to occur [5,6].

In this paper we investigate the coherent propagation of mixed higher order $4\pi-(N=2)$ solitons through dispersive nonlinear media in the presence of a resonant three level system. We show that mixed higher order solitons initially undergo a typical breakup of a 4π-SIT soliton with subsequent attenuation at the pump frequency. Simultaneously, parametric amplification of the signal field generates two pulses of area 2π identical to the pump solitons after the breakup. In the absence of dispersive nonlinear effects, we find that the cloned pair travels unaltered through the medium due to the coherent population trapping effect. In contrast, in a dispersive nonlinear medium we find that parametric amplification generates a pair of solitary waves in the signal frequency whose areas oscillate around 2π. Furthermore, for long propagation distances, the NLS component induces a typical two-soliton repulsive interaction between the cloned pair generated from the mixed SIT-NLS soliton, which is not present in the pair generated from a single SIT soliton.

II. BASIC EQUATIONS

Let us begin by considering the interaction of a three-level atom with two optical fields propagating in the z direc-

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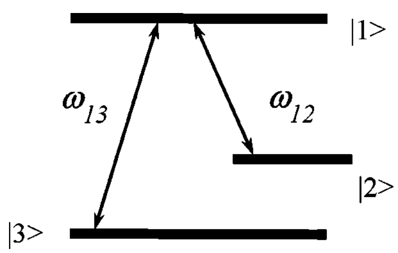

FIG. 1. Sketch of the energy levels for a Λ system.

tion and polarized in the x direction. We consider one of the optical fields as a $4\pi-(N=2)$ soliton represented by $A_{13}(z,t)$ at the pump transition ω_{13}, resonant with states $|1\rangle$ and $|3\rangle$. The other is a weak field of arbitrary profile $A_{12}(z,t)$ (signal) with a frequency corresponding to the frequency ω_{12} connecting the states $|1\rangle$ and $|2\rangle$ at the Stokes transition. Such configuration of quantum levels and fields is well known as a Λ scheme, and is illustrated in Fig. 1. The NLS equations that describe the propagation of the envelope of these fields, through a nonlinear medium are given by

$$\frac{\partial A_{12}}{\partial z}=-\frac{i\beta_{12}}{2}\frac{\partial^2 A_{12}}{\partial T^2}+i\gamma_{12}[|A_{12}|^2+2|A_{13}|^2]A_{12}+\frac{i\omega_{12}n_a}{2\varepsilon_0 c}c_2^*c_1\mu_{12}, \quad (1)$$

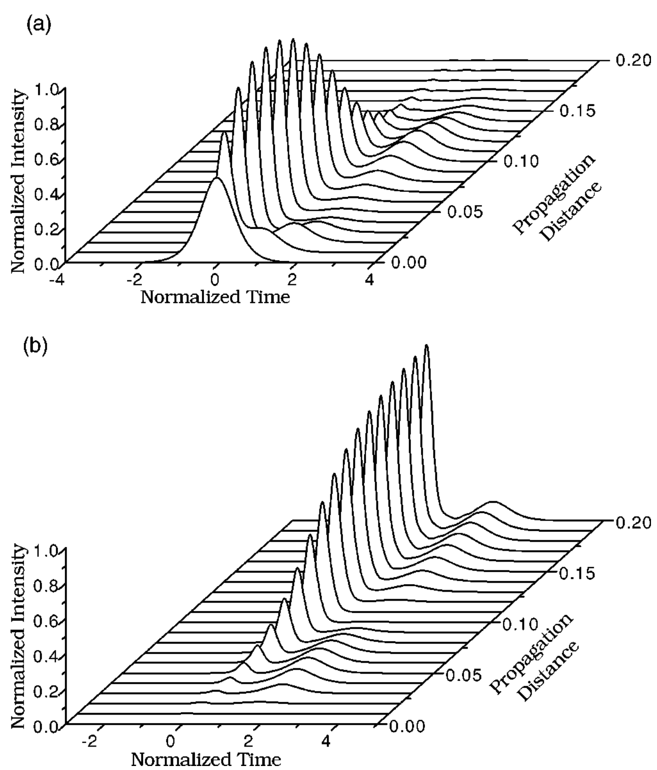

FIG. 2. Intensity profiles of the pump (a) and signal (b), respectively, as functions of the normalized propagation distance z/z_0 and normalized time τ.

