Effects of random potential on transport

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The effects of random potential on the transport of two systems, which are the motion of motor proteins along a biopolymer and the thermally assisted vortex diffusion in layered high-T_c superconductors, are investigated, respectively. It is found that the effects of the random potential on the transport process as the amplitude of random potential increased are much more remarkable than those as the correlation length of random potential increased. The amplitude and the correlation length of random potential play opposing roles in the transport of the systems.

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I. INTRODUCTION

The effects of noise on a dynamical system has been studied extensively in statics and dynamics. In statics, the noise-induced transition has been investigated in the context of nonequilibrium phenomena [1]. In dynamics, the noise-induced transport has been of growing interest. A number of recent attempts to understand broad principles of energy transduction in nonequilibrium physical and biological systems have focused on correlation ratchet systems which extract work out of fluctuations which are correlated in time [2–8]. It has been demonstrated that time correlated fluctuations interacting with spatial asymmetry are sufficient conditions to give rise to transport [2]. On the other hand, it was also shown that temporal asymmetric driving (with zero mean) can operate a correlation ratchet even when the potential has spatial symmetry [6].

The largest amount of work about the noise has been referred to the consideration of fluctuations depending on the time, very little work has been done on fluctuations depending on the state variable. Recently, the fluctuations depending on the state variable have been proposed by Dunlap and co-workers [10–12]. They had studied nonlinear mobility of a classical particle of charge q and mass m moving in an infinite one-dimensional space spanned by the coordinate x, and subjected to a potential $U(x)$ with the period L and an external electric field E, where the potential $U(x)$ is a random stationary potential which depends on the state variable. The mobility, defined as the ratio of the velocity of the charge to the field E, is given simply in terms of a finite-space correlation function [11]: $C(L,y)=\overline{\exp[U(x+y)/kT]\exp[-U(x)/kT]}$, where the overbar represents an ensemble average over realizations of the random potential $U(x)$. The assumed stationarity of the stochastic process underlying the potential ensures that the finite-space correlation function depends only on the difference y in the coordinate values [12].

According to Ref. [11], $U(x)=U_0(x)+\eta(x)$, where $U_0(x)$ is the deterministic part with the period L and $\eta(x)$ is a small fluctuation depending on the state variable and superposed on the deterministic part $U_0(x)$. Considering that the random part $\eta(x)$ takes only two values separated by 2Δ, making discontinuous jumps at random points along the one-dimensional space, and $\eta(x)=\Delta(-1)^{n(x,0)}$, where the randomness of the function $\eta(x)$ has been expressed in terms of the random function $n(x_2,x_1)$, which counts the number of jumps the potential makes between the values $+\Delta$ and $-\Delta$ in the interval between $x=x_1$ and $x=x_2$. The mean of the random function $n(x_2,x_1)$ is $\overline{n(x_2,x_1)}=|x_2-x_1|/l$, where the correlation length l is the mean distance between jumps. The probability distribution of $n(x,0)$ is Poissonian. Above properties of $n(x_2,x_1)$ allow a straightforward calculation of the correlation function for the $\eta(x)$ [13]: $\overline{\eta(x_1)\eta(x_2)}=\Delta^2\exp(-2|x_1-x_2|/l)$ and $\overline{\eta(x)}=0$, then $\eta(x)$ is called the Dichotomous (Di) potential. Therefore, the finite-space correlation function is

$$C(L,y)=\left[\cosh^2\left(\frac{\Delta}{kT}\right)-\exp\left(-\frac{2y}{l}\right)\sinh^2\left(\frac{\Delta}{kT}\right)\right]\times\int_0^L\exp\left[-\frac{U_0(x)-U_0(x+y)}{kT}\right]dx. \quad (1)$$

Now a question to be raised is how the random potentials influence the transport in some realistic systems, for instance, the motion of motor proteins along a biopolymer [4] and the thermally assisted vortex diffusion in layered high-T_c superconductors [14]. In this Brief Report, we will study the effects of Di potentials on transport of the two realistic systems. It should be pointed out that although the effects of random potential was considered in Ref. [4], yet we will consider both fluctuating force and fluctuating potential simultaneously here. Moreover, the effects of random potential was not discussed in Ref. [14].

