Time-dependent electron-ion collision frequency at arbitrary laser intensity-temperature ratio

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In superintense laser beams collisional absorption exhibits a large amplitude modulation over a laser cycle. In this paper formulas for the time-dependent electron-ion collision frequency $\nu_{ei}(t)$ are presented. On the basis of a ballistic interaction model we deduce an expression for $\nu_{ei}(t)$ which holds for an arbitrary isotropic distribution function and arbitrary anharmonic oscillatory electron motion [Eq. (4)]. For a Maxwellian we present compact formulas for the various ratios $v_{os}(t)/v_{th}$. It is shown that the strong time dependence over one laser cycle leads to the generation of intense odd harmonics. The cycle-averaged collision frequency $\overline{\nu_{ei}}(t)$ is compared with expressions derived from the more complex dielectric model. It is shown that the correct choice of cutoffs as a consequence of dynmical screening is essential.

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I. INTRODUCTION

The electron-ion collision frequency is essential in collisional absorption of laser light and in transport phenomena, such as thermal diffusion and collisional ion heating by electrons. At low laser intensities I the oscillatory velocity $v_{os} = |\boldsymbol{v}_{os}|$ is much smaller than the thermal speed $v_{th} = (kT_e/m_e)^{1/2}$, $v_{os} \ll v_{th}$. As a consequence, ν_{ei} depends on the electron temperature T_e, but not on the laser field $\boldsymbol{E}$, and is constant over a laser cycle. At high laser intensities (e.g., $I \gtrsim 10^{18}\ \mathrm{W\,cm^{-2}}$) $\hat{v}_{os} \gg v_{th}$ may hold, $\hat{v}_{os}$ oscillation amplitude, and hence ν_{ei} becomes strongly time dependent over one cycle, $\nu_{ei} = \nu_{ei}(t)$. For this strong field case cycle-averaged expressions $\overline{\nu_{ei}}$ have been presented in the literature [1–5], but no explicit formulas for the instantaneous value $\nu_{ei}(t)$ exist so far. Its knowledge is indispensable for various applications. With the present paper we close this gap in the ideal plasma domain for Coulomb logarithms $\ln\Lambda \gtrsim 1$ and for laser frequencies $\omega \neq \omega_p$, ω_p plasma frequency.

Most of the authors based the calculation of $\overline{\nu_{ei}}$ on the dielectric theory although this is unnecessary as long as collective plasma effects occuring at $\omega \simeq \omega_p$ are ignored as in Ref. [1,2]. Instead, a much simpler ballistic model is used here to obtain analytical expressions for $\nu_{ei}(t)$ in a very immediate way for an arbitrary isotropic electron distribution function and for arbitrarily unharmonic oscillatory motion at all ratios v_{os}/v_{th}. The exact evaluation of the dielectric model for zero electron temperature [4] shows that out of resonance ($\omega \neq \omega_p$) this theory merely provides a self-consistent cutoff of the bare Coulomb potential, a fact which will also come out as a by-product from our ballistic model. On the other hand, since the exact evaluation of the dielectric model for arbitrary values v_{os}/v_{th} is cumbersome—in fact, such an analysis is still missing—all authors using this model [13,5,6] limit themselves to presenting asymptotic approximations in the form of standard Coulomb logarithms.

With the help of $\nu_{ei}(t)$ we show the appearance of harmonics and estimate their strength in one situation. Finally, in the discussion, for completeness we determine $\overline{\nu_{ei}}$ from our expressions for $\nu_{ei}(t)$ and compare them with those of the dielectric theory. We shall discover a significant discrepancy which, however, disappears when dynamical screening is taken into account.

II. TIME-DEPENDENT MOMENTUM LOSS

Throughout the paper limitation is made to isotropic distribution functions $f(\boldsymbol{x},\boldsymbol{v}_e,t) = f(v_e,t)$ and the plasma is assumed to be locally homogeneous. An individual electron of velocity $\boldsymbol{v}(t) = \boldsymbol{v}_{os}(t) + \boldsymbol{v}_e$ loses momentum $\Delta\boldsymbol{p}$ in direction of $\boldsymbol{v}(t)$ in one collision with an ion of charge Z owing to a deflection by the angle ϑ. The differential Coulomb cross section σ_Ω and the collision parameter $b_\perp$ for perpendicular deflection are given by (see textbooks)

$$\sigma_\Omega = \frac{b_\perp^2}{4\sin^4\dfrac{\vartheta}{2}}, \quad b_\perp = \frac{Ze^2}{4\pi\varepsilon_0 m_e \boldsymbol{v}^2} = 7\frac{Z}{E[eV]}[\text{Å}];$$
$$\tan\frac{\vartheta}{2} = \frac{b_\perp}{b}. \tag{1}$$

Over the total cross section $\sigma = \pi b_{\max}^2$ results

$$\Delta\boldsymbol{p} = m_e \boldsymbol{v}\frac{b_\perp^2}{\sigma}\int_{\vartheta=\varepsilon}^{\pi}\frac{(1-\cos\vartheta)\sin\vartheta}{4\sin^4\dfrac{\vartheta}{2}}\,d\vartheta\,d\varphi$$

$$= 4\pi m_e \boldsymbol{v}\frac{b_\perp^2}{\sigma}\int_{\vartheta=\varepsilon}^{\pi}\frac{d\sin\dfrac{\vartheta}{2}}{\sin\dfrac{\vartheta}{2}} = 4\pi m_e \boldsymbol{v}\frac{b_\perp^2}{\sigma}\ln\frac{b_{\max}}{b_\perp}.$$

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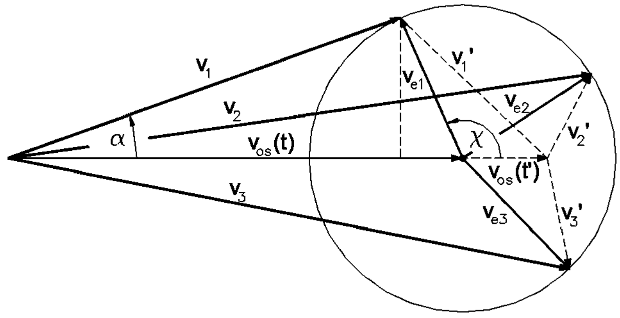

FIG. 1. Isotropic electron distribution $f(v_e)$. The resultant velocities $\boldsymbol{v}=\boldsymbol{v}_{os}+\boldsymbol{v}_e$ consist of all types of vectors $\boldsymbol{v}_i, \boldsymbol{v}_i'$ indicated in the figure for two oscillation velocities $\boldsymbol{v}_{os}(t)$ and $\boldsymbol{v}_{os}(t')$. The momentum losses $\dot{\boldsymbol{p}}$ form the angles α with $\boldsymbol{v}_{os}$; the average loss $\langle \dot{p} \rangle$ is directed along $\boldsymbol{v}_{os}$.

