Dynamics of periodically forced semiconductor laser with optical feedback

Jorge Manuel Mendez,[1] R. Laje,[1] M. Giudici,[2], J. Aliaga,[1] and G. B. Mindlin[1]
[1]*Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Ciudad Universitaria, Pabellon I, CP 1428, Buenos Aires, Argentina*
[2]*Institut Non-lineaire de Nice, Route des Lucioles 1361, 06560 Valbonne, France*
(Received 16 June 2000; published 25 May 2001)

Recently it was proposed that semiconductor lasers with optical feedback present a regime where they behave as noise driven excitable units. In this work we report on an experimental study in which we periodically force one of these lasers and we compare the results with the solutions of a simple model. The comparison is based on a topological analysis of experimental and theoretical solutions.

DOI: 10.1103/PhysRevE.63.066218 PACS number(s): 05.45.$-$a, 42.65.Sf

I. INTRODUCTION

The dynamics of semiconductor lasers with optical feedback has been studied in depth since 1989 [1], although systematic studies go back as far as 1977 [2]. This system presents a very rich dynamics [1,3–6]; in particular, for moderate to strong feedback levels three qualitatively different regimes have been observed [7,8]. For pumping current values close to the solitary laser threshold, the system is stable. As the injection current is increased, the laser become unstable and displays sudden power drops followed by a slower recovery stage. The characteristic rate of such fluctuations is much smaller compared with the typical semiconductor laser rates (carrier and photon lifetime, relaxation oscillations), hence the name low frequency fluctuations (LFF's). At higher injection currents, the laser optical linewidth broadens up to several hundreds of GHz and the so-called coherence collapse (CC) regime settles down.

The theoretical model usually used to describe the dynamics of semiconductor lasers with optical feedback is the one by Lang and Kobayashi [9]. It is obtained considering a laser operating in a single longitudinal mode and weak feedback level. According to this model, an high-dimensional chaotic attractor is at the origin of the drops [10] and therefore the nature of LFF's would be purely deterministic. On the other hand, Vaschenko and collaborators [11] have shown that LFF's are a consequence of the interplay between the time delay and the multimode operation of the laser. These experimental results could not be explained in the frame of a single mode interpretation.

Another approach to the interpretation of the LFF's is based on the recognition of the most important dynamical ingredients at the origin of the instability, i.e., to identify the role of noise and/or the bifurcation type. Following this path, it was recently proposed that semiconductor lasers with optical feedback in the LFF's regime, behave as a noise-driven excitable medium [8]. In this new dynamical scenario, the role of noise is to induce a large deterministic excursion in the phase space.

In Ref. [8] excitability is meant as the possibility of conforming pulses for perturbations above a given threshold. The distributions of the time intervals between drops and their dependency with the experimental parameters (like the feedback and the injection current of the laser) could be explained in terms of a simple model (two-dimensions and two-codimension) displaying pulse conformation for perturbations beyond a threshold [12,13]. Furthermore, recently Giacomelli and collaborators [14] have reported experimental evidence of coherence resonance in this system, enforcing the confidence on this scenario.

The first attempt to demonstrate the excitability behavior has been done periodically forcing the system with pulses of small amplitude [8] . The purpose of this procedure was to show the existence of a threshold. Forcing a system which is able to conform pulses for perturbations beyond a threshold with a periodic signal, results in a (at least) three dimensional one. The global bifurcation structure of this system presents periodic solutions, quasiperiodicity, period doubling, and chaos [15,16]. A complete demonstration of the excitability behavior of the system under study has to consider all this complexity. On the other hand, the semiconductor lasers with external optical feedback display a multimode behavior in the LFF's [11]. This multimode behavior is not being considered in the simplest excitable models, which are conceived to explain only the LFF's. In principle these modes could play an important role in the system under forcing.

In order to explore the range of validity of the simple pulse-conforming scenario, we have analyzed the dynamics of a periodically forced semiconductor laser with optical feedback, prepared in the noise-driven excitability region of its parameter space. The forcing applied was sinusoidal. Stable patterns in the time series have been obtained under this kind of forcing. We have separated periodic orbits from the experimental time series and calculated their topological numbers, like the self-linking number and the self-relative rotation rates [17–19]. Finally, we have compared the experimental results with the model. The purpose of this work is to build confidence or refuse the new scenario, at least in the accessible range of parameters. This paper is organized as follows. Section II describes the experimental setup and the measurements. Section III gives an interpretation of the results. Section IV contains the comparison between the experimental results and the model. And finally, in Sec. V, conclusions of the work are given.

