Statistical theory of dusty plasmas: Microscopic equations and Bogolyubov-Born-Green-Kirkwood-Yvon hierarchy

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Basic principles of statistical theory of dusty plasmas are formulated with regard for electron and ion absorption by dust particles. Rigorous microscopic equations are introduced and employed to derive the BBGKY hierarchy and kinetic equations. The charging processes are shown to induce a considerable modification of both microscopic and kinetic equations for plasma particles and grains. In the approximation of dominant influence of charging collisions, explicit kinetic equations are derived and applied to calculate stationary distributions of grain velocities and charges.

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I. INTRODUCTION

Recently many attempts have been made to work out the kinetic theory of dusty plasmas (see, for example, Refs. [1–8]). One of the main purposes of such theory is to describe self-consistently the grain charging dynamics associated with the absorption of plasma particles. In order to do this, it was proposed [1] to treat the grain charge as a new variable that introduces the principal possibility to study the grain charge distribution on equal footing with spatial and velocity distributions. Such formalism was successfully applied to investigate waves and fluctuations in dusty plasmas [1,5–8] with due regard for the self-consistent dynamics of grain charging in terms of various approximate versions of kinetic equations for grains and plasma particles.

Another important point of the kinetic description of dusty plasmas is to take into account the influence of electron and ion absorption by grains on the grain motion. It was shown in Ref. [9] that in spite of large difference between the masses of plasma particles and grains, such influence could be rather essential, similarly to the case of Brownian particle motion in gases. In particular, absorption of plasma particles by grains results in the bombardment force acting on grains that could make a reason for grain attraction at large distances. Being introduced in the equation for the grain density fluctuation evolution, these forces produce long-range grain correlations [5].

Obviously, in order to involve both effects caused by electron and ion absorption (charge and momentum transfer from plasma particles to grains) into a self-consistent description, appropriate collision integrals should be introduced in the kinetic equations along with the modified collision terms responsible for the elastic Coulomb collisions.

However, solving this important problem is complicated by the mutual influence of charging processes and Coulomb collisions. Moreover, dusty plasmas are usually characterized by strong coupling between plasma particles and grains that requires considerable improvement of the traditional perturbation approaches. That is why a rigorous kinetic theory of dusty plasmas with regard for consistent treatment of grain charging and particle collisions (both elastic and inelastic) has not yet been formulated. Recent progress in this field has been achieved by deriving kinetic equations in terms of various approximations. For example, in Refs. [6,7] the main attention was paid to the effects of charging collisions (i.e., to the problem of charge and momentum transfer in course of plasma particle absorption by grains), however, the explicit form of the collision integrals describing the elastic Coulomb collisions was not specified. In Ref. [8] the kinetic equation for grains was derived under the assumption that the discreteness of both electron and ion components could be neglected. Such approximation excludes from consideration the bombardment forces and the effects of grain diffusion in the charge and velocity spaces.

The purpose of this paper is to propose a consistent approach to the derivation of kinetic equations for dusty plasmas on the basis of the first principles of statistical mechanics. We start from the formulation of rigorous microscopic equations (equations for microscopic phase densities) for dusty plasmas taking into account electron and ion absorption and contact grain-grain collisions explicitly (Sec. II). Such equations differ from the traditional microscopic (Klimontovich) equations for ordinary plasmas which are continuity equations in the six-dimensional phase space. In the case under consideration, absorption of plasma particles by grains generates additional sources in the microscopic equations for both plasma particles and grains. The physical reason for such sources is evident: disappearance of plasma particles and abrupt changes of grain momenta and charges due to inelastic collisions.

The microscopic equations obtained are used to formulate the Bogolyubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy of equations for dusty plasmas (Sec. III). The additional sources entering the microscopic equations give rise to new terms in the equations of the hierarchy. In the case of systems consisting of grains only (no plasma particles), the new hierarchy reduces to the relevant hierarchy for the hard-sphere gas [10].

Possible application of the results obtained to the derivation of kinetic equations is discussed in Sec. IV. In terms of various approximations, kinetic equations derived by other authors can be reproduced, in particular, the equations proposed in Refs. [6,7] for the case of dominant influence of charging collisions. The obtained kinetic equations are applied to calculate stationary distributions of grain velocities and charges (Sec. V).

II. EQUATIONS FOR MICROSCOPIC PHASE DENSITIES IN DUSTY PLASMAS

Let us consider a dusty plasma consisting of electrons, ions, and monodispersed finite-size dust particles (grains) under the assumption that each grain absorbs all encountered electrons and ions.