The Coulomb logarithm $\ln \Lambda = \ln(b_{\max}/b_\perp)$ is discussed later. Here it is treated as a constant. The momentum loss per unit time is $\dot{\boldsymbol{p}} = \sigma n_i |\boldsymbol{v}| \Delta \boldsymbol{p}$,

$$\dot{\boldsymbol{p}} = -m_e \nu_{ei}(\boldsymbol{v}) \boldsymbol{v} = -\frac{K}{v^3}\boldsymbol{v}; \quad K = \frac{Z^2 e^4 n_i}{4\pi\varepsilon_0^2 m_e} \ln \Lambda. \quad (2)$$

By the first equality the collision frequency $\nu_{ei}(\boldsymbol{v})$ of the loss of momentum $\boldsymbol{p} = m_e \boldsymbol{v}$ is defined. To obtain the ensemble-averaged momentum loss $\langle \dot{\boldsymbol{p}} \rangle$ averaging has to be done on $\boldsymbol{v}/v^3$ over the thermal velocities $\boldsymbol{v}_e$. For an isotropic distribution function $f(v_e)$ the velocity $\boldsymbol{v}$ consists of all vector sums as sketched in Fig. 1. The quantity $\dot{\boldsymbol{p}}$ is parallel to $\boldsymbol{v}$, whereas $\langle \dot{\boldsymbol{p}} \rangle$ points along $\boldsymbol{v}_{os}$. With the angles α and χ as indicated in the figure holds

$$\left\langle \frac{\boldsymbol{v}}{v^3} \right\rangle = \frac{\boldsymbol{v}_{os}}{v_{os}} \left\langle \frac{\cos \alpha}{v^2} \right\rangle = -\left\langle \frac{\partial}{\partial \boldsymbol{v}_{os}} \frac{1}{v} \right\rangle = -\frac{\partial}{\partial \boldsymbol{v}_{os}} \left\langle \frac{1}{v} \right\rangle. \quad (3)$$

Hence, determining $\langle \dot{\boldsymbol{p}} \rangle$ is perfectly analoguous to calculating the gravitational force of a spherical mass distribution on a point mass at distance R from its center. We obtain

$$\langle \dot{\boldsymbol{p}} \rangle = m_e \nu_{ei}(t) \boldsymbol{v}_{os} = -K \frac{\partial}{\partial \boldsymbol{v}_{os}} \int_0^\infty \int_0^\pi \frac{2\pi v_e^2 \sin \chi}{v} f(v_e)\, d\chi\, dv_e$$

$$= K \frac{\boldsymbol{v}_{os}}{v_{os}^3} \int_0^{v_{os}} 4\pi v_e^2 f(v_e)\, dv_e.$$

Formally, without making recurrence to the analogy of the gravitational potential, the last step follows from $v_{os} v_e \sin \chi = -\partial v / \partial \chi$. The time-dependent collision frequency $\nu_{ei}(t)$ results as

$$\nu_{ei}(t) = \frac{K}{m_e v_{os}^3(t)} \int_0^{v_{os}} 4\pi v_e^2 f(v_e)\, dv_e. \quad (4)$$

This is our basic expression for $\nu_{ei}(t)$, valid for any $f(v_e)$ and arbitrary $v_{os}(t)$. In particular, if for a dilute plasma in thermal equilibrium $f(v_e)$ is a Maxwellian

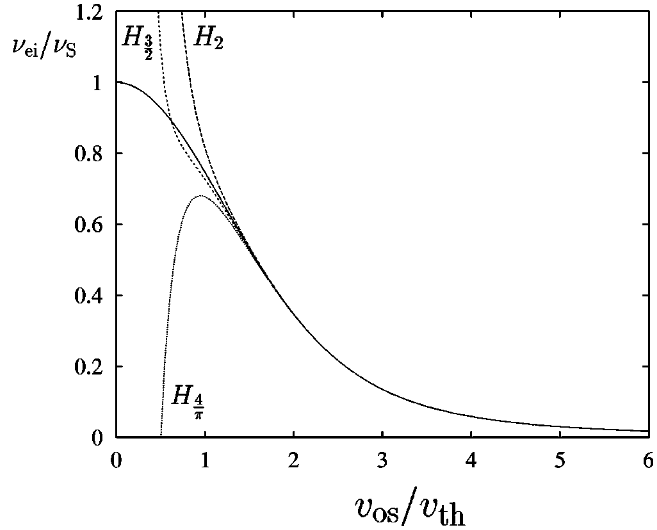

FIG. 2. The collision frequency ν_{ei} of Eq. (5) normalized to Silin's formula ν_S (solid line), upper and lower bounds H_2 and $H_{4/\pi}$ (dashed) and best fit $H_{3/2}$ from Eq. (10) (dotted line).

$$f_M(v_e) = \left(\frac{\beta}{\pi}\right)^{3/2} e^{-\beta v_e^2}; \quad \beta = \frac{m_e}{2kT_e}$$

and when setting $w = \beta^{1/2} v$, $w_{os} = \beta^{1/2} v_{os} = v_{os}/\sqrt{2} v_{th}$, ν_{ei} becomes

$$\nu_{ei}(t) = \frac{K}{m_e v_{os}^3(t)} \frac{4}{\pi^{1/2}} \int_0^{w_{os}} w^2 e^{-w^2} dw. \quad (5)$$

For $v_{os} \ll v_{th}$ and $\exp(-w_{os}^2) \simeq 1$, ν_{ei} reduces to the time-independent expression ν_S

$$\nu_S = \frac{K}{m_e v_{os}^3} \frac{4}{\pi^{1/2}} \int_0^{w_{os}} w^2 dw = \frac{1}{3} \sqrt{\frac{2}{\pi}} \frac{K}{m_e v_{th}^3};$$
$$v_{th} = \left(\frac{kT_e}{m_e}\right)^{1/2}, \quad (6)$$

in agreement with Ref. [1]. We notice that $\nu_{ei}(t)$ from Eq. (4) behaves regular for all ratios v_{os}/v_{th}. For $v_{os} \gg v_{th}$,

$$\nu_{ei}(t) = \frac{K}{m_e v_{os}^3(t)}. \quad (7)$$

The collision frequency from Eq. (5) normalized to Silin's expression ν_S for vanishing v_{os}/v_{th} from Eq. (6) is plotted in Fig. 2.

