Dichromatic nonlinear eigenmodes in slab waveguide with $\chi^{(2)}$ nonlinearity

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The existence of purely nonlinear eigenmodes in a waveguiding structure composed of a slab with quadratic nonlinearity surrounded by (non)linear claddings is reported. Modes having bright and dark solitonlike shapes and consisting of two mutually locked harmonics are identified. Asymmetrical modes are shown to exist in symmetrical environments. Constraints for the existence of the modes are derived in terms of parameters of guiding structure materials.

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In the past several years, much research has been performed on the dynamics of nonlinear waves in media with quadratic, or $\chi^{(2)}$, nonlinearity. The interest in these investigations is stimulated by their high potential for such applications as all-optical signal processing, amplification, etc. (see, e.g., [1–4] and references therein). From a fundamental point of view, the discovery of the solitonic regimes of beam/pulse propagation in $\chi^{(2)}$ media is of great significance. The existence of various soliton families in such an environment was first theoretically predicted in [5–9]. Subsequently, quadratic solitons were experimentally observed both in bulk media and in film waveguides [10,11].

The great interest that is always accorded to wave-guiding structures is due to the possibility of high concentrations of optical power in them and, as a consequence, an easier observation of nonlinear effects. Such investigations were mostly performed in the waveguides with cubic nonlinearity. In particular, analytical theories describing nonlinear modes with different symmetries supported by guiding structures were developed [12–18]. Apart from this, some interesting nonlinear effects, including the existence of nonlinear impurity modes [19] as well as surface modes in media with inversion symmetry [20,21], were studied in guided structures with quadratic nonlinearity [3,16,22–24]. Recently, it was shown that quadratic waveguides are able to support two-component eigenmodes, which are stationary modes without energy exchange between composing harmonics [25]. In bulk $\chi^{(2)}$ media, such modes were studied in [26] for the cw case. The eigenmodes found in [25] have constituted nonlinearly perturbed linear modes and therefore they have trigonometric form. These modes transform into pure linear ones in the case of vanishing nonlinearity.

In this paper, we aim to demonstrate the existence and study the structure of pure nonlinear eigenmodes in waveguides with cascaded $\chi^{(2)}$ nonlinearity. These modes being of genuine nonlinear nature have no counterparts in the linear limit and represent new families of dichromatic eigenstates of nonlinear waveguides. In contrast to modes studied in [25], they have solitonlike form.

We consider a structure consisting of a nonlinear dielectric film of width l disposed between two semi-infinite (non)linear dielectric media. The constituent media are characterized by dielectric constants ε_i and quadratic nonlinear coefficients $\chi_i^{(2)}=\chi_i$, where $i=1,2,3$ for $y>l$, $l>y>0$, and $y<0$, respectively. When considering some of the media as linear, we will set the corresponding nonlinear coefficient χ_i equal to zero. It should be noted that in general it is impossible to split the field into noninteracting TE and TM modes. Nevertheless, depending on an associated class of symmetry, some crystals can support TE or TM modes [9]. In [9], one can also find a detailed description of the corresponding materials and their symmetry properties. In what follows, we consider the scalar interaction of TE polarized fundamental and second-harmonic guided waves copropagating along the x axis, which is assumed to coincide with the optics axis of the waveguide material.

Looking for stationary harmonic solutions, we substitute the electric field in the form of a sum of the fundamental waves (FW) and the second harmonic (SH) waves,

$$E=A(y)e^{iq_1x-i\omega t}+B(y)e^{iq_2x-2i\omega t}+\text{c.c.}, \tag{1}$$

into the Maxwell equations. After that, we arrive at a system of two equations for the harmonics amplitudes, which is solvable provided that the condition $q_2=2q_1\equiv 2q$ is satisfied. Thus the set of equations for the amplitudes A and B takes the following form:

$$\begin{aligned}&\frac{\partial^2 A}{\partial y^2}+[\varepsilon_i(\omega)-q^2]A+2\chi_i A^*B=0,\\ &\frac{\partial^2 B}{\partial y^2}+4[\varepsilon_i(2\omega)-q^2]B+4\chi_i A^2=0.\end{aligned} \tag{2}$$

Here we have introduced dimensionless variables $y\rightarrow k_0 y$ and $q\rightarrow q/k_0$, where $k_0=\omega/c$ is the wave vector of the FW in vacuum.