In the case under consideration, the microscopic phase density (microscopic distribution function) associated with the ith plasma particle (electron or ion) can be written as

$$N_{i\sigma}(X,t)=\delta(X-X_{i\sigma}(t))\theta(t_{i\sigma}-t). \tag{1}$$

Here, $X\equiv(\mathbf{r},\mathbf{v})$, $X_{i\sigma}(t)\equiv(\mathbf{r}_{i\sigma}(t),\mathbf{v}_{i\sigma}(t))$ is the phase trajectory, $t_{i\sigma}$ is the time of ith particle collision with any grain, the subscript σ labels plasma particle species ($\sigma=e,i$), and $\theta(x)$ is the Heaviside step function,

$$\theta(x)=\begin{cases}1, & x\geqslant 0,\\ 0, & x<0.\end{cases} \tag{2}$$

The microscopic phase density describing the subsystem of plasma particles of species σ is given by

$$N_{\sigma}(X,t)=\sum_{i=1}^{N_\sigma} N_{i\sigma}(X,t)=\sum_{i=1}^{N_\sigma}\delta(X-X_{i\sigma}(t))\theta(t_{i\sigma}-t), \tag{3}$$

where N_σ can be regarded as the number of particles existing until time t (unabsorbed particles).

The above definition of microscopic distribution differs from the traditional one by the presence of the Heaviside step function $\theta(t_{i\sigma}-t)$ describing plasma particle disappearance at the time instants of their collisions with grains.

According to Eqs. (1) and (2) the value of $N_{i\sigma}(X,t)$ at the instant of plasma particle collision could be treated as

$$N_{i\sigma}(X,t_{i\sigma})=\lim_{\varepsilon\to 0}N_{i\sigma}(X,t_{i\sigma}-\varepsilon)=\lim_{\varepsilon\to 0}\delta(X-X_{i\sigma}(t_{i\sigma}-\varepsilon)), \tag{4}$$

i.e., we define this value as the limit of $N_\sigma(X,t)$ on the left.

Combining the derivatives of $N_{i\sigma}(X,t)$ over t, $\mathbf{r}$, and $\mathbf{v}$, and taking into account that plasma particle trajectories are governed by the equations of motion

$$\frac{d\mathbf{r}_{i\sigma}(t)}{dt}=\mathbf{v}_{i\sigma}(t);\qquad \frac{d\mathbf{v}_{i\sigma}(t)}{dt}=\frac{1}{m_\sigma}\mathbf{F}_\sigma(\mathbf{r}_{i\sigma}(t),t), \tag{5}$$

where $\mathbf{F}_\sigma(\mathbf{r},t)=e_\sigma\mathbf{E}(\mathbf{r},t)$ is the Lorentz force generated by the microscopic electric field $\mathbf{E}(\mathbf{r},t)$, it is easy to show that $N_{i\sigma}(X,t)$ satisfies the following equation:

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial\mathbf{r}}+\frac{1}{m_\sigma}\mathbf{F}_\sigma\frac{\partial}{\partial\mathbf{v}}\right\}N_{i\sigma}(X,t) =-\delta(X-X_{i\sigma}(t))\delta(t-t_{i\sigma}). \tag{6}$$

Since $t_{i\sigma}$ is the time of ith particle collision with some definite grain, we can use the following relation:

$$\begin{aligned}\delta(t-t_{i\sigma})&=\delta(|\mathbf{r}_{i\sigma}(t)-\mathbf{r}_g(t)|-a)\left|\frac{\partial|\mathbf{r}_{i\sigma}(t)-\mathbf{r}_g(t)|}{\partial t}\right|_{t=t_{i\sigma}}\\ &=\delta(|\mathbf{r}_{i\sigma}(t)-\mathbf{r}_g(t)|-a)\\ &\quad\times|\mathbf{e}_{\mathbf{r}_{i\sigma}(t)-\mathbf{r}_g(t)}(\mathbf{v}_{i\sigma}(t)-\mathbf{v}_g(t))|,\end{aligned} \tag{7}$$

where $\mathbf{e}_\mathbf{r}=\mathbf{r}/r$, $\mathbf{r}_g(t)$ and $\mathbf{v}_g(t)$ are the coordinate and velocity of the grain with which the ith particle collides.

It is useful to note that for particles approaching the grain, we have

$$\frac{\partial}{\partial t}|\mathbf{r}_{i\sigma}(t)-\mathbf{r}_g(t)|_{t=t_{i\sigma}}=\mathbf{e}_{\mathbf{r}_{i\sigma}(t)-\mathbf{r}_{g(t)}}(\mathbf{v}_{i\sigma}(t)-\mathbf{v}_g(t))|_{t=t_{i\sigma}}<0. \tag{8}$$

This means that $\vartheta(t_{i\sigma})>\pi/2$, where $\vartheta(t)$ is the angle between the vectors $\mathbf{r}_{i\sigma}(t)-\mathbf{r}_g(t)$ and $\mathbf{v}_{i\sigma}(t)-\mathbf{v}_g(t)$.