II. EXPERIMENT

The experimental setup is shown in Fig. 1. It is very similar to the one used by Yacomotti *et al.* [13]. The semicon-

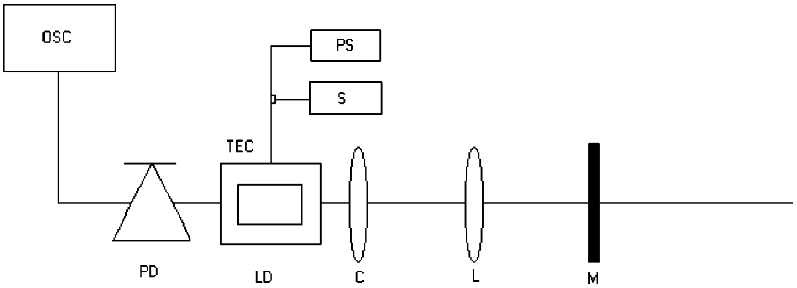

FIG. 1. Experimental setup: LD: laser diode; PD: photodiode; OSC: oscilloscope; C: collimator; L: lens; M: mirror; TEC: thermoelectric cooler; PS: power source; S: wave form generator.

ductor laser used is a single transverse mode Sharp LTO30MD/MF emitting at a nominal wavelength of 750 nm; its current threshold has been measured to be at 36.66 mA. In our experiment the laser has been thermally stabilized up to 0.01 °C. An high reflectivity mirror ($>90\%$) is placed in front of the laser edge, at a distance of 130 cm, in order to return into the laser cavity part of the light emitted. A collimator and an antireflection-coated lens are placed into the cavity in order to reduce the beam divergence and to mode match the returned beam with the emitted beam. The intensity output is detected by a 1-GHz bandwidth photodiode and the signal is analyzed with a HP54616B 500-MHz oscilloscope. The system has been prepared in the region of parameters where it behaves as an excitable system; the pumping current has been set close the solitary laser threshold value and the feedback level was moderately strong, gauged by a threshold reduction of 5–10 %. In this condition the laser output displays noise-induced LFF's characterized by a recovery stage of approximately 250 ns. From the work of Yacomotti *et al.* [13], the probability distributions of time intervals between drops can be used to specify accurately the point of the parameter space at which we have been operating. We have prepared the system in such a way to obtain a monomodal distribution of times between dropouts events. The distribution has a most probable time of 0.85 μs and an average value of 0.9 μs. According to the model presented in Ref. [13], this situation corresponds to have prepared the system in a state far from the saddle-loop global bifurcation. This prevents the inevitable noise present in the system to anticipate this bifurcation and therefore to affect noticeably the dynamics of the system.

We have applied an electrical sinusoidal signal into the semiconductor laser via the pumping current. The forcing has been generated by a HP 33021A wave form generator and overlapped to the dc biasing current of the laser through a bias *T*. We have analyzed the response of the laser as the amplitude and frequency of the forcing current is changed.

We have obtained an interesting behavior of the semiconductor laser for periods ranging from 140 ns to 1 μs and for amplitudes from 200 to 500 mVpp. It appears that the dynamics of the system is more affected by changes in the period of the modulation than by variations of the amplitude. In Fig. 2 we report the behavior of the system when the modulation amplitude is fixed at 210 mVpp and the period is changed. For forcing period between 1 μs and 710 ns, the response of the system seems to be phase-locked with rotation number 1:1. In other words, the time series is periodic with the period of the forcing and presents one dropout for every period of the forcing time. We define a generic behavior $q{:}p$ as the periodical pattern in which the time-series have q dropouts every p periods of the forcing time. For periods between 670 and 410 ns the time series presents coexistence of two different behaviors. In some time windows the behavior is 1:1 but, in other windows, it changes to another periodic pattern, represented by one dropout for every two periods of the forcing (1:2). When the period is between 400 and 330 ns the system seems to be perfectly locked, with rotation number of 1:2. The time series has a periodic pattern with one drop event every two periods of the forcing. At lower forcing periods, the system jumps mainly between three different periodicities: 1:2, 1:3, and 1:4. Furthermore, we have identified a 2:3 periodic pattern at forcing periods between 590 and 530 ns. The main effect of increasing the forcing amplitude is to decrease the value of the forcing period at which the behavior with low q dominates. For example, when the system is forced with amplitude of 400 mVpp, the 1:1 regime appears already at a forcing period of 600 ns. In Fig. 3, we show a map of the dynamical behaviors experimentally observed in the system for different forcing amplitude A and forcing periods T.