We consider two cases, namely nonlinear film surrounded (i) by two linear media ($\chi_1=\chi_3=0$) and (ii) by two nonlinear media. For case (i), in [25] the existence of eigenmodes with profile approximately described by trigonometric functions was shown. As was mentioned above, these modes transform into linear waveguide modes in the limit of vanishing nonlinearity. Along with these approximate trigonometric solutions, the set (2) possesses different families of solitonlike solutions [5–9]. In order to treat the problem analytically, we consider here solutions having an exact analytical form. One can easily check by direct substitution that the functions

$$A=a\,\mathrm{sech}^2[\eta(y-y_0)],\quad B=b\,\mathrm{sech}^2[\eta(y-y_0)],\tag{3a}$$

where

$$a=\pm b/\sqrt{2},\quad b=3\eta^2/\chi_i,\quad \eta^2=-\Delta\varepsilon_i/3>0,$$
$$q^2=\varepsilon_i(2\omega)-\Delta\varepsilon_i/3,\tag{3b}$$

and

$$A=a\{1-\tfrac{3}{2}\,\mathrm{sech}^2[\eta(y-y_0)]\},$$
$$B=b\{1-\tfrac{3}{2}\,\mathrm{sech}^2[\eta(y-y_0)]\},\tag{4a}$$

where

$$a=\pm b/\sqrt{2},\quad b=-2\eta^2/\chi_i,\quad \eta^2=\Delta\varepsilon_i/3>0,$$
$$q^2=\varepsilon_i(2\omega)-\Delta\varepsilon_i/3\tag{4b}$$

are solutions of the set (2). These solitonlike solutions have only one free parameter y_0 describing the position of the solution's center. Contrary to the homogeneous bulk media, where, due to the translation symmetry, this parameter can be arbitrarily chosen, in waveguides it is strictly defined by boundary conditions. The material parameter $\Delta\varepsilon_i=\varepsilon_i(\omega)-\varepsilon_i(2\omega)=[n_i(\omega)+n_i(2\omega)]\Delta q_i/2\approx n_i(\omega)\Delta q_i$ characterizes the dispersion properties of the corresponding medium, where $n_i(\omega)$ and $\Delta q_i=2[n_i(\omega)-n_i(2\omega)]$ are the refractive index and the mismatch, respectively. Note that the solution (3) having the shape of a bright soliton exists only in media with negative mismatch, while the solution (4) being an analog of a dark soliton exists in media with positive mismatch.

Let us start with case (i). The field pattern of the whole system in this case has the following form:

$$\begin{aligned}A&=A_1e^{\mu_1(l-y)},\quad B=B_1e^{2\mu_2(l-y)}\quad \text{for } l\leqslant y;\\ A&=A_3e^{\mu_3 y},\quad B=B_3e^{2\mu_4 y}\quad \text{for } y\leqslant 0;\end{aligned}\tag{5}$$

$$\mu_{1,3}^2=q^2-\varepsilon_{1,3}(\omega),\quad \mu_{2,4}^2=q^2-\varepsilon_{1,3}(2\omega),\tag{6}$$

with A and B taken from Eqs. (3a) or (4a) for $0\leqslant y\leqslant l$.