With regard for Eqs. (2), (7), and (8) the equation for $N_{i\sigma}(X,t)$ takes the form

$$\begin{aligned}&\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial\mathbf{r}}+\frac{1}{m_\sigma}\mathbf{F}_\sigma\frac{\partial}{\partial\mathbf{v}}\right\}N_{i\sigma}(X,t)\\ &\quad=-\int d\mathcal{X}'\,\delta(\mathcal{X}'-\mathcal{X}_g(t))\delta(|\mathbf{r}-\mathbf{r}'|-a)\\ &\qquad\times|\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')|N_{i\sigma}(X,t)\end{aligned}$$

or

$$\begin{aligned}&\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial\mathbf{r}}+\frac{1}{m_\sigma}\mathbf{F}_\sigma\frac{\partial}{\partial\mathbf{v}}\right\}N_{i\sigma}(X,t)\\ &\quad=\int_{(\vartheta>\pi/2)}d\mathcal{X}'\,\delta(\mathcal{X}'-\mathcal{X}_g(t))\delta(|\mathbf{r}-\mathbf{r}'|-a)\\ &\qquad\times\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')N_{i\sigma}(X,t).\end{aligned} \tag{9}$$

Here, ϑ is the angle between $\mathbf{r}-\mathbf{r}'$ and $\mathbf{v}-\mathbf{v}'$,

$$\delta(\mathcal{X}-\mathcal{X}_g(t))\equiv\delta(\mathbf{r}-\mathbf{r}_g(t))\delta(\mathbf{v}-\mathbf{v}_g(t))\delta(q-q_g(t)),$$

i.e., we introduce an extended phase variable $\mathcal{X}\equiv(\mathbf{r},\mathbf{v},q)$ which includes the grain charge q as a new variable [1]. In such extended phase space, the trajectory $\mathcal{X}_g(t)$ determines the microscopic state of the grain. Integration over $\mathcal{X}'$ in Eqs. (9) with regard for the condition $\vartheta>\pi/2$ is equivalent to the treatment of the contact value $N_{i\sigma}(X,t_{i\sigma})$ in terms of Eq. (4).

Since only one grain can be at the phase point $\mathcal{X}$ at the instant $t_{i\sigma}$ (i.e., the ith plasma particle can collide only with one grain), the expression $\delta(\mathcal{X}'-\mathcal{X}_g(t))$ in Eqs. (9) can be replaced by the complete microscopic grain density

$$N_g(\mathcal{X},t)=\sum_{k=1}^{N_g}\delta(\mathcal{X}-\mathcal{X}_k(t)) \tag{10}$$

and thus, the equation for the complete microscopic phase density of plasma particles (3) can be written as

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{1}{m_\sigma}\mathbf{F}_\sigma\frac{\partial}{\partial \mathbf{v}}\right\}N_\sigma(X,t)$$
$$=-\int d\mathcal{X}'|\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')|$$
$$\times\delta(|\mathbf{r}-\mathbf{r}'|-a)N_g(\mathcal{X}',t)N_\sigma(X,t). \tag{11}$$

The right-hand part of Eq. (11) with regard for the condition $\vartheta>\pi/2$ can be interpreted as a microscopic flux of particles of species σ through the grain surface, i.e., the flux of particles absorbed by the grain.

It should be noted that in view of the fact that grain trajectories have discontinuities of the first order (detailed discussion of this point is given below), the microscopic phase density $N_g(\mathcal{X},t)$ at time $t=t_{i\sigma}$ (such time instants correspond to the condition $|\mathbf{r}_{i\sigma}-\mathbf{r}'|=a$) in Eq. (11) should be treated in the same sense as $N_\sigma(X,t_{i\sigma})$, i.e.,

$$N_g(\mathcal{X}',t_{i\sigma})=\lim_{\varepsilon\to 0}N_g(\mathcal{X}',t_{i\sigma}-\varepsilon).$$

The next problem is to derive the microscopic equation for the grain phase density. Before doing this we have to define such density for the times of grain collisions with plasma particles or other grains. The point is that at the collision instant the grain trajectory $\mathcal{X}_k(t)$ changes abruptly and thus the microscopic distribution (10) becomes indefinite and nonintegrable. In order to get rid of nonintegrability we divide the time axis into time segments between collisions (such division is individual for each grain) and write the grain phase trajectory as a sum of its relevant parts. For example, for the ith grain we have

$$\mathcal{X}_i(t)=\mathcal{X}_i^{(0)}(t)\,\theta(t-t_0)\,\theta(t_1-t)+\mathcal{X}_i^{(1)}(t)\,\theta(t-t_1)\,\theta(t_2-t)$$
$$+\mathcal{X}_i^{(2)}(t)\,\theta(t-t_2)\,\theta(t_3-t)+\cdots. \tag{12}$$