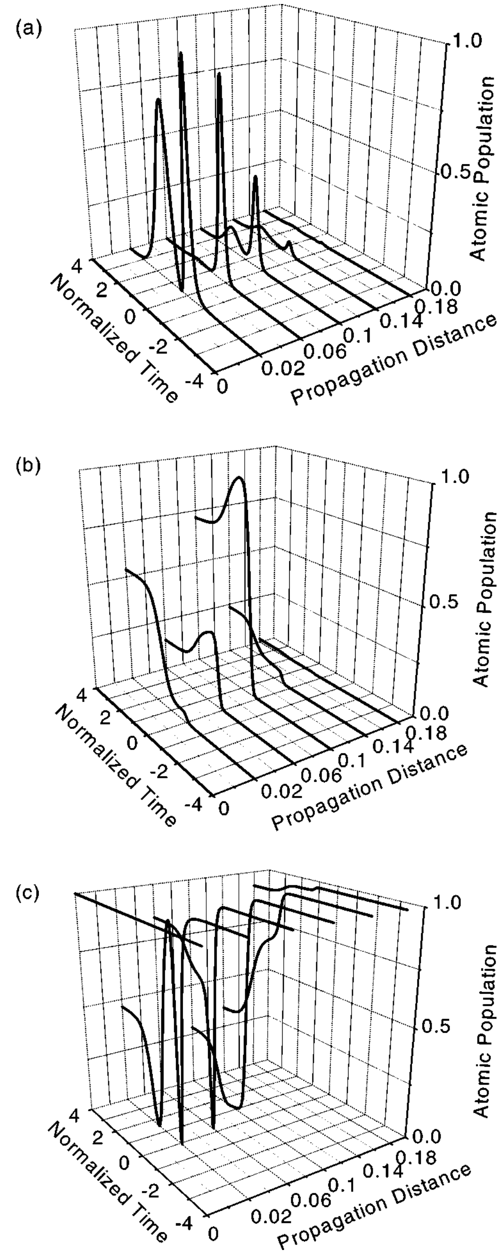

FIG. 3. Dynamics of the atomic populations in level $|1\rangle$ (a), level $|2\rangle$ (b), and level $|3\rangle$ (c), illustrating the coherent population trapping effect.

$$\frac{\partial A_{13}}{\partial z}=-\frac{i\beta_{13}}{2}\frac{\partial^2 A_{13}}{\partial \tau^2}+i\gamma_{13}[|A_{13}|^2+2|A_{12}|^2]A_{13}+\frac{i\omega_{13}n_a}{2\varepsilon_0 c}c_3^*c_1\mu_{13}, \quad (2)$$

where β_{ij} is the group velocity dispersion parameter, which should be negative for the generation of the NLS soliton, and γ_{ij} is related to the Kerr nonlinearity. The probability amplitudes $c_j(z,t)$ of the atomic levels $|j\rangle$ for this system within the rotating wave approximation are written as

$$\frac{\partial c_1}{\partial t}=\frac{i}{\hbar}[c_2\mu_{12}A_{12}+c_3\mu_{13}A_{13}], \quad (3)$$