Applying ordinary boundary conditions to the two interfaces, namely the continuity of the electric field and its y-derivative, we get the following set of equations for the FW amplitudes for solution (3a):

$$A_1=a\,\mathrm{sech}^2[\eta(l-y_0)],\tag{7a}$$

$$\mu_1A_1=2a\eta\,\mathrm{sech}^2[\eta(l-y_0)]\tanh[\eta(l-y_0)],\tag{7b}$$

$$A_3=a\,\mathrm{sech}^2(\eta y_0),\tag{7c}$$

$$\mu_3A_3=2a\eta\,\mathrm{sech}^2(\eta y_0)\tanh(\eta y_0).\tag{7d}$$

An analogous set of four equations appears for the SH. Thus we have a set of eight equations for five free parameters of the mode A_1, A_3, B_1, B_3, and y_0. Three excessive equations, in the case of solvability of the set, give three constraints for seven system parameters $\varepsilon_i(\omega)$, $\varepsilon_i(2\omega)$, and l. The nonlinear coefficient χ in the considered case of linear claddings can be removed from Eqs. (2) by proper normalization of the field amplitudes, i.e., $(A,B)\rightarrow\chi(A,B)$.

Equations (7a) and (7c) can be considered as definitions of amplitudes A_1 and A_3. Then the two other equations can be rewritten as

$$\mu_1=2\eta\tanh[\eta(l-y_0)],\tag{8a}$$

$$\mu_3=2\eta\tanh(\eta y_0).\tag{8b}$$

Conditions of compatibility of Eqs. (8) with analogous equations for SH are

$$\mu_1=2\mu_2\quad\text{and}\quad\mu_3=2\mu_4.\tag{9}$$

This gives us two of the three expected constraints,

$$\varepsilon_1(2\omega)-\Delta\varepsilon_1/3=\varepsilon_2(2\omega)-\Delta\varepsilon_2/3=\varepsilon_3(2\omega)-\Delta\varepsilon_3/3.\tag{10}$$

Finally, set (8) gives both the position of the mode's maximum y_0,

$$y_0=\frac{1}{\eta}\tanh^{-1}\left(\frac{\mu_3}{2\eta}\right),\tag{11}$$

and the third constraint,

$$l\eta=\tanh^{-1}\left(\frac{\mu_1}{2\eta}\right)+\tanh^{-1}\left(\frac{\mu_3}{2\eta}\right).\tag{12}$$

Note that from Eqs. (3b), (6), and (10) it follows that

$$\mu_{1,3}^2=-4\Delta\varepsilon_{1,3}/3>0.\tag{13}$$

Along with the values of increments of the field decay, these relations define the sign of mismatch in each cladding. Consider a symmetric waveguide when media one and three are identical, thus $\varepsilon_1=\varepsilon_3$. In this case, Eqs. (11) and (12) imply $y_0=l/2$, i.e., the maximum of the mode field is located in the middle of the slab. Constraint (12) takes the form

$$\Delta\varepsilon_1=\Delta\varepsilon_2\tanh^2(l\eta/2).\tag{14}$$

It is useful to rewrite constraints (10) and (14) in the following form:

$$\begin{aligned}\varepsilon_1(\omega)-\varepsilon_2(\omega)&=4[\varepsilon_1(2\omega)-\varepsilon_2(2\omega)]\\&=(4|\Delta\varepsilon_2|/3)\mathrm{sech}^2(l\eta/2)>0,\end{aligned}\tag{15}$$

which permits us to define dielectric constants of the cladding in terms of the waveguide core parameters $\varepsilon_2(\omega)$, $\varepsilon_2(2\omega)$, and l. For example, at $\Delta\varepsilon_2=-0.01$ and $l=10$ we get $\Delta\varepsilon_1=-0.008$ and $\varepsilon_1(\omega)=\varepsilon_2(\omega)+0.0123$.

Thus one may conclude that for a bright solitonlike mode to exist, (i) the mismatches of the slab and cladding media must be negative [see Eqs. (3b) and (13)] and (ii) refractive indices of cladding media for both frequencies must be greater than the corresponding refractive indices in the core [see Eq. (15)]. The second condition means that such a nonlinear mode exists when the structure under consideration does not support any guided modes in the linear limit. It is also worth noticing that conditions (9) mean that the increments of field decay out of the slab are equal for both harmonics.