Here t_0 is the evolution initiation time, $t_1<t_2<t_3\ldots$ are the times of the first, second, etc. collision of the ith grain with any other particle,

$$\mathcal{X}_i^{(n)}(t)\equiv(\mathbf{r}_i(t),\mathbf{v}_i^{(n)}(t),q_i^{(n)}), \tag{13}$$

$$\mathbf{r}_i(t)=\mathbf{r}_{i0}+\int_{t_0}^{t}dt'\,\mathbf{v}_i(t')$$
$$=\mathbf{r}_{i0}+\int_{t_0}^{t_1}dt'\,\mathbf{v}^{(0)}(t')+\int_{t_1}^{t_2}dt'\,\mathbf{v}^{(1)}(t')+\cdots, \tag{14}$$

$$\mathbf{v}_i^{(0)}(t)=\mathbf{v}_{i0}+\frac{q_{i0}}{m_g}\int_{t_0}^{t}dt'\,\mathbf{E}(\mathbf{r}_i(t),t),$$
$$\mathbf{v}_i^{(n)}(t)=\mathbf{v}_i^{(n-1)}(t_n)+\delta\mathbf{v}_i^{(n)}+\frac{q_i}{m_g}\int_{t_n}^{t}dt'\,\mathbf{E}(\mathbf{r}_i(t'),t'),$$
$$n>0, \tag{15}$$

$$q_i^{(0)}=q_{i0},$$
$$q_i^{(n)}=q_i^{(n-1)}+\delta q_i^{(n)},\quad n>0, \tag{16}$$

$\mathbf{r}_{i0}$, $\mathbf{v}_{i0}$ and q_{i0} are initial phase coordinates of the ith grain, $\delta\mathbf{v}_i^{(n)}$ and $\delta q_i^{(n)}$ are the jumps of the grain velocity and charge due to the nth collision of the ith grain with other particles which depend on the species of the particle involved in the nth collision, namely

$$\delta q_i^{(n)}=\begin{cases} e_\sigma & \text{for a collision with a plasma particle,}\\ \frac{1}{2}(q'-q_i^{(n-1)}) & \text{for a collision with another grain with the charge } q', \end{cases} \tag{17}$$

$$\delta\mathbf{v}_i^{(n)}=\begin{cases} -\dfrac{m_\sigma}{m_\sigma+m_g}(\mathbf{v}_i^{(n-1)}(t_n)-\mathbf{v}'_\sigma), & \text{for a collision with a plasma particle with the velocity } \mathbf{v}'_\sigma,\\ -\mathbf{e}_{\mathbf{r}_i(t_n)-\mathbf{r}'}(\mathbf{e}_{\mathbf{r}_i(t_n)-\mathbf{r}'}[\mathbf{v}_i^{(n-1)}(t_n)-\mathbf{v}']), & \text{for a collision with another grain with the velocity } \mathbf{v}'. \end{cases} \tag{18}$$

When deriving Eqs. (17) and (18) we assumed the grain masses and sizes to be equal and the two-grain collisions result in the equalization of their charges.

The function that reproduces $\delta(\mathcal{X}-\mathcal{X}_i(t))$ at any point where $\delta(\mathcal{X}-\mathcal{X}_i(t))$ is defined, can be written as

$$N_{ig}(\mathcal{X},t)=\sum_{n=0}^{N_{\mathrm{col}}(t)}\delta(\mathcal{X}-\mathcal{X}_i^{(n)}(t))\,\theta(t_{n+1}-t)\,\theta(t-t_n), \tag{19}$$

where $N_{\rm col}(t)$ is the number of ith grain collisions till time t. The representation (19) has the advantage that the function $N_{ig}(\mathcal{X},t)$ becomes integrable in contrast to the expression for $\delta(\mathcal{X}-\mathcal{X}_i(t))$.

Taking the derivatives of $N_{ig}(\mathcal{X},t)$ over t, $\mathbf{r}$, $\mathbf{v}$ we can show that

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{1}{m_g}\mathbf{F}_g\frac{\partial}{\partial \mathbf{v}}\right\}N_{ig}(\mathcal{X},t)=\delta(\mathcal{X}-\mathcal{X}_i(t))\delta(t-t_0)-\sum_{n=1}^{N_{\rm col}}\delta(t-t_n)[\delta(\mathcal{X}-\mathcal{X}_i^{(n-1)}(t_n))-\delta(\mathcal{X}-\mathcal{X}_i^{(n)}(t_n))], \tag{20}$$

where

$$\mathbf{F}_g=\mathbf{F}_g(\mathbf{r},q,t)=q\mathbf{E}(\mathbf{r},t),\quad \mathcal{X}_i^{(n)}(t_n)=\mathcal{X}_i^{(n-1)}(t_n)+\delta\mathcal{X}_i^{(n)},\quad \delta\mathcal{X}_i^{(n)}\equiv(0,\delta\mathbf{v}_i^{(n)},\delta q^{(n)}).$$

Similarly to the case of plasma particles we can use the relation

$$\delta(t-t_n)=\delta(|\mathbf{r}_i(t)-\mathbf{r}_c(t)|-\tilde{a})|\mathbf{e}_{\mathbf{r}_i(t)-\mathbf{r}_c(t)}(\mathbf{v}_i(t)-\mathbf{v}_c(t))|,$$

where $\mathbf{r}_c(t)$ and $\mathbf{v}_c(t)$ are the coordinate and velocity of the particle (electron, ion, or grain) with which the ith grain is involved into nth collision, $\tilde{a}=a$ for grain-plasma collisions and $\tilde{a}=2a$ for grain-grain collisions.