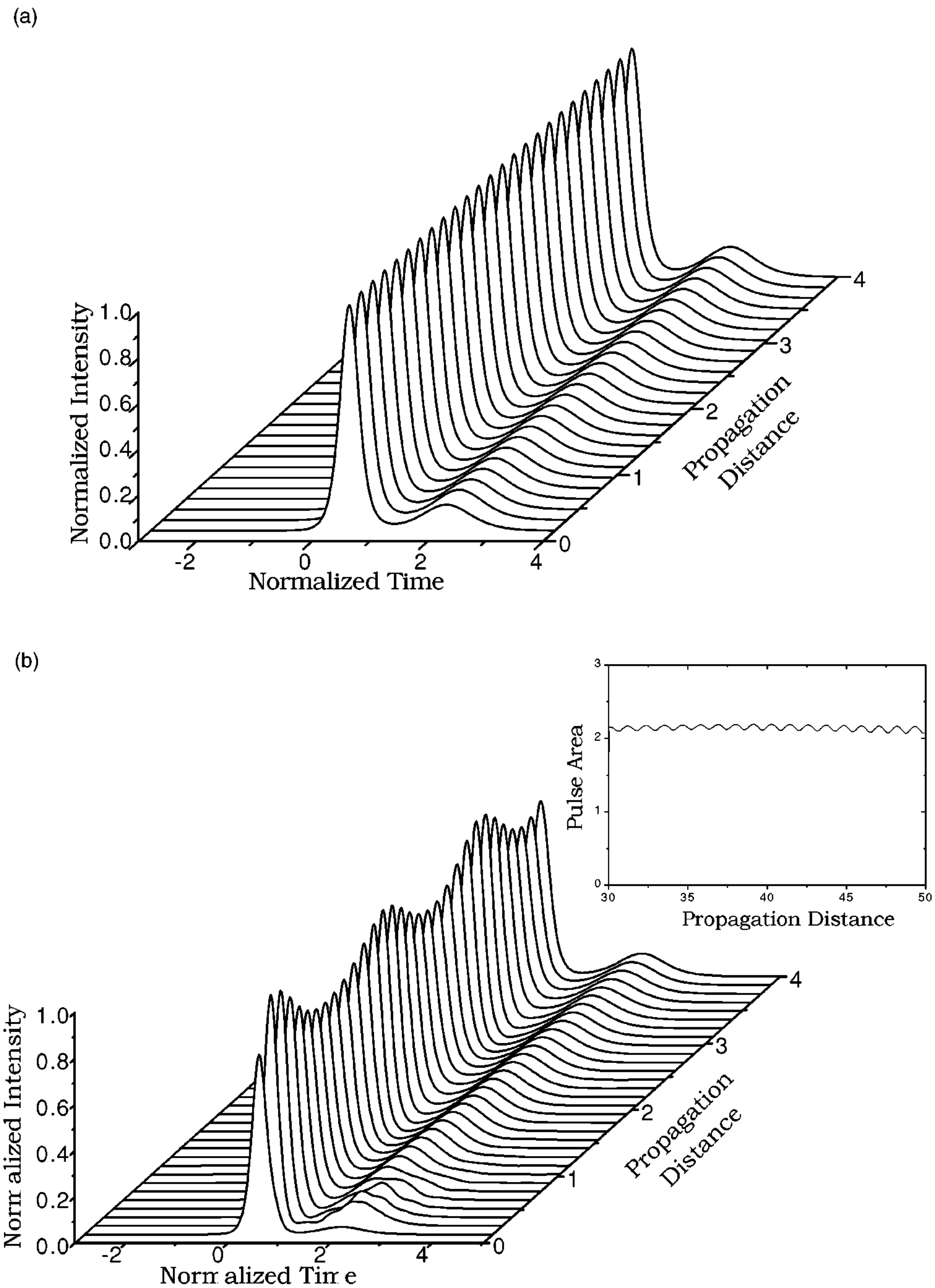

FIG. 4. Propagation under coherent population trapping: (a) in the absence of dispersive nonlinear effects, and (b) in their presence. Inset: oscillation displayed by the area of the taller pulse during propagation. The pulse area is divided by π.

$$\frac{\partial c_2}{\partial t} = \frac{i}{\hbar}[c_1 \mu_{12} A_{12}^*], \tag{4}$$

$$\frac{\partial c_3}{\partial t} = \frac{i}{\hbar}[c_1 \mu_{13} A_{13}^*]. \tag{5}$$

Here $\mu = (\mu_{12} + \mu_{13})\mathbf{x}$, with μ_{12} and μ_{13} as the electrical dipole moments related to the associated permitted transitions. By solving this set of equations using a combination of the Runge-Kutta method to determine the quantum probability coefficients and of the beam propagation method for the evolution of the optical fields, one is able to study higher order soliton propagation properties in the presence of group velocity dispersion, and self- and cross-phase modulation.

III. RESULTS AND DISCUSSION

We consider the initial conditions $c_1 = 0$, $c_2 = 0$, and $c_3 = 1$, so that the population is in the ground state to start with, and the input pump and signal pulses are written as

$$A_{13}(\tau) = \sqrt{P_0}\,\mathrm{sech}\,\tau,$$

$$A_{12}(\tau) = 0.05\sqrt{P_0}\,\mathrm{sech}\,\tau,$$

where $\tau = (t - z/v_g)/T_0$, with T_0 the pulse width and v_g an average group velocity. The parameters must be arranged in such a way that the input power P_0 satisfies the coexistence condition [12]

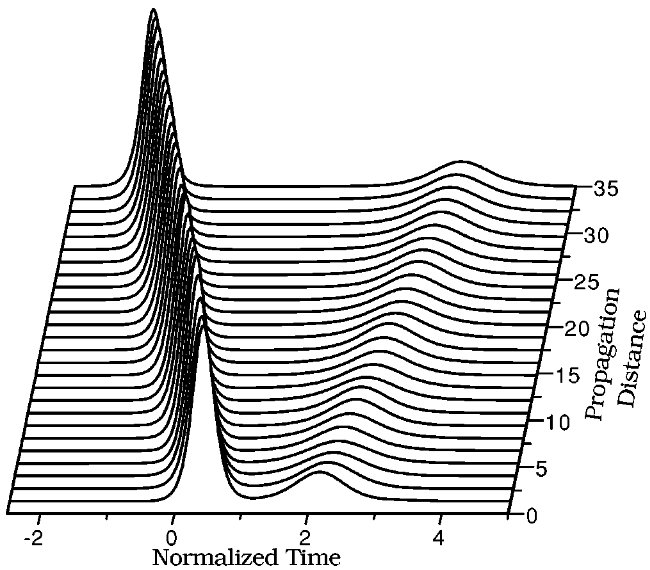

FIG. 5. Asymptotic behavior illustrating pair repulsion due to soliton interaction.