III. INTERPRETATION

It is a typical strategy in nonlinear dynamics to find the simplest equations compatible with the dynamical scenario we want to describe. A simple model able to describe pulse conformation for perturbations beyond a threshold has been proposed by Eguia *et al.* [12]. Also, they have demonstrated that it gives a good statistical description of the times between dropouts events.

In order to compare the model with the experimental results, the periodically forcing is added as shown:

$$x' = y, \tag{3.1}$$

$$y' = x - y - x^3 + xy + \epsilon_1 + \epsilon_2 x^2 + A\cos(\omega t), \tag{3.2}$$

with $(x,y) \in R^2$ and $\epsilon_1, \epsilon_2 \in R^+$, A is the forcing amplitude, and ω the forcing frequency. The comparison between the parameters dependence of the real system with the dependence on ϵ_1 and ϵ_2 of the model allows for identifying ϵ_1 with the laser pumping current and ϵ_2 with the feedback level [13]. Accordingly, the forcing term has been written in the model as a term added to ϵ_1. Without the forcing, the proposed system presents four qualitatively different behaviors. In Fig. 4, we display the mapping of the different behaviors in the parameter space (ϵ_2, ϵ_1). The organizations of the invariant manifolds are also given: we note that, in region

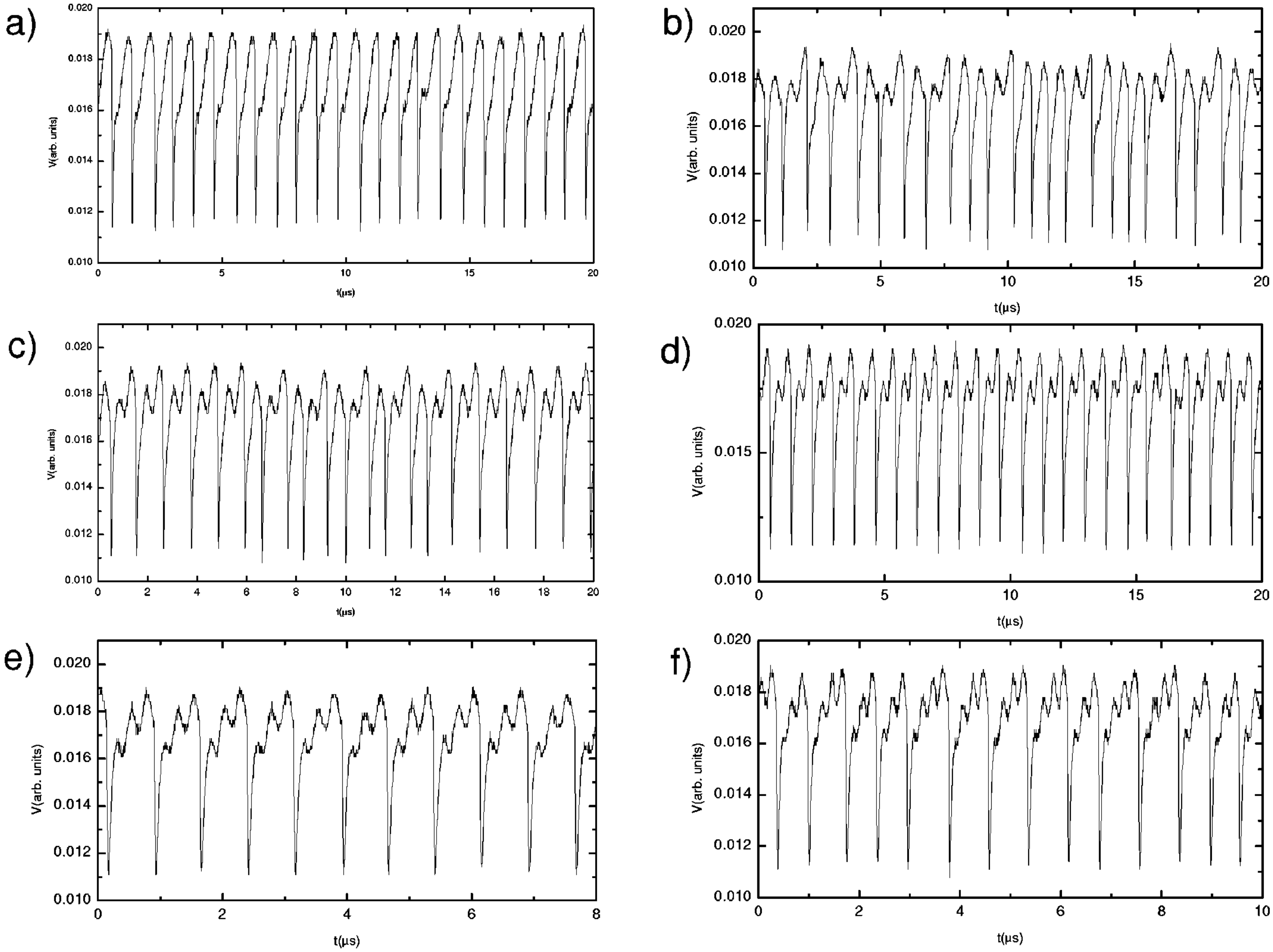

FIG. 2. Time series corresponding to every one of the experimentally observed behaviors. (a) 1:1 pattern. (b) Intermittencies between the 1:1 and the 1:2 patterns. (c) 2:3 pattern. (d) 1:2 pattern. (e) 1:3 pattern. (f) Intermittencies between the 1:3 and the 1:4 pattern.

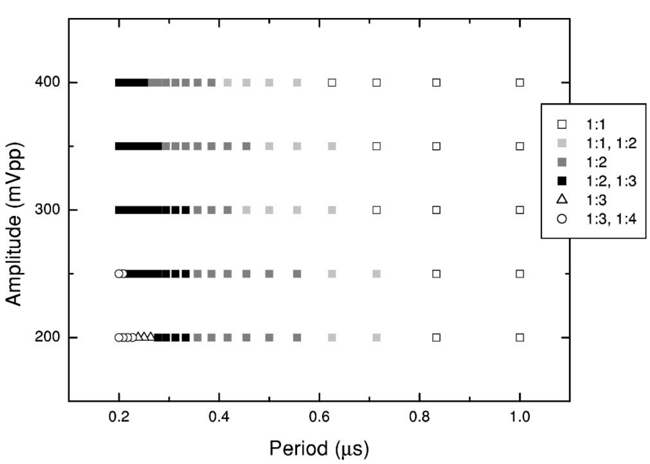

FIG. 3. The experimental map of the regions in which the different periodical patterns have been observed. The amplitude is in unit of millivolts peak to peak (mVpp).

II, the system exhibits the dynamics we have observed experimentally without applying the forcing.

As already pointed out by Feingold *et al.* [15], when an excitable system is periodically forced, the solutions are very similar to the ones of a periodically forced oscillator. The bifurcation diagram can be represented by mapping the re-

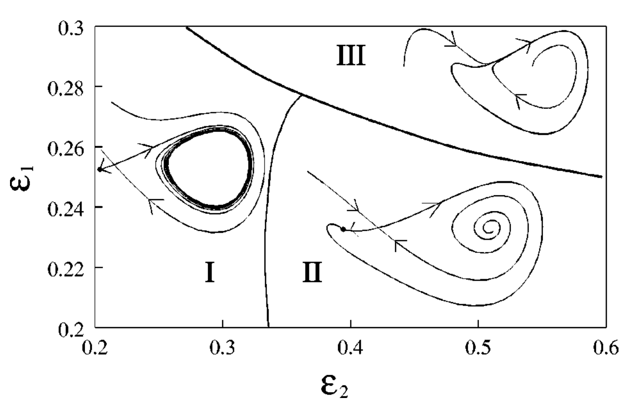

FIG. 4. Bifurcation diagram and phase portrait for Eqs. (3.1) and (3.2) (without the periodical forcing). Figure from Ref. [13].