II. THE MOTOR PROTEINS

For the motor proteins, Astumian and Bier [4] proposed a motion model of the motor proteins in a periodic piecewise

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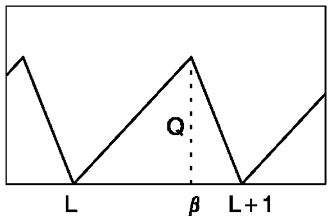

FIG. 1. The periodic piecewise ratchet potential.

linear potential, predictions of which are consistent with the experimental data given by Svoboda *et al.* [9] for a single protein molecule moving along a biopolymer. In an overdamped environment and after scaling the viscosity away, the motion model of protein molecule can be described by the following Langevin equation in the case of dimensionless form $dx/dt=-\partial U(x,t)/\partial x+\xi(t)$, where $\xi(t)$ represents the Gaussian white noise $\langle \xi(t)\rangle=0$ and $\langle \xi(t)\xi(s)\rangle=2kT\delta(t-s)$.

The potential $U(x,t)$ undergoes a fluctuation, two cases had been respectively discussed in Ref. [4]. One is fluctuating force $\partial U(x,t)/\partial x=\partial U(x,t)/\partial x+F(t)$, and the net force $F(t)$ fluctuates between $+\Delta F$ and $-\Delta F$ in its period $\mathcal{T}$. The other is fluctuating barrier $\partial U(x,t)/\partial x=\partial[U(x,t)+u(x,t)]/\partial x$, and $u(x,t)$ can take the values $+\Delta u$ and $-\Delta u$. However, here we consider the two cases simultaneously: $\partial U(x,t)/\partial x=\partial[U_0(x)+\eta(x)]/\partial x-F(t)$, where $U_0(x)$ is a periodic piecewise ratchet potential with the barrier height Q, the period L, and the parameter β (see Fig. 1). $\eta(x)$ is a Di potential, and the term $F(t)$ is a slow forcing of square wave of amplitude A with the period $\mathcal{T}$. In order to get fluctuation induced flow it is essential that one side is steeper that the other, i.e., that the parameter $\beta\neq L/2$. Here we take $\beta>L/2$, then the probability flux J of the system driven by the Di potential is given by

$$J(F)=\frac{kT[1-\exp(-FL/kT)]}{u_1+u_2+u_3+u_4}, \tag{2}$$

where

$$u_1=\beta\cosh^2\left(\frac{\Delta}{kT}\right)\left(\frac{Q}{kT\beta}-\frac{F}{kT}\right)^{-1}\times\left\{\exp\left[-\left(\frac{F}{kT}-\frac{Q}{kT\beta}\right)L\right]-1\right\}, \tag{3}$$

$$u_2=-\beta\sinh^2\left(\frac{\Delta}{kT}\right)\left(-\frac{F}{kT}-\frac{2}{l}+\frac{Q}{kT\beta}\right)^{-1}\times\left\{\exp\left[-\left(\frac{F}{kT}+\frac{2}{l}-\frac{Q}{kT\beta}\right)L\right]-1\right\}, \tag{4}$$

$$u_3=-(L-\beta)\cosh^2\left(\frac{\Delta}{kT}\right)\left(\frac{Q}{kT(L-\beta)}+\frac{F}{kT}\right)^{-1}\times\left\{\exp\left[-\left(\frac{F}{kT}+\frac{Q}{kT(L-\beta)}\right)L\right]-1\right\}, \tag{5}$$

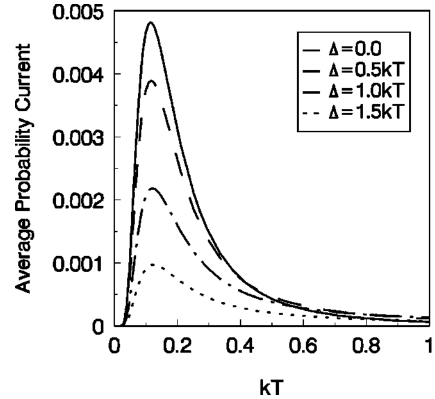

FIG. 2. The average probability current for Di potential vs kT for various amplitude Δ of the Di potential. $A=0.1$, $L=1$, $\beta=0.6$, $Q=0.2$, and $l=0.2L$.

$$u_4=(L-\beta)\sinh^2\left(\frac{\Delta}{kT}\right)\left(\frac{F}{kT}+\frac{2}{l}+\frac{Q}{kT(L-\beta)}\right)^{-1}\times\left\{\exp\left[-\left(\frac{F}{kT}+\frac{2}{l}+\frac{Q}{kT(L-\beta)}\right)L\right]-1\right\}. \tag{6}$$

For the slow fluctuation $F(t)$ of square wave of amplitude A [2], the average probability current $J_{\rm av}$ over the period $\mathcal{T}$ of the fluctuation $J_{\rm av}=(1/\mathcal{T})\int_0^{\mathcal{T}}J[F(t)]dt=(1/2)[J(A)+J(-A)]$ can be obtained.