Consider now the existence of the eigenmode based on the solution (4), which is valid for the core material with positive mismatch. A procedure analogous to that described above leads (i) to expressions similar to Eqs. (7a) and (7c) for field amplitudes in claddings,

$$\begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} (1 - \tfrac{3}{2}\,\mathrm{sech}^2[\eta(l-y_0)]),$$
$$\begin{pmatrix} A_3 \\ B_3 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} (1 - \tfrac{3}{2}\,\mathrm{sech}^2(\eta y_0)); \quad (16)$$

(ii) to the constraints (9) and (10); and (iii) to the equations

$$\frac{\mu_1}{6\eta} = \frac{\tanh[\eta(l-y_0)]\{1-\tanh^2[\eta(l-y_0)]\}}{1-3\tanh^2[\eta(l-y_0)]} > 0, \quad (17a)$$

$$\frac{\mu_3}{6\eta} = \frac{\tanh(\eta y_0)[1-\tanh^2(\eta y_0)]}{1-3\tanh^2(\eta y_0)} > 0, \quad (17b)$$

which, being analogs of Eqs. (8), define the parameter y_0 and give the third constraint. Equation (13) is valid in this case as well, thus ''dark'' modes exist when the core material has positive mismatch while claddings have a negative mismatch.

It can be proven that in the symmetric case, i.e., at $\varepsilon_1 = \varepsilon_3$, set (17) has a single solution $y_0 = l/2$ and leads to the relation

$$\sqrt{|\Delta\varepsilon_1|} = 3\sqrt{\Delta\varepsilon_2}\,\frac{\tanh(\eta l/2)[1-\tanh^2(\eta l/2)]}{1-3\tanh^2(\eta l/2)} > 0, \quad (18)$$

which in turn gives the condition $\eta l < 2\tanh^{-1}(1/\sqrt{3}) \approx 1.317$. This inequality, or equivalently $\mathrm{sech}^2(l\eta/2) > \frac{2}{3}$, means that only the central part of the dark solution, which is situated between its two zeros, participates in the mode formation. Thus the field in the waveguide core has no nodes. There is also no solution with the field amplitudes equal to zero on the interfaces $y = 0,l$. Otherwise, such a solution would be an example of an optical compacton with the field exactly equal to zero outside the waveguide core. Nevertheless, by proper choice of material parameters, the field amplitude $A = A_1 = A_3$ can be made very small. In this case, $A/a \approx \sqrt{\Delta\varepsilon_2/3|\Delta\varepsilon_1|} \ll 1$ and practically all of the optical power propagates in the core. To illustrate the restrictions for material parameters resulting from relations (14) and (18), the dependences of the parameter $\beta = |\Delta\varepsilon_1/\Delta\varepsilon_2|$ on $|\Delta\varepsilon_2|$ for ''bright'' and ''dark'' modes are shown in Fig. 1.

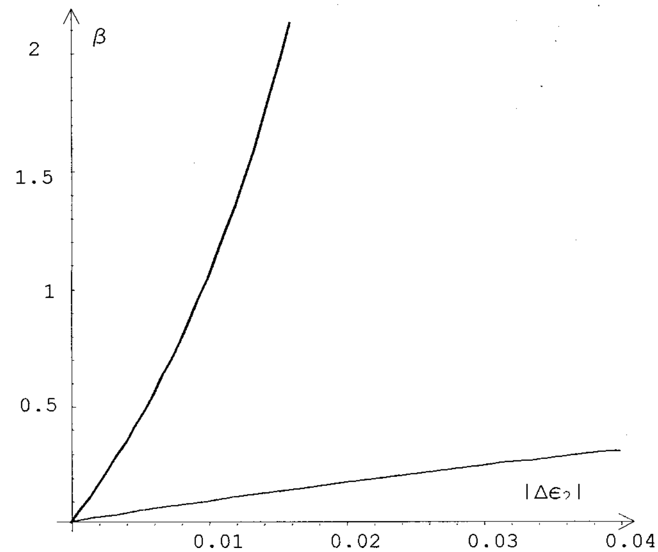

FIG. 1. Parameter $\beta = |\Delta\varepsilon_1/\Delta\varepsilon_2|$ as a function of $|\Delta\varepsilon_2|$ for $l = 10$. Thin line, ''bright'' mode; thick line, ''dark'' mode.