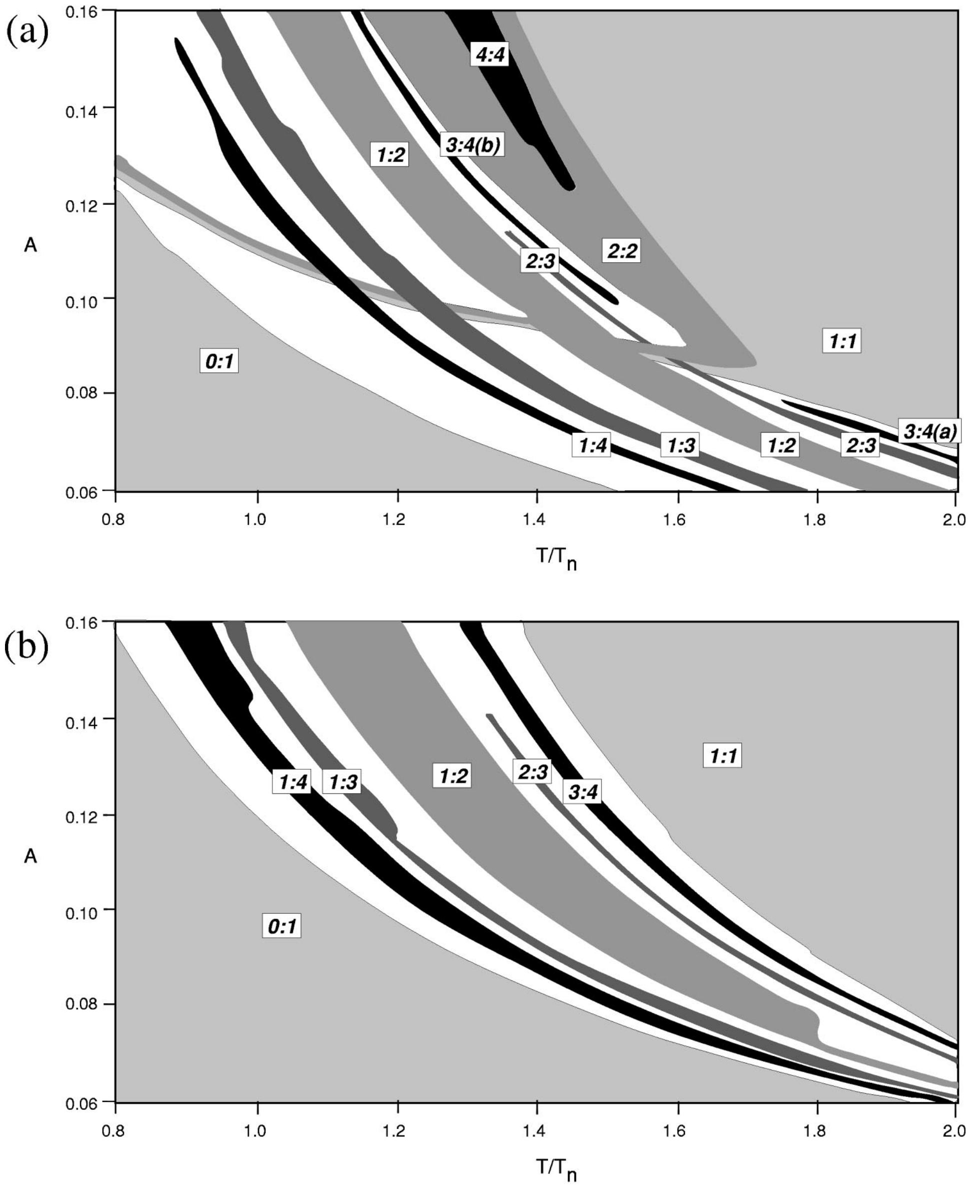

FIG. 5. Regions of the (A,T) plane where the periodic orbits with rotation number q/p are stable. (a) $\epsilon_1=0.25$ and $\epsilon_2=0.50$, (b) $\epsilon_1=0.21$ and $\epsilon_2=0.70$. $T_n=10.0$ in arbitrary units.