For $t_0\to-\infty$, we obtain an equation for $N_{ig}(\mathcal{X},t)$ given by

$$\begin{aligned}\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{1}{m_g}\mathbf{F}_g\frac{\partial}{\partial \mathbf{v}}\right\}N_{ig}(\mathcal{X},t)=&-\sum_{\sigma=e,i}\int dX'\,\delta(|\mathbf{r}-\mathbf{r}'|-a)\{|\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')|N_{ig}(\mathbf{r},\mathbf{v},q,t)\\&-|\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}'-\delta\mathbf{v}_\sigma)|N_{ig}(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_\sigma,q-e_\sigma,t)\}N_\sigma(X',t)\\&-\int d\mathcal{X}'\,\delta(|\mathbf{r}-\mathbf{r}'|-2a)|\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')|\{N_{ig}(\mathbf{r},\mathbf{v},q,t)N_g(\mathbf{r}',\mathbf{v}',q',t)\\&-N_{ig}(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_g,q-\delta q,t)N_g(\mathbf{r}',\mathbf{v}'+\delta\mathbf{v}_g,q',t)\}.\end{aligned} \tag{21}$$

Here

$$\delta\mathbf{v}_\sigma\equiv\delta\mathbf{v}_\sigma(\mathbf{v},\mathbf{v}')=-\frac{m_\sigma}{m_g}(\mathbf{v}-\mathbf{v}'),\quad \delta\mathbf{v}_g\equiv\delta\mathbf{v}_g(\mathbf{v},\mathbf{v}')=\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')),\quad \delta q\equiv\delta q(q,q')=q'-q. \tag{22}$$

The equation for the complete microscopic density

$$N_g(\mathcal{X},t)=\sum_{i=1}^{N}N_{ig}(\mathcal{X},t) \tag{23}$$

is given by

$$\begin{aligned}\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{1}{m_g}\mathbf{F}_g\frac{\partial}{\partial \mathbf{v}}\right\}N_g(\mathcal{X},t)=&-\sum_{\sigma=e,i}\int d\mathcal{X}'\,\delta(|\mathbf{r}-\mathbf{r}'|-a)[|\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')|N_g(\mathcal{X},t)-|\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\delta\mathbf{v}_\sigma-\mathbf{v}')|\\&\times N_g(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_\sigma,q-e_\sigma)]N_\sigma(X',t)-\int d\mathcal{X}'\,\delta(|\mathbf{r}-\mathbf{r}'|-2a)|\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')|\\&\times\{N_g(\mathcal{X},t)N_g(\mathcal{X}',t)\quad -N_g(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_g,q-\delta q,t)N_g(\mathbf{r}',\mathbf{v}'+\delta\mathbf{v}_g,q',t)\}.\end{aligned} \tag{24}$$

The last term in Eq. (24) describes the contribution of elastic grain-grain contact collisions which in contrast to elastic Coulomb collisions could not be taken into account by the terms with the microscopic force in the left-hand part of the equation.

It should be noted that in view of the condition of the type (8), integration over X' and $\mathcal{X}'$ in the right-hand part of Eq. (24) is restricted to the domain $\vartheta\geqslant\pi/2$ where ϑ is the angle between $\mathbf{r}-\mathbf{r}'$ and $\mathbf{v}-\mathbf{v}'$ (or, $\mathbf{v}-\mathbf{v}'-\delta\mathbf{v}_\sigma$).

Thus, we have derived rigorous microscopic equations (11) and (24) which describe particle motion in microscopic electric fields with regard for plasma particles absorption by grains and contact grain-grain collisions. These equations are valid for arbitrary stages of the system evolution and take into account the discreteness of all the plasma components.

If we formally regard $N_g(\mathcal{X},t)$ and $N_\sigma(X,t)$ as conventional (nongeneralized) functions and assume that $|e_\sigma|\ll q$ and $\mathbf{v}-\delta\mathbf{v}_\sigma=\mathbf{v}(1+m_\sigma/m_g)-(m_\sigma/m_g)\mathbf{v}'\simeq\mathbf{v}-(m_\sigma/m_g)\mathbf{v}'$, then we can expand Eq. (24) in terms of e_σ and $(m_\sigma/m_g)\mathbf{v}'$. The result is given by

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{1}{m_g}\mathbf{F}_g\frac{\partial}{\partial \mathbf{v}}\right\}N_g(\mathcal{X},t)=\frac{\partial}{\partial q}\left\{\frac{\partial}{\partial q}[QN_g(\mathcal{X},t)]-IN_g(\mathcal{X},t)\right\}+\frac{\partial}{\partial v_i}\left\{\frac{\partial}{\partial v_j}[D_{ij}N_g(\mathcal{X},t)]-\frac{1}{m_g}F_{bi}N_g(\mathcal{X},t)\right.$$
$$\left.-\frac{q}{m_g}F_{\mathrm{ch}i}\frac{\partial N_g(\mathcal{X},t)}{\partial q}\right\}-\int d\mathcal{X}''\,\delta(|\mathbf{r}-\mathbf{r}'|-2a)|\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')|[N_g(\mathcal{X},t)N_g(\mathcal{X}'',t)$$
$$-N_g(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_g,q-\delta q,t)N_g(\mathbf{r}',\mathbf{v}'+\delta\mathbf{v}_g,q',t)], \tag{25}$$