Expansion of Eq. (5) in terms of w_{os} by successive partial integration may be useful in practice for $v_{os}/v_{th} \lesssim \sqrt{2}$,

$$\nu_{ei}(t) = \frac{1}{3} \sqrt{\frac{2}{\pi}} \frac{K}{m_e v_{th}^3} \exp\left(-\frac{v_{os}^2}{2 v_{th}^2}\right) \left\{ 1 + \frac{1}{5} \left(\frac{v_{os}}{v_{th}}\right)^2 + \frac{1}{5\times 7} \left(\frac{v_{os}}{v_{th}}\right)^4 + \frac{1}{5\times 7\times 9} \left(\frac{v_{os}}{v_{th}}\right)^6 + \cdots \right\}, \quad (8)$$

although it converges for arbitrarily large values v_{os}/v_{th}. At $v_{os}=v_{th}$ the bracket amounts to 1 + 0.23. The major correction relative to ν_{ei} from Eq. (6) is due to the exponential factor $\exp(-v_{os}^2/2v_{th}^2)=\exp(-0.5)=0.6$. Because of the uncertainty of the Coulomb logarithm due to dynamical screening it is reasonable to use the simplified formula

$$\nu_{ei}(t)=\frac{1}{3}\sqrt{\frac{2}{\pi}}\frac{K}{m_e v_{th}^3}\exp\left(-\frac{v_{os}^2}{2v_{th}^2}\right), \tag{9}$$

for $v_{os}\lesssim v_{th}$. An upper and a lower bound for the integral in Eq. (5), indicated by H_2 and $H_{4/\pi}$, can be given in terms of elementary functions (see, for example, Ref. [6]),

$$H_2=\frac{\pi^{1/2}}{2}-\left[\frac{1}{w_{os}+(w_{os}^2+2)^{1/2}}+w_{os}\right]e^{-w_{os}^2},$$

$$H_{4/\pi}=\frac{\pi^{1/2}}{2}-\left[\frac{1}{w_{os}+\left(w_{os}^2+\frac{4}{\pi}\right)^{1/2}}+w_{os}\right]e^{-w_{os}^2}.$$

They are shown in Fig. 2. The accuracy becomes acceptable for $w_{os}>1/\sqrt{2}$ (deviation less than 8%). A better fit to the integral is given by $H_{3/2}$ where the number 2 in the denominator of H_2 is replaced by the value 3/2. It satisfies $H_{4/\pi}<H_{3/2}<H_2$ and oscillates around the integral. It exhibits a maximum error of only 4% in the whole domain $w_{os}\geqslant 0.5$ and starts becomming acceptable already for $w_{os}>0.5/\sqrt{2}$ (see Fig. 2). Hence, for $v_{os}/v_{th}>0.5$ (i.e., $w_{os}\geqslant 0.5/\sqrt{2}$) the time-dependent collision frequency is well approximated by

$$\nu_{ei}(t)=\frac{2}{\pi^{1/2}}\frac{K}{m_e v_{os}^3(t)}\times\left\{\frac{\pi^{1/2}}{2}-\left[\frac{1}{w_{os}+\left(w_{os}^2+\frac{3}{2}\right)^{1/2}}+w_{os}\right]e^{-w_{os}^2}\right\}. \tag{10}$$

This is simpler than Eq. (5) since the integral is removed. For $v_{os}/v_{th}<0.5$ Eq. (6) can be used.

A. Unharmonic electron motion

In all treatments in Refs. [1–4] $\boldsymbol{v}_{os}(\boldsymbol{x},t)$ was identified with a harmonic oscillatory motion of a single electron. As outlined in the foregoing paragraph, due to the statistical interactions with the ions a friction term, i.e., an additional ensemble-averaged force, of the form $m_e\nu_{ei}\boldsymbol{v}_{os}$ appears which enters in the equation of motion and has to compete with the inertial term $m_e d\boldsymbol{v}_{os}/dt$. No problem arises as long as the friction term is small, which is equivalent to the assumption $\nu_{ei}(t)\ll\omega$. In the case, however, where $\nu_{ei}(t)\gtrsim\omega$ the electron drift motion must be determined self-consistently from the first moment of a suitable kinetic equation, generally of Vlasov-Boltzmann type,

$$m_e\frac{d\boldsymbol{u}_{os}}{dt}+m_e\nu_{ei}'\boldsymbol{u}_{os}=-e(\boldsymbol{E}+\boldsymbol{u}_{os}\times\boldsymbol{B}), \tag{11}$$

where $\boldsymbol{u}_{os}=\langle\boldsymbol{v}\rangle$. This raises the question of the validity of the foregoing derivation of the friction force. The only restriction we made on $\boldsymbol{v}_{os}$ was that it must be the same for all electrons. At first glance this is generally not guaranteed when ν_{ei} exceeds the field frequency ω. Here, however, we have limited ourselves to cases with $\ln\Lambda\gtrsim 1$. This means that the majority of electron-ion collisions are small angle deflections. As a consequence the individual electron drifts $\boldsymbol{v}_{os}$ remain close to their statistical ensemble $\langle\boldsymbol{v}_{os}\rangle=\langle\boldsymbol{v}\rangle$ for a sufficiently long time and hence, the identifications

$$\boldsymbol{v}_{os}=\boldsymbol{u}_{os},\quad \nu_{ei}'(t)=\nu_{ei}(t)$$

are justified for all values of ν_{ei} (provided $\ln\Lambda\gtrsim 1$).