We plot the average probability current versus kT for different values of amplitude Δ and correlation length l of the Di potential in Figs. 2 and 3, respectively. The figures show that the average probability current is a peaked function of temperature of the bath, thus there is an optimal temperature for the driving. In addition, the effects of Di potential on the average probability current are very clearly in the figures, the peak value decreases as the amplitude Δ of Di potential is increased (Fig. 2), while the peak value increases as the correlation length l of Di potential is increased (Fig. 3). The variation of the average probability current as increasing of the amplitude Δ of Di potential is much more remarkable than that as increasing the correlation length l of Di potential.

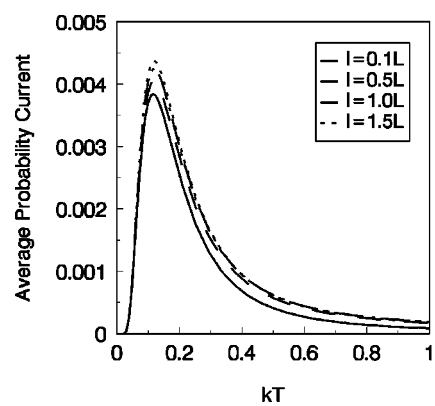

FIG. 3. The average probability current for Di potential vs kT for various correlation length l of the Di potential. $A=0.1$, $L=1$, $\beta=0.6$, $Q=0.2$, and $\Delta=0.5kT$.

III. THE VORTEX DIFFUSION IN HIGH-T_c SUPERCONDUCTORS

For the vortex diffusion in high-T_c superconductors, Chen and Dong [14] used a dynamical equation to study the single vortex thermally activated motion in the direction perpendicular to the layers as seem to a Brownian particle moving in a sinusoidal pinning potential, and they showed that the power law *I-V* characteristics and continuous crossover from flux creep to flux flow in high-T_c superconductors can be interpreted in a natural way. Under the overdamped situation (or the effective mass is very small), the Langevin equation for a flux line of length a in the dimensionless form [14] is $(a\mu)dx/dt=-U'(x)+F+\xi(t)$, where the pinning potential $U(x)$ is caused by the parallel Cu-O planes (ab planes), M is effective mass of the flux line, μ is damping constant and $\xi(t)$ represents the thermally fluctuating force $\langle\xi(t)\rangle=0$ and $\langle\xi(t)\xi(s)\rangle=[2kT/(a\mu)]\delta(t-s)$, $F=(1/c)i\phi_0 a$ is the driving force with i being electric current density, and ϕ_0 as the superconducting flux quantum $hc/2e$. Here the x axis is taken along the normal of the planes.

When one assumes that the pinning potential $U(x)$ is a random potential $U(x)\rightarrow U_p(x)+\eta(x)$, where $U_p(x)=(\alpha/2)\sin(2\pi x/L)$ [15], which is a good approximation to the intrinsic pinning caused by the layered structure when the magnetic field **H** is parallel to the layers, α is the height of the pinning potential well, L is the period of the distance between two planes, and the term $\eta(x)$ is considered as Di potential. Now the current J for the vortex diffusion in high-T_c superconductors is [14]

$$J=\frac{kT[1-\exp(-FL/kT)]}{a\mu\int_0^L \exp(-Fy/kT)C(L,y)dy}, \tag{7}$$

where $C(L,y)=L[\cosh^2(\Delta/kT)-\exp(-2y/l)\sinh^2(\Delta/kT)]I_0[\alpha\sin(\pi y/L)/kT]$, and $I_0(x)$ is the modified Bessel function. Due to the motion of flux lines, an induction electric field E can be produced: $E=(B/c)\langle\dot{x}\rangle$. Note that the average velocity $\langle\dot{x}\rangle$ is related to the current J according to $\langle\dot{x}\rangle=LJ$, and the flux-flow resistivity ρ_0 is $\rho_0=B\phi_0/\mu c^2$ when there is no pinning. A simple formula of ρ_0 is given by Bardeen and Stephen [16]: $\rho_0=\rho_n H/H_{c_2}$ with ρ_n being the normal state resistivity and H_{c_2} being the upper critical field (below H_c represents the lower critical field). If we introduce $E_0=ckT_c\rho_n/\phi_0 aL$, $i_0=ckT_c/\phi_0 aL$, and a reduced temperature $\tau=T/T_c$, then from Eq. (7) we have