where

$$Q=\sum_\sigma \frac{e_\sigma^2}{2}\int dX'N_\sigma(X',t)\,\delta(|\mathbf{r}-\mathbf{r}'|-a)|\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')|,$$

$$I=\sum_\sigma e_\sigma\int dX'N_\sigma(X',t)\,\delta(|\mathbf{r}-\mathbf{r}'|-a)|\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')|,$$

$$D_{ij}=\sum_\sigma \frac{1}{2}\left(\frac{m_\sigma}{m_g}\right)^2\int dX'v_i'v_j'N_\sigma(X',t)\,\delta(|\mathbf{r}-\mathbf{r}'|-a)|\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')|, \tag{26}$$

$$\mathbf{F}_b=\sum_\sigma m_\sigma\int dX'\mathbf{v}'N_\sigma(X',t)\,\delta(|\mathbf{r}-\mathbf{r}'|-a)|\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')|,$$

$$\mathbf{F}_{\mathrm{ch}}=-\sum_\sigma \frac{e_\sigma}{q}m_\sigma\int dX'\mathbf{v}'N_\sigma(X',t)\,\delta(|\mathbf{r}-\mathbf{r}'|-a)|\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')|.$$

Neglecting the last term in Eq. (25) associated with contact grain-grain collisions and assuming that the quantities D_{ij}, Q and $(1/m_g)\mathbf{F}_{\mathrm{ch}}$ are small, we obtain

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{1}{m_g}\mathbf{F}_g\frac{\partial}{\partial \mathbf{v}}+\frac{1}{m_g}\mathbf{F}_{\mathrm{b}}\frac{\partial}{\partial \mathbf{v}}+I\frac{\partial}{\partial q}\right\}N_g(\mathcal{X},t)=0. \tag{27}$$

Here the velocity dependence of the bombardment force $\mathbf{F}_{\mathrm{b}}$ is also disregarded. With the accuracy up to this force, Eq. (27) reproduces the equation derived in Ref. [8] under the assumption that the discreteness of plasma particles is unimportant.

It should be noted, however, that though the quantities D_{ij}, Q and $\mathbf{F}_{\mathrm{ch}}$ are really small, they can be important for the description of grain dynamics. This becomes evident if one draws attention to the analogy between grains in plasmas and Brownian particles in fluids. In the latter case, the difference of particle masses is also very large, however, the Brownian particle motion cannot be described disregarding its diffusion in the velocity space. This makes the reason why equations of the type (27) cannot be used for the consistent description of the grain subsystem.

Finally, we notice, that Eq. (27) can also be derived by combining the derivatives of the microscopic grain density defined by Eq. (10) [i.e., disregarding the nonintegrability of $N_g(\mathcal{X},t)$ at the collision instance] over $\mathbf{r}$, $\mathbf{v}$ and q. Such derivation as well as the expansion of Eq. (24) in terms of e_σ and $(m_\sigma/m_g)\mathbf{v}$ with the accuracy up to the first order introduces a considerable inaccuracy: the contribution of discrete changes of the grain phase trajectory is disregarded. For Eq. (27) to be valid, both the function $N_g(\mathcal{X},t)$ and its derivatives must be integrable for any time, as in the case of continuously changing $q_i(t)$ and $\mathbf{v}_i(t)$. We can assume such continuity on the basis of physical reasons, but in this case the time- and phase-variable differentials dt, $d\mathbf{v}$, dq cannot be smaller than the relevant physically infinitesimal intervals during which many collisions with grains occur and we can use smoothened trajectories instead of microscopic trajectories. This means that Eq. (24) loses its microscopic nature.

III. BBGKY HIERARCHY FOR DUSTY PLASMAS

Once we have the equations for the microscopic phase density, it is possible to derive the BBGKY hierarchy for the distribution functions of the system under consideration. Before doing this we rewrite Eqs. (11) and (24) in a unified form using the notation for the microscopic phase density of all particle species given by

$$N_\sigma(\mathcal{X},t)=\begin{cases}N_\sigma(X,t)\,\delta(q-e_\sigma), & \sigma=e,i,\\ N_g(\mathcal{X},t), & \sigma=g.\end{cases} \tag{28}$$