At $\hat{v}_{os}/v_{th}>1$ the friction term shows a strong time dependence. Hence, the motion induced by a harmonic driver in Eq. (11) becomes unharmonic and, in concomitance, higher harmonics appear in the current density $\boldsymbol{j}_e$, in $\boldsymbol{E}$ and in $\boldsymbol{B}$. The unharmonicity is expected to be particularly strong in the dense plasma when T_e is low and $v_{os,\max}$ largely exceeds v_{th}. To evaluate its strength we choose solid state density ($n_i\simeq 6\times 10^{22}$ cm^{-2}), $Z=5$ and $T_e\simeq 100$ eV are chosen. It is further assumed that $\boldsymbol{E}$ is such that $v_{os,\max}$ approximately equals v_{th}. Then $\nu_{ei}(t)$ from Eq. (9) can be used: $4\times 10^{15}\leqslant\nu_{ei}(t)\leqslant 10^{16}$ s^{-1}. For a Nd laser, $\nu_{ei}\gg\omega$ holds. As a consequence the inertial term in Eq. (11) can be omitted and

$$\boldsymbol{v}_{os}(t)=-\frac{e}{m_e\nu_{ei}(t)}\hat{\boldsymbol{E}}\cos\omega t,\quad (\boldsymbol{v}_{os}\times\boldsymbol{B}\simeq 0)$$

is obtained in this collision-dominated case (dc behavior). This is a transcendental equation of the form

$$\boldsymbol{w}_{os}e^{-w_{os}^2}=\boldsymbol{F}\cos\omega t;\quad \boldsymbol{F}=-\frac{3\sqrt{\pi}e\hat{\boldsymbol{E}}v_{th}^2}{2K}. \tag{12}$$

Since $\boldsymbol{w}_{os}(t+\pi/\omega)=-\boldsymbol{w}_{os}(t)$ it follows that $\boldsymbol{w}_{os}$ contains odd harmonics only,

$$\boldsymbol{w}_{os}(t)=\boldsymbol{w}_1\cos\omega t+\boldsymbol{w}_3\cos 3\omega t+\boldsymbol{w}_5\cos 5\omega t+\cdots.$$

Expanding the exponential in Eq. (12),

$$\boldsymbol{w}_{os}\left(1-w_{os}^2+\frac{1}{2}w_{os}^4-\frac{1}{3!}w_{os}^6+\frac{1}{4!}w_{os}^8\mp\cdots\right)=\boldsymbol{F}\cos\omega t,$$

and keeping in mind that we have to renormalize, i.e., the sum of terms showing a $\cos\omega t$ dependence must add up to F and the sum of all higher harmonics must vanish. One obtains for

$$\cos\omega t:\quad 1-\frac{3}{4}+\frac{10}{32}-\frac{35}{384}=0.472,$$

$$\cos 3\omega t: \quad -\frac{1}{4}+\frac{5}{32}-\frac{21}{384}=-0.148,$$

if all contributions from w_{os}^8 and higher powers of w are neglected. Thus, the ratio $|\boldsymbol{w}_3|/|\boldsymbol{w}_1|$ is as large as 0.31. As a consequence, the skin depth in the overdense plasma is increased and deviates for one more reason from a pure exponential decay (another reason is skin layer absorption).

B. Dynamic screening and cutoffs

The bare Coulomb cross section for momentum transfer diverges proportional to $\ln b$. Therefore, in the expression of $\Delta\boldsymbol{p}$ one has to introduce an upper cutoff $b=b_{\max}$. This fact reveals that an important piece of physics is missing. In fact, a proper treatment would yield the cutoff self-consistently. In the published literature there exist essentially two kinds of, in principle, more complete derivations: the classical Vlasov treatment (for example, Refs. [1,3], Catto and Speziale [2]; all for $\omega_p \ll \omega$) and a quantum treatment for $\omega_p \ll \omega$ with Volkov states and von Neumann equation [5], Volkov states and standard time-dependent perturbation theory (Shima and Yatom [2]) and, recently, a quantum kinetic approach based on the Kadanoff-Baym equation has been presented [7]. To make the Vlasov treatment feasible the straight orbit approximation is introduced. In the quantum treatment this assumption is equivalent to the use of Volkov states. In both cases this simplification needs a lower cutoff $b=b_{\min}$ (often erroneously called ''parameter of closest approach'') whereas no divergence appears for large b values. In the test particle model used here no divergence arises for $b\to 0$ either, if large angle deflections are included.

There seems to be universal agreement only on $b_{\max}$ for $\hat{v}_{os} \ll v_{th}$ and $\omega_p \gg \omega$ in which case $b_{\max}$ is identified with the thermal Debye length $\lambda_D=v_{th}/\omega_p$. In the underense plasma, $\omega_p \ll \omega$, screening is considered unimportant and λ_D is substituted by $\lambda_\omega=v_{th}/\omega$. Silin [1], Shima and Yatom [2], Catto and Speziale [2], and Decker *et al.* [3] make use of λ_ω in the Coulomb logarithm also for arbitrarily large $\hat{v}_{os}/v_{th}$ in contrast to Refs. [5] and [7]. To clarify the situation on physical grounds one can make use of the so-called oscillator model [8]. At zero temperature, $T_e=0$, the small deflection $\boldsymbol{\delta}(t)$ from the straight orbit $\boldsymbol{x}=\boldsymbol{v}_0 t$, an electron undergoes in a collision with an ion, obeys the oscillator equation

$$\frac{\partial^2}{\partial t^2}\boldsymbol{\delta}+\omega_p^2\boldsymbol{\delta}=-\frac{e}{m_e}\boldsymbol{E}_c(t), \tag{13}$$

where $\boldsymbol{E}_c$ is the bare Coulomb field. In passing nearby an ion the electron starts oscillating at the plasma frequency ω_p owing to the attraction. Summing the oscillation energies over all impact parameters one arrives at the energy irreversibly extracted from one beam electron per unit time, i.e., to $\nu_{ei}(t)$ in the dielectric approximation for monoenergetic drift motion of the electrons. The restoring force $-m_e\omega_p^2\boldsymbol{\delta}$ stands for the self-consistent shielding of the ion by the surrounding electrons. It removes the logarithmic divergence. When suppressing it one is led back to the ballistic model for nearly straight orbits. In fact, from

$$\frac{\partial^2}{\partial t^2}\boldsymbol{\delta}=-\frac{e}{m_e}\boldsymbol{E}_c(\boldsymbol{v}_e,t)$$

the cross section σ_Ω of Eq. (1) follows, on the basis of which $\nu_{ei}(t)$ was calculated, however without the restriction to small angle deflections. The oscillator model corresponds to the situation $\hat{v}_{os}^2/v_{th}^2 \gg 1$ and $\omega_p^2 \gg \omega^2$. Comparing the momentum transfer $\Delta\boldsymbol{p}$ obtained from the oscillator model with the expression for $\Delta\boldsymbol{p}$ in this paper, agreement is found if $b_{\max}=v_0/\omega_p$ is set. The same cutoff was deduced in an alternative way by de Ferrariis and Arista and Peter and Meyer-ter-Vehn when studying the energy loss of a charged particle moving at $v_0 \gg v_{th}$ [9], in agreement also with Refs. [7] and [10] in the asymptotic limit. Since with decreasing ratio v_{os}/v_{th} there is a continuous transition in $b_{\max}$ from v_{os}/ω_p to λ_D we propose