gions in the parameter space (A,T) where the orbits with rotation number q/p are stable (Arnold tongues). A periodic orbit of rotation number q/p is meant to be a periodic orbit with q spikes every p multiples of the forcing period. Changing the parameters ϵ_1 and ϵ_2 within region II, the bifurcation diagram presents, essentially, the same regions. In Fig. 5, we show the bifurcation diagram for two different points in the parameter space (ϵ_1,ϵ_2), but always located in the region II of Fig. 4. As has already been said, ϵ_2 could be related with the feedback level. For parameter values within region II, but not close to the homoclinic bifurcation, the system displays a dynamics similar to the one observed experimentally. We have extracted for every one of the important phase-locking regions, a deterministic attracting periodic orbit, to be compared with the experiment. How could deterministic information be extracted from the noisy experimental time series? We conjecture that the dominant segments of high recurrence in the noisy time series are the fingerprints of deterministic periodic orbits of the forced system. For example, in Figs. 2(a) and (c), the 1:1 and the 1:2 orbits could be recognized. Every one of the identified periodic orbits corresponds to different resonances of the forced deterministic system. Considering this hypothesis, the q:p periodic patterns experimentally observed could be identified with the resonances with rotation number q/p. Consequently, we have chosen the rotation number of the dominant segments of the time series to characterize the behavior of the system. So, the determin-

TABLE I. Topological invariant numbers for the experimental periodic orbits.

Orbits	Period	SRRR	SLN
1:1	1	0	0
2:3	3	$(-2/3)^2,0$	-4
1:2	2	$(-1/2),0$	-1
1:3	3	$(-1/3)^2,0$	-2
1:4	4	$(-1/4)^3,0$	-3

TABLE II. Topological invariant numbers for the periodic orbits from the model.

Orbits	Period	SRRR	SLN
1/1	1	0	0
2/3	3	$(-2/3)^2$,0	-4
1/2	2	$(-1/2)$,0	-1
1/3	3	$(-1/3)^2$,0	-2
1/4	4	$(-1/4)^3$,0	-3

istic skeleton may be constructed with the q:p periodic patterns experimentally found. There are points in the experimental phase portrait where the system is not exactly in only one resonance behavior. Looking at the time series corresponding to these points, the system seems to be jumping between different resonances. For example in Fig. 2(c) the periodic orbit 2:3 can be clearly recognized, but also the system jumping between the 1:2 and the 2:3 resonances can be seen. We claim that noise is responsible for these kind of intermittencies. In general, for parameters values such that