$$\frac{E}{E_0}=\frac{\tau L^2 H[1-\exp(-i/i_0\tau)]}{H_{c_2}\int_0^L \exp(-iy/i_0 L\tau)C(L,y)dy}. \tag{8}$$

One can chooses the form of the pinning intensity α [17] as $\alpha=(H_c^2/8\pi f)\xi_c\xi_{ab}(\phi_0/B)^{1/2}$, where f is a numeric constant and is approximately 6 [18], ξ_{ab} and ξ_c are the correlation lengths in ab plane and perpendicular to the plane, respectively, and are proportional to $(T_c-T)^{-1/2}$.

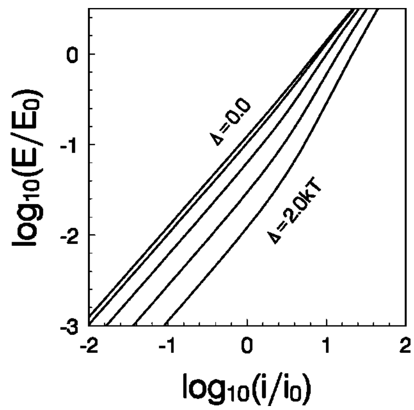

FIG. 4. Calculated electric field vs current density for various amplitudes of the Di potential, from $\Delta=0.0kT$ to $\Delta=2.0kT$ at $0.5kT$ intervals. The reduced temperature $\tau=0.99$ and the correlation length of the Di potential $l=0.5L$. The other parameter values are given by Ref. [14] (the same in Fig. 5).

By virtue of Eq. (8), we can discuss the effects of random potentials on the linear $\log_{10}(E/E_0)-\log_{10}(i/i_0)$ characteristic. The linear characteristic has been plotted for various values of the amplitudes Δ and the correlation lengths l of the Di potential in Figs. 4, 5. When correlation length l of random potential is fixed, the variation of the linear characteristic (see Fig. 4) for increasing the amplitude Δ of random potential is equivalent to that in Ref. [14] for decreasing the reduced temperature (or for decreasing the temperature of the system). It means that increasing the amplitude of random potential would make decreasing the temperature of system, or make the system far away from the transition temperature. On the other hand, when the amplitude of random potential is fixed, the variation of the linear characteristic (see Fig. 5) for increasing the correlation length l of random potential is opposite to that in Ref. [14] for decreasing the reduced temperature (or for decreasing the temperature of the system). It means that increasing the correlation length l of random potential would make increasing the temperature of system, or make the system closing the transition temperature.

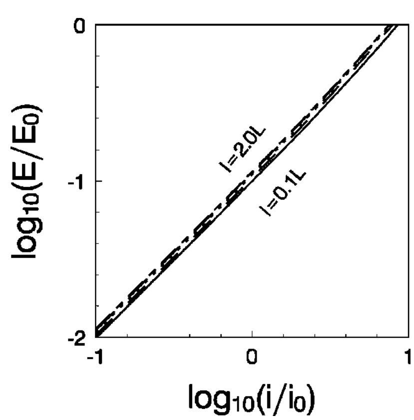

FIG. 5. Calculated electric field vs current density for various correlation lengths of the Di potential: (from right to left) $l=0.1L$, $l=0.5L$, $1.0L$, $1.5L$, and $2.0L$. The reduced temperature $\tau=0.99$ and the amplitude of the Di potential $\Delta=0.5kT$.

In summary, the effects of Di potential on the transport of motor proteins [4] and vortex diffusion in superconductors [14] have been discussed by using of the definition of stochastic potential in Ref. [11]. It has been shown that the amplitude and the correlation length of Di potential play opposing roles in the transport of the nonlinear systems. It should be pointed out that the random potential could be the Ornstein-Uhlenbeck (OU) process which is defined by the infinite sum of Di potentials [11], one can show that the effects of the OU potential on the transport are same as these of Di potential.

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