$$b_{\max}(t)=\frac{[v_{os}^2(t)+v_{th}^2]^{1/2}}{\omega_p}, \tag{14}$$

which shows the correct asymptotic behavior on both extremes of v_{os}, i.e., $v_{os}\to 0$ and $v_{os}\to\infty$. The velocity averaging is thereby done on $\boldsymbol{v}^2=(\boldsymbol{v}_{os}+\boldsymbol{v}_e)^2$ since it is standard to identify $b_{\min}$ with the averaged reduced de Broglie wavelength $\lambda_B=\hbar/m_e\langle v\rangle$ as soon as λ_B is larger than $2b_\perp$. Thus $\Lambda=b_{\max}/b_{\min}$ depends quadratically on v. The justification of such a procedure is found in Ref. [9], de Ferrariis. For the rest we follow the common practice to substitute $\ln\Lambda$ in all integrals over $\boldsymbol{v}_e$ by its average $\langle \ln\Lambda\rangle$ and, since this average is not easily calculated, it is substituted by $\ln\langle\Lambda\rangle$ and, for simplicity, indicated by $\ln\Lambda$.

At this point we want to make a remark on $b_{\min}$. Time and again $b_{\min}$ is identified with the ''closest approach'' of the colliding electron, especially when $b_{\min}$ is to be identified with $b_\perp$. This makes no sense since the closest apprach is $b=0$, corresponding to a $\chi=\pi$ deflection. Sometimes a weaker statement is used which says that $b_\perp$ discriminates straight orbits from bent ones. Again, this is a weak argument. The real justification is as follows (Ref. [4], p. 215): If a collision parameter $b=b_0$ is introduced to discern between straight and bent orbits and the momentum loss for $b<b_0$ is calculated exactly for v fixed the resulting Coulomb logarithm is

$$\ln\Lambda=\ln\frac{b_0}{b_\perp}+\ln\frac{b_{\max}}{b_0}=\ln\frac{b_{\max}}{b_\perp}.$$

The beauty is that the somehow arbitrary parameter b_0 cancels as long as $b_{\max}\gg b_\perp$ holds. The quantity $b_\perp$ appearing in Λ is not more than a useful symbol to write the Rutherford cross section in a concise way.

On the basis of the oscillator model a physical interpretation of the cutoff $b_{\max}=v_0/\omega_p$ can be given that is very appealing. From Jackson's treatment of Coulomb collisions

[11] we learn that the interaction lasts for a time $\tau=2b/v$. Collisions with $b>v_0/\omega_p$ do not contribute. Inserting $b=v/\omega_p$ in the interaction time formula shows that

$$\tau_{int}\leqslant\frac{2}{\omega_p}=\frac{\tau_p}{\pi},\quad \tau_p=2\pi/\omega_p,$$

i.e., only those interactions do irreversible work which undergo not more than one third of a plasma oscillation during collision. For times $\tau_{int}\leqslant\tau_p/\pi$ most of the electrons do not appreciably change their velocity during a collision as long as $\omega<\omega_p$ holds, i.e., the plasma is overdense. An alternative interpretation of this behavior is as follows. A harmonic system oscillating at frequency ω_0, whether classical or quantum mechanical, can, in the weak coupling limit, only be excited by a frequency ω_0. For $\tau>\tau_{int}$ the frequency spectrum of the driver does almost no component contain at $\omega=\omega_0$. This tells us that in the underdense situation with $\omega_p\ll\omega$ and $v_{os}/v_{th}\gg 1$ the correct cutoff is expected to be $b_{\max}=v_{os}/\omega$ since $\boldsymbol{\delta}$ oscillates at ω rather than ω_p. There exists an exact solution of Eq. (13) for $\boldsymbol{v}_0=\boldsymbol{v}_{os}(t)$ instead of $\boldsymbol{v}_0=$const in terms of modified Bessel functions folded with an Anger function [4]. For $v_{os}/\omega b_{\min}\gg 1$ and $\omega_p\lesssim\omega/5$ the leading term is just $b_{\max}=v_{os}/\omega$, in agreement also with Refs. [5] and [2], Shima and Yatom. As a consequence, in Eq. (14) ω_p has to be replaced by max(ω,ω_p).

Later the cycle-averaged value of the Coulomb logarithm $\overline{\ln\Lambda}$ will be needed which traditionally (and in our case also for comparison with existing expressions) is identified with $\ln\bar{\Lambda}$. With $b_{\min}=\lambda_B$ we obtain

$$\ln\bar{\Lambda}=\ln\frac{m_e(\hat{v}_{os}^2/2+v_{th}^2)}{\hbar\max(\omega,\omega_p)},\quad \lambda_B>2b_\perp. \tag{15}$$

For $b_{\min}=b_\perp$ the velocity dependence is v^3. Owing to the slow change of the logarithm it is acceptable to use again the averaging factor 1/2, hence

$$\ln\bar{\Lambda}=\ln\frac{4\pi\varepsilon_0 m_e(\hat{v}_{os}^2/2+v_{th}^2)^{3/2}}{Ze^2\max(\omega,\omega_p)},\quad \lambda_B\leqslant 2b_\perp. \tag{16}$$

C. Cycle-averaged collision frequency $\bar{\nu}_{ei}$

In the published literature expressions for $\bar{\nu}_{ei}$ with harmonic $v_{os}(t)$ only are available. It may be interesting to see whether the simple ballistic model is able to reproduce formulas for $\bar{\nu}_{ei}$ obtained from the dielectric theory. To this aim correct averaging over one field period must be done. Multiplying Eq. (11) with $\boldsymbol{v}_{os}$ yields

$$\frac{d}{dt}\frac{m_e}{2}\boldsymbol{v}_{os}^2+m_e\nu_{ei}\boldsymbol{v}_{os}^2=-e\boldsymbol{v}_{os}\boldsymbol{E}.$$

In the steady state case cycle averaging leads to

$$m_e\overline{\nu_{ei}\boldsymbol{v}_{os}^2}=-e\overline{\boldsymbol{v}_{os}\boldsymbol{E}}.$$

The term on the right-hand side represents the true energy absorption per cycle. The reversible fraction of absorbed power contained in $-e\boldsymbol{v}_{os}\boldsymbol{E}$ vanishes after time averaging. A meaningful definition of ν_{ei} is achieved by setting

$$m_e\overline{\nu_{ei}\boldsymbol{v}_{os}^2}=m_e\bar{\nu}_{ei}\,\overline{\boldsymbol{v}_{os}^2}=2\bar{\nu}_{ei}\bar{E}_{kin}.$$