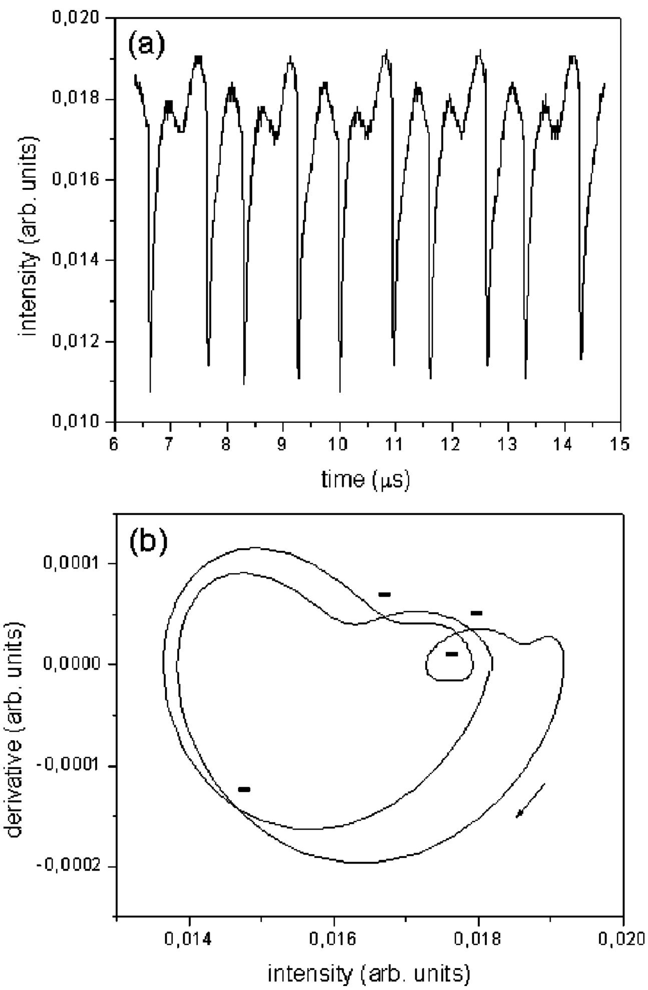

FIG. 6. (a) Experimental time series with the 2:3 pattern. (b) Embedding of the corresponding time series. The mathematical symbols display the orientation of the crossing. The topological numbers were computed in the (x,t) projection.

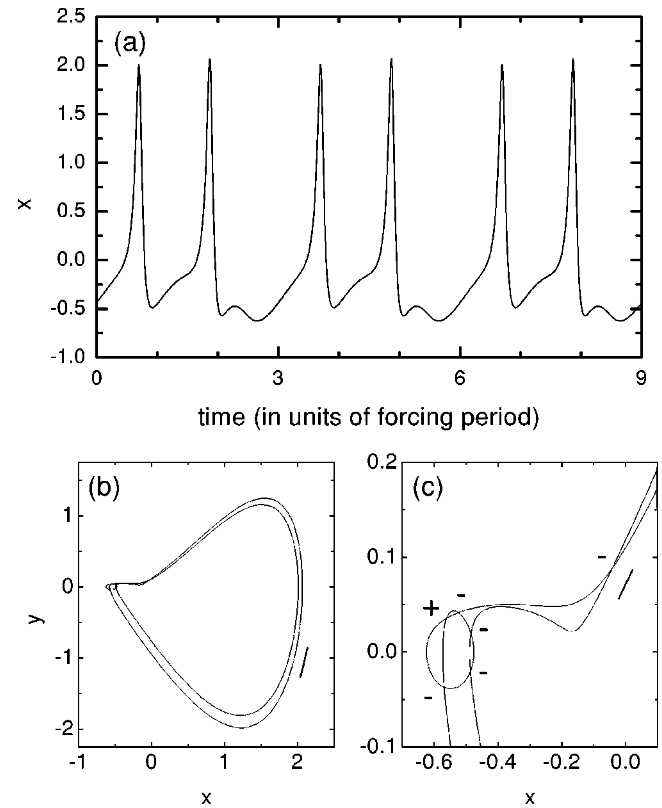

FIG. 7. (a) Time series displaying the orbit with rotation number equal to 2/3. (b) Embedding of the corresponding orbit. (c) Enlargement of the bad resolve zone of (b). The mathematical symbols display the orientation of the crossing. Also in the theoretical case, the actual computation of topological numbers was performed in (x,t) space.

the system is at the edge of an Arnold tongue, a small amount of noise may unlock the system. By consequence, the time series presents only segments of phase-locked behavior. Within a tongue the system might also unlock, although less frequently. These intuitive arguments are confirmed by the numerical simulations: we have observed that the jumping among different phase-locking behaviors occurs more frequently for parameter values located close to the edge of an Arnold tongue. In conclusion, taking into account the effect of noise, the bifurcation structure of the experiment is consistent with the theoretical predictions of the model.

IV. COMPARISON BETWEEN THE EXPERIMENTAL DATA AND THE SOLUTIONS OF THE MODEL

Experimental periodic orbits have been extracted from the time series by the method of close returns [18,19]. We have looked for periodic orbits at multiples of the forcing period. The practical way to find close return segments is to color code the distances $d_{i,p}$ between the points of the time series x_i and x_p. If the distance is smaller than a given ϵ, a black point is plotted at (i,p). In such a plot, a periodic recurrence originates horizontal lines. In this way the segments have been identified. Using this method, we have separated the following periodic orbits: 1:1, 1:2, 1:3, 2:3, and 1:4. In order to use a topological description of the periodic orbits, a three dimensional embedding is needed. We have used the convenient embedding (x,x',t). The time series was filtered using an adjacent averaging. In spite of the noise present, two topological invariant numbers have successfully been calculated. We have calculated the self-relative rotation rates (SRRR) and the self-linking number (SLN) for every one of the orbits. In Table I the results are shown. We have controlled the stability of the result, calculating the topological numbers for five embedded orbits extracted from different segments. In Table II the results of SRRR and SLN for the most relevant periodic orbits of the model are shown. In Fig. 6, we display the 2:3 pattern and the corresponding embedding for the experimental measurements whereas in Fig. 7 the results for the simulation are shown. The topological organization of the periodic orbits extracted from the experiment and from the model is equivalent.