In terms of this notation the unified microscopic equation can be rewritten as

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{1}{m_\sigma}\mathbf{F}_\sigma^{\rm ext}\frac{\partial}{\partial \mathbf{v}}-\sum_{\sigma'=e,i,g}\int d\mathcal{X}'\hat{V}_{\sigma\sigma'}(\mathcal{X},\mathcal{X}')N_\sigma(\mathcal{X}',t)\right\}N_\sigma(\mathcal{X},t)$$

$$=-\sum_{\sigma'=e,i,g}\int d\mathcal{X}'\{W^{(1)}_{\sigma\sigma'}(\mathbf{r}-\mathbf{r}',\mathbf{v}-\mathbf{v}')N_\sigma(\mathcal{X},t)N_{\sigma'}(\mathcal{X}',t)-W^{(2)}_{\sigma\sigma'}(\mathbf{r}-\mathbf{r}',\mathbf{v}-\mathbf{v}'-\delta\mathbf{v}_{\sigma\sigma'}-\delta\mathbf{v}'_{\sigma\sigma'})$$

$$\times N_\sigma(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_{\sigma\sigma'},q-\delta q_{\sigma'},t)N_{\sigma'}(\mathbf{r}',\mathbf{v}'+\delta\mathbf{v}'_{\sigma\sigma'},q',t)\},\quad \sigma=e,i,g. \tag{29}$$

Here, we introduce the Lorentz force $\mathbf{F}_\sigma^{\rm ext}$ associated with the external electromagnetic field, if present, and the notation

$$\hat{V}_{\sigma\sigma'}(\mathcal{X},\mathcal{X}')=\frac{qq'}{m_\sigma}\frac{\partial}{\partial \mathbf{r}}\frac{1}{|\mathbf{r}-\mathbf{r}'|}\frac{\partial}{\partial \mathbf{v}},$$

$$W^{(1)}_{\sigma\sigma'}(\mathbf{r},\mathbf{v})=-\mathbf{e}_{\mathbf{r}}\mathbf{v}\delta(r-a_{\sigma\sigma'})\theta(\vartheta-\pi/2)(\delta_{\sigma g}+\delta_{\sigma' g}-\delta_{\sigma g}\delta_{\sigma' g}),\quad \vartheta=(\widehat{\mathbf{r},\mathbf{v}}),$$

$$W^{(2)}_{\sigma\sigma'}(\mathbf{r},\mathbf{v})=-\mathbf{e}_{\mathbf{r}}\mathbf{v}\delta(r-a_{\sigma\sigma'})\theta(\vartheta-\pi/2)\delta_{\sigma g},\quad a_{\sigma\sigma'}=a_\sigma+a_{\sigma'},\quad a_\sigma=\begin{cases}0, & \sigma=e,i,\\ a, & \sigma=g,\end{cases} \tag{30}$$

$$\delta q_\sigma\equiv\delta q_\sigma(q,q')=\begin{cases}e_\sigma, & \sigma=e,i,\\ q'-q, & \sigma=g,\end{cases}$$

$$\delta\mathbf{v}_{\sigma\sigma'}\equiv\delta\mathbf{v}_{\sigma\sigma'}(\mathbf{v},\mathbf{v}')=\delta_{\sigma g}\delta\mathbf{v}_{\sigma'}(\mathbf{v},\mathbf{v}'),\quad \delta\mathbf{v}'_{\sigma\sigma'}\equiv\delta\mathbf{v}'_{\sigma\sigma'}(\mathbf{v},\mathbf{v}')=\delta_{\sigma g}\delta_{\sigma' g}\delta\mathbf{v}_g(\mathbf{v},\mathbf{v}'),$$

$$\delta\mathbf{v}_{\sigma'}(\mathbf{v},\mathbf{v}')=\begin{cases}-\dfrac{m_{\sigma'}}{m_g}(\mathbf{v}-\mathbf{v}'), & \sigma'=e,i\\ \mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')), & \sigma'=g,\end{cases}\qquad \delta_{\sigma g}=\begin{cases}0, & \sigma=e,i,\\ 1, & \sigma=g.\end{cases}$$

The formal representation of Eq. (29) can be further simplified in terms of phase variables supplemented with the component that describes particle species.

For example, in terms of the variable

$$\xi\equiv(\mathcal{X},\sigma)\equiv(\mathbf{r},\mathbf{v},q,\sigma),\quad \sum_{\sigma=e,i,g}\int d\mathcal{X}\equiv\int d\xi,\quad N_\sigma(\mathcal{X},t)\equiv N(\xi,t),$$

Eq. (29) reduces to

$$\left\{\frac{\partial}{\partial t}+\hat{L}(\xi)\right\}N(\xi,t)=-\int d\xi'\{[W^{(1)}(\xi,\xi')-\hat{V}(\xi,\xi')]N(\xi,t)N(\xi',t)$$

$$-W^{(2)}(\xi-\Delta_{\xi\xi'},\xi'+\Delta'_{\xi\xi'})N(\xi-\Delta_{\xi\xi'},t)N(\xi'+\Delta'_{\xi\xi'},t)\}, \tag{31}$$

where

$$\hat{L}(\xi)\equiv\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{1}{m_\sigma}\mathbf{F}_\sigma^{\rm ext}\frac{\partial}{\partial \mathbf{v}},\quad W^{(1),(2)}(\xi,\xi')\equiv W^{(1),(2)}_{\sigma\sigma'}(\mathbf{r}-\mathbf{r}',\mathbf{v}-\mathbf{v}'), \tag{32}$$

$$\Delta_{\xi\xi'}\equiv\Delta_{\sigma\sigma'}(\delta\mathbf{v}_{\sigma\sigma'},\delta q_{\sigma\sigma'})=(0,\delta\mathbf{v}_{\sigma\sigma'},\delta q_{\sigma\sigma'},0),\quad \delta q_{\sigma\sigma'}=\delta_{\sigma g}\delta q_{\sigma'},\quad \Delta'_{\xi\xi'}\equiv\Delta'_{\sigma\sigma'}(\delta\mathbf{v}'_{\sigma\sigma'})=(0,\delta\mathbf{v}'_{\sigma\sigma'},0,0).$$

Representation (32) is the most convenient one for a formal transformation of microscopic equations, in particular, for the derivation of the BBGKY hierarchy.