It guarantees the correct determination of energy absorption from the corresponding averaged quantities and, for $\hat{v}_{os}\ll v_{th}$ where ν_{ei} does not vary with time, $\bar{\nu}_{ei}=\nu_{ei}$ results. Hence

$$\bar{\nu}_{ei}=\frac{\overline{\nu_{ei}\boldsymbol{v}_{os}^2}}{\overline{\boldsymbol{v}_{os}^2}}=\frac{K}{2\bar{E}_{kin}}\overline{\frac{1}{v_{os}(t)}\int_0^{v_{os}}4\pi v_e^2 f(v_e)dv_e}. \tag{17}$$

Again, this expression holds for arbitrary $f(v_e)$ and arbitrary $v_{os}(t)$. For $f(v_e)=f_M(v_e)$ it becomes

$$\bar{\nu}_{ei}=2\left(\frac{\beta}{\pi}\right)^{1/2}\frac{K}{\bar{E}_{kin}}\overline{\frac{1}{w_{os}}\int_0^{w_{os}}w^2e^{-w^2}dw}. \tag{18}$$

This formula is accessible to a numerical evaluation only, especially if one keeps in mind that for $\nu_{ei}(t)>\omega$ w_{os} is no longer sinusoidal. In an underdense plasma, however, it is nearly harmonic in general. If $v_{os}(t)=\hat{v}_{os}|\cos\omega t|$ is set one obtains in the range $\hat{v}_{os}<v_{th}$ from Eq. (17) for a Maxwellian distribution

$$\bar{\nu}_{ei}=\frac{1}{3}\sqrt{\frac{2}{\pi}}\frac{Z^2e^4n_i}{4\pi\varepsilon_0^2m_e^2v_{th}^3}\left\{1-\frac{9}{40}\left(\frac{\hat{v}_{os}}{v_{th}}\right)^2+\frac{15}{448}\left(\frac{\hat{v}_{os}}{v_{th}}\right)^4-\frac{35}{9216}\left(\frac{\hat{v}_{os}}{v_{th}}\right)^6\pm\cdots\right\}\ln\Lambda.$$

It shows that the Spitzer-Braginskii collision frequency Eq. (6) which was derived for vanishing drift velocity is still a not too bad approximation for $\hat{v}_{os}=v_{th}$, i.e., $\hat{w}_{os}=1/\sqrt{2}$ (19% error). Now $\bar{\nu}_{ei}$ in the opposite case $\hat{w}_{os}>1$ is determined. One may start from Eq. (18) and expand it in Taylor series

$$\bar{\nu}_{ei}=\left(\frac{\beta}{\pi}\right)^{1/2}\frac{\bar{K}}{\bar{E}_{kin}}\frac{\omega}{2\pi}\int_0^{2\pi/\omega}dt\times\left\{\sum_{n\geqslant 1}(-1)^n\left(\frac{1}{2n+1}-1\right)\frac{w_{os}^{2n}}{n!}\right\}.$$

In the published literature for $\bar{\nu}_{ei}$ only expressions with harmonic $w_{os}(t)$ exist. Therefore, in order to compare, again use must be made of a harmonic time dependence, $w_{os}(t)=\hat{w}_{os}\cos\omega t$. The integrals can be done analytically (see, for example, Ref. [12]):

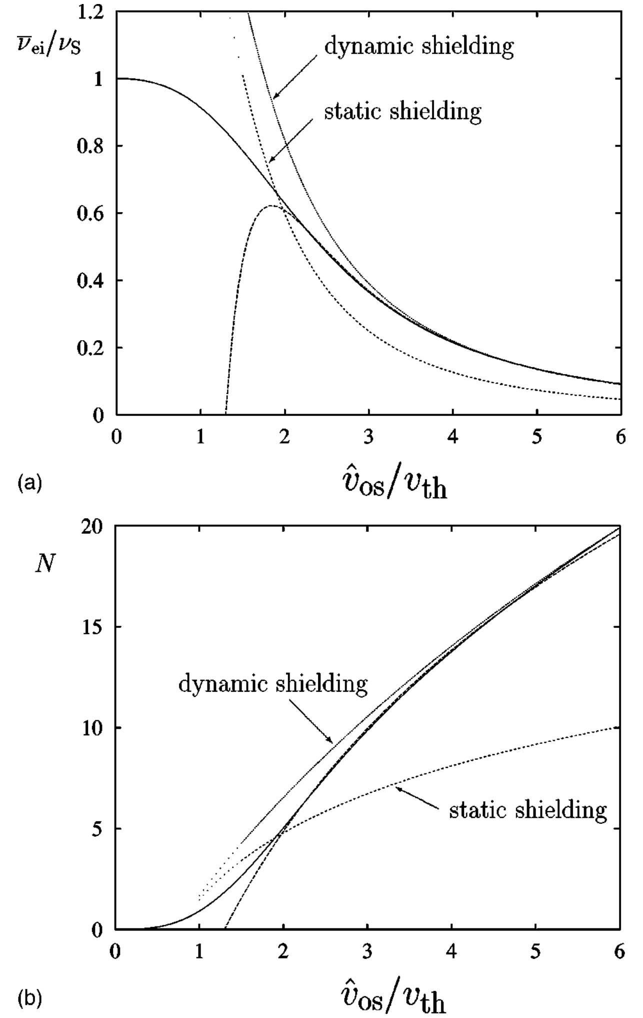

FIG. 3. (a) Normalized time-averaged collision frequency $\overline{\nu}_{ei}/\nu_S$. Solid: Sum of Eq. (19) times 3; dashed: present fit, first line of Eq. (20); static shielding according to Eq. (21); dotted: its modification by the dynamically shielded upper cutoff $b_{\max}$ from Eq. (14). The Coulomb logarithm is $\ln \Lambda_{os}=3$. (b) $N=(\overline{\nu}_{ei}/\nu_S)\hat{v}_{os}^3/v_{os}^3$, with $\overline{\nu}_{ei}/\nu_S$ from (a).