V. CONCLUSIONS

In this work, we have studied the dynamics of the periodically forced semiconductor laser with optical feedback. We conjectured the system as a noise driven one able to conform pulses for perturbations beyond a threshold [12,13], and we compared topologically the observed solutions. Experimentally, we have analyzed the semiconductor laser with optical feedback forced by a sinusoidal signal. Considering the periodic patterns given by the system under the forcing, as the fingerprints of periodic orbits of the forced deterministic system, we have compared the experimental results with the solutions of the simple model proposed by Eguia *et al.* [12]. We have shown that the topological organization of the periodic orbits experimentally identified is equivalent, within the parameter region explored, to the one of the proposed scenario.

ACKNOWLEDGMENTS

This work was partially funded by Fundación Antorchas, UBA, CONICET, and Fomec.

[1] J. Sacher, W. Elsässer, and E. O. Göbel, Phys. Rev. Lett. **63**, 2224 (1989).

[2] J. Risch and C. Voumard, J. Appl. Phys. **48**, 2083 (1977).

[3] T. Heil, I. Fischer, and W. Elsaber, Phys. Rev. A **58**, R2672 (1998).

[4] R. W. Tkach and A. R. Chraplyvy, and J. Lightwave, J. Lightwave Technol. **LT-4**, 1655 (1986).

[5] D. Lenstra, B. Verbeek, and A.J. den Boef, IEEE J. Quantum Electron. **QE-21**, 674 (1985).

[6] J. Mork, J. Mark, and B. Tromborg, IEEE J. Quantum Electron. **28**, 93 (1992).

[7] P. Besnard, B. Meziane, and G. M. Stéphan, IEEE J. Quantum Electron. **29**, 1271 (1993).

[8] M. Giudici, C. Green, G. Giacomelli, U. Nespolo, and J. R. Tredicce, Phys. Rev. E **55**, 6414 (1997).

[9] R. Lang and K. Kobayashi, IEEE J. Quantum Electron. **QE-16**, 347 (1980).

[10] G. H. M. Van Tartwijk, A. M. Levine, and D. Lenstra, IEEE J. Sel. Top. Quantum Electron. **1**, 466 (1995); I. Fischer, G. H. M. Van Tartwijk, A. M. Levine, W. Elsasser, E. O. Gobel, and D. Lenstra, Phys. Rev. Lett. **76**, 220 (1996).

[11] G. Vaschenko, M. Giudici, J. J. Rocca, C. S. Menoni, J. R. Tredicce, and S. Balle, Phys. Rev. Lett. **81**, 5536 (1998).

[12] M. C. Eguia, G. B. Mindlin, and M. Giudici, Phys. Rev. E **58**, 2636 (1998).

[13] A. M. Yacomotti, M. C. Eguia, J. Aliaga, O. E. Martinez, G. B. Mindlin, and A. Lipsich, Phys. Rev. Lett. **83**, 292 (1999).

[14] G. Giacomelli, M. Giudici, S. Balle, and J. R. Tredicce, Phys. Rev. Lett. **84**, 3298 (2000).

[15] M. Feingold, D. L. Gonzalez, O. Piro, and H. Viturro, Phys. Rev. A **37**, 4060 (1988).

[16] J. C. Alexander, E. J. Doedel, and H. G. Othmer, SIAM (Soc. Ind. Appl. Math.) J. Appl. Math. **50**, 1373 (1990).

[17] H. G. Solari, M. A. Natiello, and G. B. Mindlin, *Nonlinear Dynamics* (IOP Publishing, London, 1996).

[18] G. B. Mindlin, H. G. Solari, M. A. Natiello, R. Gilmore, and X. Hou, J. Nonlinear Sci. **1**, 147 (1991).

[19] G. B. Mindlin and R. Gilmore, Physica D **58**, 229 (1992).