Now we continue with case (ii), for which all three media are assumed to be nonlinear. The solution to set (2) in claddings should be taken in the form of a bright soliton (3), because it is the only solution having a zero asymptotic value at $y \to \pm\infty$. In the core region, fields with both ''bright'' (3) and ''dark'' (4) shape can form the modes. Thus we suppose that mismatches in claddings are negative and take the ''bright'' solutions $\{A_i, B_i\} = \{a_i, b_i\}\mathrm{sech}^2[\eta_i(y-y_i)]$ in all three media provided the core material mismatch is also negative, and we replace the ''bright'' solution by the ''dark'' one $\{A_2, B_2\} = \{a_2, b_2\}\{1 - \frac{3}{2}\,\mathrm{sech}^2[\eta_2(y-y_2)]\}$ in the core region when its mismatch is positive. The condition in which the wave vector q is common for the claddings and the core in both cases again gives us the constraints (10). The boundary conditions applied to the FW field after some simplifications lead, in the case of a ''bright'' solution, to the following equations:

$$a_1\,\mathrm{sech}^2[\eta_1(l-y_1)] = a_2\,\mathrm{sech}^2[\eta_2(l-y_2)], \quad (19a)$$

$$\eta_1\tanh[\eta_1(l-y_1)] = \eta_2\tanh[\eta_2(l-y_2)], \quad (19b)$$

$$a_3\,\mathrm{sech}^2(\eta_3 y_3) = a_2\,\mathrm{sech}^2(\eta_2 y_2), \quad (19c)$$

$$\eta_3\tanh(\eta_3 y_3) = \eta_2\tanh(\eta_2 y_2). \quad (19d)$$

Since the relations $a_1/b_1 = a_2/b_2 = a_3/b_3$ are satisfied regardless of the sign of a_i [see Eq. (3b)], the corresponding set of equations for the SH coincide with set (19) and do not bring additional restrictions, except that χ_2 and χ_i should have equal signs. Analysis of set (19) shows that its solutions are

$$y_1=l-\frac{1}{\eta_1}\tanh^{-1}\gamma_1<l,\quad 0<y_2=\frac{1}{\eta_2}\tanh^{-1}\left(\frac{\eta_3}{\eta_2}\gamma_3\right)<l,\quad y_3=\frac{1}{\eta_3}\tanh^{-1}\gamma_3>0, \tag{20}$$

$$l=\frac{1}{\eta_2}\left[\tanh^{-1}\left(\frac{\eta_3}{\eta_2}\gamma_3\right)+\tanh^{-1}\left(\frac{\eta_1}{\eta_2}\gamma_1\right)\right], \tag{21}$$

and

$$y_1=l+\frac{1}{\eta_1}\tanh^{-1}\gamma_1>l,\quad y_2=\frac{1}{\eta_2}\tanh^{-1}\left(\frac{\eta_3}{\eta_2}\gamma_3\right)>l,$$

$$y_3=\frac{1}{\eta_3}\tanh^{-1}\gamma_3>0\quad \text{at}\quad \gamma_3\eta_3>\gamma_1\eta_1, \tag{22a}$$

$$y_1=l-\frac{1}{\eta_1}\tanh^{-1}\gamma_1<l,$$

$$y_2=\frac{-1}{\eta_2}\tanh^{-1}\left(\frac{\eta_3}{\eta_2}\gamma_3\right)<0,$$

$$y_3=\frac{-1}{\eta_3}\tanh^{-1}\gamma_3<0\quad \text{at}\quad \gamma_3\eta_3<\gamma_1\eta_1, \tag{22b}$$

$$l=\frac{1}{\eta_2}\left|\tanh^{-1}\left(\frac{\eta_3}{\eta_2}\gamma_3\right)-\tanh^{-1}\left(\frac{\eta_1}{\eta_2}\gamma_1\right)\right| \tag{23}$$

where

$$\gamma_i=\left[\frac{\beta_i-\alpha_i}{\beta_i(1-\alpha_i)}\right]^{1/2},\quad \alpha_i=\frac{\chi_i}{\chi_2},\quad \beta_i=\left|\frac{\Delta\varepsilon_i}{\Delta\varepsilon_2}\right|,\quad i=1,3.$$