$$P_{0(N=2-NLS)}=P_{0(4\pi-SIT)}$$

We refer to Figs. 2(a) and 2(b), where we depict the intensity profiles of the pump and the signal respectively, as functions of the normalized propagation distance z/z_0 and of the normalized time τ. Figure 2(a) shows that during the first stage due to the NLS component, a $N=2$ soliton is excited, but for a very short time, as the SIT component quickly dominates inducing the pump pulse to breakup into a pair of 2π solitary waves. Subsequently, we note that while both waves undergo strong attenuations, the signal is simultaneously amplified into two 2π solitary waves with the same properties as the waves just absorbed at the pump frequency, demonstrating that the energy of the pump has been transferred to the former pair. The numerical simulations indicate that this process of energy transfer occurs together with the population transfer to the initial state $|3\rangle$, as illustrated in Fig. 3. For $z/z_0 \geqslant 0.2$, one finds that $c_1 \rightarrow 0$, $c_2 \rightarrow 0$, and $c_3 \rightarrow 1$, which means that the populations in levels $|1\rangle$ and $|2\rangle$ become zero and the entire population settles in level $|3\rangle$, characterizing population trapping.

It is important to compare our results with the results in the literature. The three-level configuration provides the cloning process of the pair, a result previously obtained for a SIT soliton [8]. It should be noted that this result would not have occurred in a two-level system [12]. Furthermore, an important physical aspect is observed by considering the combined SIT-NLS effect after the energy transfer process is completed. At this stage, the pump is gone and the pair travels unaltered in the absence of the NLS component, as depicted in Fig. 4(a). By contrast, in the presence of nonlinear dispersive effects [Fig. 4(b)], the amplitude of the taller pulse displays a small oscillation whose period is approximate z_0, so that one may identify this periodicity with the characteristic NLS soliton period [inset of Fig. 4(b)]. Consequently the areas of both pulses oscillate around the 2π value.

The asymptotic behavior is illustrated in Fig. 5 where one can clearly see a repulsive interaction between the pair. At this propagation distance, the pump is already fully depleted and the population is trapped in level $|3\rangle$. Therefore the coupled set of equations (1)–(5) is reduced to one single ordinary nonlinear Schrödinger equation for the amplified signal whose initial condition is given by a two-soliton solution. The properties of such a solution were defined at the beginning of the propagation by the pump and signal interaction from which the soliton pair originated. We can see that the taller pulse moves forward with respect to the time coordinate frame employed here, while the smaller one suffers a delay. This means that the taller soliton goes faster than the group velocity, and moves ahead of the smaller one, which propagates with a velocity smaller than the group velocity. According to analytical, experimental, and numerical work on soliton interactions [14–16], the nature of the interaction should be determined by the phase difference δ between the pair and a repulsive interaction should develop for $\pi/2<\delta<\pi$, cousing the pulses to separate monotonically. Using a variational approach [17], one may show that the phase difference between the pair is proportional to the difference between their squared amplitudes, that is, $\delta = \gamma z(|A_1|^2-|A_2|^2)$. By varying the ratio of the pump intensity relative to the signal intensity, one may change the relative amplitudes of the resulting pair, controlling the phase difference between them and, with this, the nature of the soliton interaction, which might even become attractive. Work in this direction is currently being developed.

IV. CONCLUSIONS

In conclusion, we have investigated the propagation of two fields through three-level media in a Λ scheme, within the framework of Maxwell-Bloch equations together with proper NLS equations for the fields. Through numerical simulations we have found an attenuation of a high order soliton $4\pi-(N=2)$ concomitant with the formation of two $2\pi-(N=1)$ solitary waves at the Stokes signal whose areas oscillate around the 2π value. The taller pulse of the pair exhibits an oscillation period of z_0, revealing a signature of the original $N=2$ pump pulse. Furthermore, after the total depletion of the pump, we show clearly, through the dynamics of the populations at each level, that coherent population trapping is established in such a way that the whole population is kept at the lower level while the other two levels remain empty. In this regime, the soliton pair undergoes a repulsive interaction, developing different velocities, in contrast to the propagation, where nonlinear and dispersive effects are neglected.

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