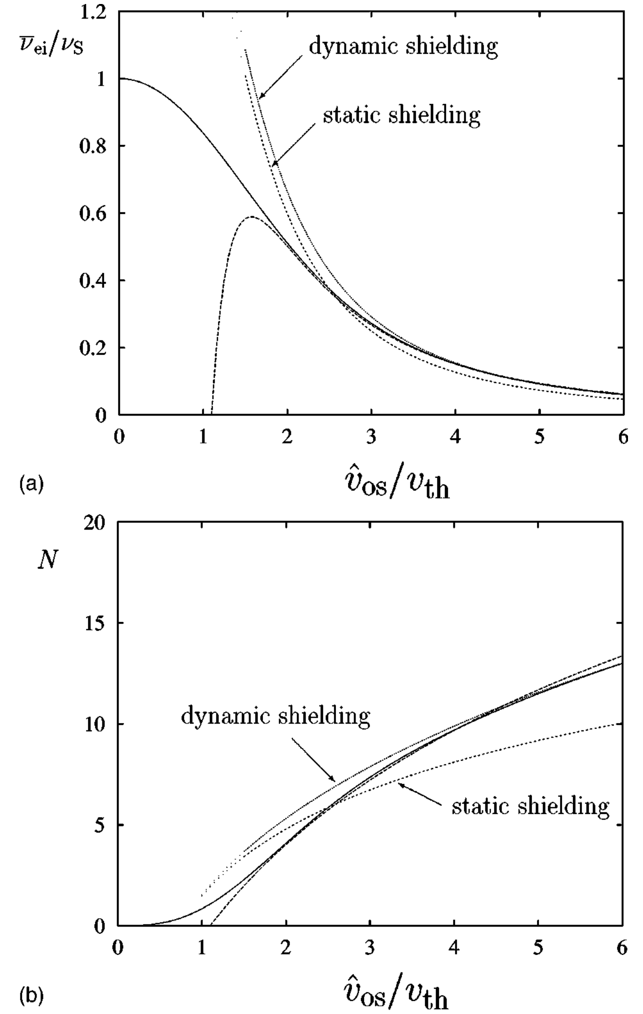

FIG. 4. (a), (b) $\overline{\nu}_{ei}/\nu_S$ and $N=(\overline{\nu}_{ei}/\nu_S)\hat{v}_{os}^3/v_{os}^3$, as in Fig. 3, except using the second line of Eq. (20) and $\ln \Lambda_{th}=10$.

$$\overline{\nu}_{ei}=4\frac{\beta^{3/2}}{\pi^{1/2}}\frac{\overline{K}}{m_e}\sum_{n\geqslant 1}(-1)^{n+1}\frac{2n}{2n+1}\frac{(2n)!}{2^{2n}(n!)^3}\hat{w}_{os}^{2n-2}. \tag{19}$$

This expression is valid for arbitrary ratios $\hat{v}_{os}/v_{th}$. For vanishing $\hat{v}_{os}$ the sum reduces to the constant term 1/3 from $n=1$ and hence reproducing again Spitzer's result Eq. (6). For $\hat{w}_{os}\gg 1$ it is not very practical because the series converges slowly. Instead, in the domain $\hat{w}_{os}>1$ a best fit to the series (19) in terms of a function $A\ln(\hat{v}_{os}/2v_{th}+C)/(\hat{v}_{os}/v_{th})^3$ is made, with the constants A,C to be adjusted. In Figs. 3(a) and 4(a) the sum from Eq. (19) times the normalized average Coulomb logarithm $\ln\overline{\Lambda}/\ln\Lambda_{th}$, $\Lambda_{th}=v_{th}/\omega_p$, is evaluated in the interval $0\leqslant\hat{v}_{os}/v_{th}\leqslant 6$. Above $\hat{v}_{os}/v_{th}\simeq 1.5$ the fitted curve agrees best with the series for the parameters A = 2.7 and C = 0.35 in the case of the thermal Coulomb logarithm $\ln \Lambda_{th}=3$ [Fig. 3(a)]. For $\ln\Lambda_{th}=10$ the best fit is A = 1.8, C = 0.45 [Fig. 4(a)]. In order to notify the deviations at large values $\hat{v}_{os}/v_{th}$ we plot in Figs. 3(b) and 4(b) the curves from (a) times $(\hat{v}_{os}/v_{th})^3$ for $\ln\Lambda_{th}=3$ and $\ln\Lambda_{th}=10$. Since they are also satisfactory we are led to the simple approximations for $\overline{\nu}_{ei}$ as follows:

$$\ln\Lambda_{th}=3:\ \overline{\nu}_{ei}=2\sqrt{\frac{2}{\pi}}\frac{Z^2e^4n_i}{4\pi\varepsilon_0^2m_e^2\hat{v}_{os}^3}\times 2.7\ln\left(\frac{\hat{v}_{os}}{2v_{th}}+0.35\right)\ln\Lambda_{th}, \tag{20}$$

$$\ln\Lambda_{th}=10:\ \overline{\nu}_{ei}=2\sqrt{\frac{2}{\pi}}\frac{Z^2e^4n_i}{4\pi\varepsilon_0^2m_e^2\hat{v}_{os}^3}\times 1.8\ln\left(\frac{\hat{v}_{os}}{2v_{th}}+0.45\right)\ln\Lambda_{th}.$$

In the published literature asymptotic expressions with static shielding are given as follows [3,1],

$$\overline{\nu_{ei}}=\frac{Z^2e^4n_i}{\pi^2\varepsilon_0^2m_e^2\hat{v}_{os}^3}\left(\ln\frac{\hat{v}_{os}}{2v_{th}}+1\right)\ln\Lambda_{th},$$

$$\Lambda_{th}=\frac{\lambda_D}{\max(b_\perp,\lambda_B/2)};\quad \hat{v}_{os}>2v_{th}. \tag{21}$$

In Figs. 3 and 4 the corresponding graphs are shown for $\ln\Lambda_{th}=3$ and 10 (dashed lines). In Fig. 3 the discrepancy between the two expressions amounts to a factor of almost 2 at high ratios $\hat{v}_{os}/v_{th}$. If however the thermal Coulomb logarithm $\ln\Lambda_{th}$ is substituted by the dynamically screened $\ln\overline{\Lambda}_{th}+\ln(1+\hat{v}_{os}^2/2v_{th}^2)$ the agreement with the fit in the first line of Eq. (20) becomes almost perfect (dotted lines in Figs. 3,4).

The fit used in this paper differs from the asymptotic form $\ln(\hat{v}_{os}/2v_{th})+1$ of Eq. (21), used also by Silin [1], Shima and Yatom [2], Catto and Speziale [2] and Decker *et al.* [3]. Both fits have their advantages. Equation (21) perfectly reproduces Eq. (19) for $\hat{v}_{os}/v_{th}\gg1$, however, only after the correction presented here. Figures 3 and 4 show that applicability of Eq. (21) corrected is guaranteed for $\hat{v}_{os}/v_{th}\simeq3$. For most applications ν_{ei} is needed at ratios $\hat{v}_{os}/v_{th}$ ranging typically from 1 to 5. The fit chosen here does not reproduce exactly the practically insignificant asymptotic limit; however, it agrees better in the intermediate regime as Figs. 3 and 4 show.