In addition, for γ_i to be real-valued, either inequalities $0<\alpha_i<1,\alpha_i<\beta_i$ or $\alpha_i>\beta_i,\alpha_i>1$ should be satisfied. Equations (21) or (23) give an additional constraint to constraint (10) for the system parameters, where unlike the analogous constraints (12) and (18), the nonlinear coefficients χ_i are involved. It can be inferred from Eq. (20) that for this solution the mode maximum is located in the core and that the fields monotonically decay in the claddings. In the particular case γ_1 (or γ_3)$=0$, we get $y_2=y_1=l$ (or $y_2=y_3=0$), i.e., the mode maximum coincides with one of the core-cladding interfaces. Contrary to Eq. (20), for solution (22) the mode maximum is situated outside the core. For the symmetrical waveguide, only solution (20) survives and Eqs. (20) and (21) imply $y_2=l/2$ and $y_3+y_1=l$. Thus the corresponding mode is symmetric with respect to the middle of the core. In this case, constraint (21) can be rewritten as a dependence of the parameter $\beta=\beta_1=\beta_3$ on other material parameters in the following form: $\beta=\alpha+(1-\alpha)\tanh^2(l\sqrt{|\Delta\varepsilon_2|/12})$, where $\alpha=\alpha_1=\alpha_3$.

Finally, an analogous analysis of the corresponding set of equations in the case of a ‘‘dark’’ solution shows that under the condition

$$l=\frac{1}{\eta_2}[\tanh^{-1}(p_1)+\tanh^{-1}(p_3)], \tag{24}$$

the positions of the solution maxima are

$$y_1=l-\frac{1}{\eta_1}\tanh^{-1}\left(\frac{3\eta_2p_1(1-p_1^2)}{\eta_1(1-3p_1^2)}\right),\quad y_2=\frac{1}{\eta_2}\tanh^{-1}(p_3),$$

$$y_3=\frac{1}{\eta_3}\tanh^{-1}\left(\frac{3\eta_2p_3(1-p_3^2)}{\eta_3(1-3p_3^2)}\right). \tag{25}$$

The parameters $p_{1,3}$ are the solutions of algebraic equations

$$27(1-\alpha_i)p_i^6-27(\beta_i+2-\alpha_i)p_i^4+9(2\beta_i+3-\alpha_i)p_i^2+\alpha_i-3\beta_i=0 \tag{26}$$

with additional conditions

$$p_i^2<\tfrac{1}{3}\quad \text{for}\quad \alpha_i>0\quad \text{and}\quad \tfrac{1}{3}<p_i^2<1\quad \text{for}\quad \alpha_i<0. \tag{27}$$

Parameters α_i and β_i were defined in Eq. (23). Among the six solutions of Eq. (26), the only solutions that provide positiveness of l in Eq. (24) and real-valued y_i in Eq. (25) should be chosen. It is important to emphasize that in the case of the symmetric waveguide, due to the equalities $\alpha_1=\alpha_3=\alpha$ and $\beta_1=\beta_3=\beta$, the parameters p_1 and p_3 satisfy a single equation, but they are not necessarily equal. That can lead to a nontrivial consequence. In the case of the existence of two or more roots $p^{(j)}$ of Eq. (26) satisfying all the restrictions originating from Eqs. (24), (25), and (27), one can choose $p_1=p^{(j)}$ and $p_3=p^{(n)}$ with $j\neq n$. At such a choice, $p_1\neq p_3$ and it follows from Eqs. (24) and (25) that $y_2\neq l/2$ and $y_3+y_1\neq l$. The modes with such parameters y_i are asymmetric ones with respect to the middle plane $y=l/2$ of the guiding structure. These modes are reminiscent of the asymmetric modes found for the first time in [12] in the case of linear film surrounded by media with Kerr nonlinearity. The nature of the asymmetric modes is purely nonlinear and they are absent in symmetric linear structures. Omitting details here, we briefly summarize some results of the analysis of system (24)–(27) and show that the above-mentioned two-component asymmetric modes indeed can exist. To illustrate this, consider the case $\alpha>0$. There is only a single solution $p^{(1)}$ to the set (24)–(27) in the parameter domain $0<\alpha<3\beta$, $\beta<3$. Hence $p_1=p_3=p^{(1)}$ and we arrive at a symmetric mode with $y_2=l/2$ and $y_3+y_1=l=(2/\eta_1)\tanh^{-1}(p^{(1)})$. For instance, at $\alpha=\beta=0.1$, the waveguide core width and parameters y_i are $l\approx 3$ (or using dimension variables $l\approx 0.48\lambda$, where λ is the wavelength of the FW in vacuum), $y_1\approx -61$, $y_2\approx 1.5$, and $y_3\approx 64$; at $\alpha=\beta=0.5$, they are $l\approx 6.4$, $y_1\approx -23.3$, $y_2\approx 3.2$, and $y_3\approx 29.7$. The field of the modes in claddings is described by the tails of the ‘‘bright’’ solutions and therefore decays monotonically outside the waveguide core. At $\beta=3$, a bifurcation occurs and for $\beta>3$ in the domain $3\beta<\alpha<\alpha_{\text{cr}}\approx 2\beta-5+2\sqrt{\beta^2-2\beta+13}$ of the parameter plane (α, β), along with the root $p^{(1)}$, two additional roots $0<p^{(2)}<p^{(1)}$ and $p^{(3)}=-p^{(2)}$ appear. If in Eqs. (25) we choose $p_1=p_3=p^{(1)}$

($p^{(2)}$), we get symmetric modes. While choosing $p_1 = p^{(1)}$ and $p_3 = \pm p^{(2)}$ or vice versa $p_3 = p^{(1)}$ and $p_1 = \pm p^{(2)}$, we arrive at two pairs of asymmetric modes, where at $p_{1,3} = -p^{(2)}$ a negative root comes into the play in combination with the greater positive root. It can be inferred from Eqs. (24) and (25) that $0 < y_2 < l$ when two positive roots are chosen and $y_2 > l$ or $y_2 < 0$ when the negative root is used. In the latter case, the center of the mode is shifted out of the waveguide core. As an example, consider $\beta = 5$. Then $\alpha_{\mathrm{cr}} \approx 15.58$, and taking $\alpha = 15.4$ we get $p^{(1)} \approx 0.298$ and $p^{(2,3)} \approx \pm 0.157$. Equations (24) and (25) imply that the parameters of two asymmetric modes are as follows: $l \approx 18.04, y_1 \approx 8.58, y_2 \approx 6.13, y_3 \approx 3.9$ and $l \approx 5.79, y_1 \approx -3.67, y_2 \approx -6.13, y_3 \approx -3.9$. For two complementary asymmetric modes that are mirror images of the two modes described above, the parameters are $y_i^c = l - y_i$.

In conclusion, we have analytically studied the nonlinear eigenmode formation in a slab waveguide when the core material exhibits a quadratic nonlinearity while claddings are either linear or nonlinear. These modes represent stationary coupled states formed by FW and SH waves with mutually balanced up- and down-conversion processes. The existence of eigenmodes based on solutions with different topology, viz., ''bright'' and ''dark'' ones, is demonstrated. The system of equations defining the mode's parameters is derived and resolved. The conditions required for the modes to exist are obtained, e.g., the fulfilling of constraints (10) and a negative mismatch of cladding materials are necessary for the existence of all considered modes. The asymmetric modes are found in a symmetric waveguide in the case in which the core and the claddings are supposed to be nonlinear and the field in the core is described by the ''dark'' solitonlike solution (4). The domain in system parameter space where this nonlinear break of symmetry can take place is determined. We expect that the consideration of TM or combined TM-TE composing fields will bring new interesting features of nonlinear mode formation in guiding structures.

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