From their 2D numerical simulations Decker *et al.* [3] find an enhancement of $\overline{\nu_{ei}}$ for $\hat{v}_{os}/v_{th}\gg1$ (see their Fig. 5) which they attribute to ''correlated collisions.'' Such a correlation is certainly present, however, in our opinion (i) no distinction exists between oscillation amplitudes $x_0>\lambda_D$ and $x_0<\lambda_D$ and (ii) the increase in $\overline{\nu_{ei}}$ can entirely be explained by using the correct cutoffs from Eqs. (15) and (16), leading to correction factors $f_c=1+\ln(\hat{v}_{os}^2/2v_{th}^2+1)/\ln\Lambda_{th}$ and $f_c=1+\frac{3}{2}\ln(\hat{v}_{os}^2/2v_{th}^2+1)/\ln\Lambda_{th}$. So, for $\hat{v}_{os}/v_{th}=6.8$ they find an enhancement by a factor of 1.33. The corresponding f_c factors range from 2.1 to 1.3 for $\ln\Lambda_{th}$ varying from 3 to 10 (we do not know their actual Λ_{th} values). For $\hat{v}_{os}/v_{th}=11.5$, $\overline{\nu_{ei}}$ is increased by 1.8; our f_c factors vary from 3.1 to 1.4 for $3\leq\ln\Lambda_{th}\leq10$. The authors of the present paper believe that most of the difference between simulation and Eq. (21) has its origin in the dynamic shielding.

III. DISCUSSION

The ballistic model combined with the rigorous solution of the oscillator model for $\boldsymbol{v}_0=$const and $\boldsymbol{v}_0$ sinusoidal [4] is able to yield reliable time-dependent collision frequencies for Coulomb logarithms $\ln\Lambda\gtrsim1$. When cycle-averaged, expressions are obtained which agree with generally believed formulas, however, only after correcting them by appropriate cutoffs, partly known from the literature, whose physical interpretation is given in this paper. Generally only asymptotic formulas are deduced from the more general theories owing to their complexity, and no limits are given from what parameters $\hat{v}_{os}/v_{th}$ on they become acceptable (e.g., Refs. [1–3]). As a by-product of the present paper in one case (Ref. [3]) such limits can immediately be deduced from Figs. 3,4.

There is a general remarque to be made. In the standard procedure of inverse bremsstrahlung absorption a Fourier expansion in space and time is undertaken which means that a peaked function is approximated by periodic elements. It is evident that this is not adequate. Fourier representation only removes the differential operators by translating them into algebraic relations, but then one is left with a sum of multiple fast oscillating integrals of time-consuming evaluation and difficult interpretation. Finally, it has to be mentioned that the treatment of collisions in nonideal plasmas (density effects, negative Coulomb logarithms, large angle and multiple scattering) is not yet in an advanced stage, however, promising progress has been made [7,13].

IV. SUMMARY

We have used a ballistic model to calculate inverse bremsstrahlung absorption for any laser intensity-temperature ratio. Formulas for the time-dependent collision frequency $\nu_{ei}(t)$ are presented. For $v_{os}(t)$ periodic and an arbitrary electron distribution function $f(v_e)$ or a Maxwellian, $f=f_M$, respectively, we obtain

$$\nu_{ei}(t)=\frac{K}{m_ev_{os}^3(t)}\int_0^{v_{os}(t)}4\pi v_e^2f(v_e)dv_e,$$

$$K=\frac{Z^2e^4n_i}{4\pi\varepsilon_0^2m_e}\ln\Lambda,\quad f=f_M,\quad v_{os}/v_{th}<0.7:$$

$$\nu_{ei}(t)=\frac{1}{3}\sqrt{\frac{2}{\pi}}\frac{K}{m_ev_{th}^3}\exp\left(-\frac{v_{os}^2}{2v_{th}^2}\right),$$

$$f=f_M,\quad v_{os}/v_{th}\geq0.7:$$

$$\nu_{ei}(t)=\frac{2}{\pi^{1/2}}\frac{K}{m_ev_{os}^3(t)}\times\left\{\frac{\pi^{1/2}}{2}-\left[\frac{1}{w_{os}+\left(w_{os}^2+\frac{3}{2}\right)^{1/2}}+w_{os}\right]e^{-w_{os}^2}\right\}.$$

From cycle averaging the following expressions are deduced for $\overline{\nu_{ei}}$:

$$\overline{\nu_{ei}}=\frac{\overline{K}}{2\overline{E}_{kin}}\overline{\frac{1}{v_{os}(t)}\int_0^{v_{os}(t)}4\pi v_e^2f(v_e)dv_e},$$

$$v_{os}=\hat{v}_{os}|\cos\omega t|,\quad \omega\neq\omega_p;\quad f=f_M:$$

$$\overline{\nu_{ei}}=4\frac{\beta^{3/2}}{\pi^{1/2}}\frac{\overline{K}}{m_e}\sum_{n\geqslant 1}(-1)^{n+1}\frac{2n}{2n+1}\frac{(2n)!}{2^{2n}(n!)^3}\hat{w}_{os}^{2n-2}.$$

$2\lesssim\hat{v}_{os}\lesssim 10$: Eqs. (20) and (21) may be used.

$$\hat{v}_{os}\leqslant 1.5v_{th}:$$

$$\overline{\nu_{ei}}=\frac{1}{3}\sqrt{\frac{2}{\pi}}\frac{Z^2e^4n_i}{4\pi\varepsilon_0^2m_e^2v_{th}^3}\left\{1-\frac{9}{40}\left(\frac{\hat{v}_{os}}{v_{th}}\right)^2+\frac{15}{448}\left(\frac{\hat{v}_{os}}{v_{th}}\right)^4\right\}\overline{\ln\Lambda}.$$

The time-dependent collision frequencies, Eqs. (4),(19), deduced from a ballistic model are valid for arbitrary oscillatory motions and arbitrary isotropic distribution functions f in the domain of ideal plasmas ($\ln\Lambda\gtrsim 1$). There is agreement between $\overline{\nu_{ei,\mathrm{ballistic}}}$ and $\overline{\nu_{ei,\mathrm{dielectric}}}$. Dynamic screening is essential. Corresponding formulas for $\nu_{ei}(t)$ and $\overline{\nu_{ei}}$ in nonideal quantum plasmas have been obtained recently [7].

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