The first equation of this hierarchy is obtained by statistical averaging of Eq. (31). We have

$$\left\{\frac{\partial}{\partial t}+\hat{L}(\xi)\right\}f_1(\xi,t)=-\int d\xi'\{[W^{(1)}(\xi,\xi')-\hat{V}(\xi,\xi')]f_2(\xi,\xi',t)-W^{(2)}(\xi-\Delta_{\xi\xi'},\xi'+\Delta'_{\xi\xi'})f_2(\xi-\Delta_{\xi\xi'},\xi'+\Delta'_{\xi\xi'},t)\}, \tag{33}$$

where $f_1(\xi,t)$ and $f_2(\xi,\xi',t)$ are the single particle and binary distribution functions defined as

$$f_1(\xi,t)\equiv\langle N(\xi,t)\rangle, \quad f_2(\xi,\xi',t)=\langle N(\xi,t)N(\xi',t)\rangle-\delta(\xi-\xi')f_1(\xi,t), \tag{34}$$

the angular brackets imply time averaging over the physically infinitesimal interval τ_{ph} which introduces the level of the kinetic description.

Combination of Eq. (31) and a similar equation for $N(\xi',t)$, when averaged, results in

$$\begin{aligned}&\left\{\frac{\partial}{\partial t}+\hat{L}(\xi)+\hat{L}(\xi')-[\hat{V}(\xi,\xi')+\hat{V}(\xi',\xi)]\right\}f_2(\xi,\xi',t)+\{[W^{(1)}(\xi,\xi')+W^{(1)}(\xi',\xi)]f_2(\xi,\xi',t)\\&\quad-W^{(2)}(\xi-\Delta_{\xi\xi'},\xi'+\Delta'_{\xi\xi'})f_2(\xi-\Delta_{\xi\xi'},\xi'+\Delta'_{\xi\xi'},t)-W^{(2)}(\xi'-\Delta_{\xi'\xi},\xi+\Delta'_{\xi'\xi})f_2(\xi+\Delta'_{\xi'\xi},\xi'-\Delta_{\xi'\xi},t)\}\\&=-\int d\xi''\{[\hat{V}(\xi,\xi'')+\hat{V}(\xi',\xi'')+W^{(1)}(\xi,\xi'')+W^{(1)}(\xi',\xi'')]f_3(\xi,\xi',\xi'',t)-W^{(2)}(\xi-\Delta_{\xi\xi''},\xi''+\Delta'_{\xi\xi''})\\&\quad\times f_3(\xi-\Delta_{\xi\xi''},\xi',\xi''+\Delta'_{\xi\xi''},t)-W^{(2)}(\xi'-\Delta_{\xi'\xi''},\xi''+\Delta'_{\xi'\xi''})f_3(\xi,\xi'-\Delta_{\xi'\xi''},\xi''+\Delta'_{\xi'\xi''},t)\}.\end{aligned} \tag{35}$$

Here

$$\begin{aligned}f_3(\xi,\xi',\xi'',t)&=\langle N(\xi,t)N(\xi',t)N(\xi'',t)\rangle-\delta(\xi-\xi')f_2(\xi',\xi'',t)\\&\quad-\delta(\xi-\xi'')f_2(\xi,\xi';t)-\delta(\xi'-\xi'')f_2(\xi'',\xi,t)-\delta(\xi-\xi')\delta(\xi-\xi'')f_1(\xi,t).\end{aligned} \tag{36}$$

The distribution function of arbitrary multiplicity is governed by the equation

$$\begin{aligned}&\left\{\frac{\partial}{\partial t}+\sum_{i=1}^{S}\hat{L}(\xi_i)-\sum_{i=1}^{S}\sum_{j\neq i=1}^{S}\hat{V}(\xi_i,\xi_j)\right\}f_S(\xi_1\ldots\xi_S;t)+\sum_{i=1}^{S}\sum_{j\neq i=1}^{S}[W^{(1)}(\xi_i,\xi_j)f_S(\xi_1\ldots\xi_S;t)\\&\quad-W^{(2)}(\xi_i-\Delta_{ij},\xi_j+\Delta'_{ij})f_S(\xi_1\ldots\xi_i-\Delta_{ij}\ldots,\xi_j+\Delta'_{ij}\ldots,\xi_S;t)]\\&=\sum_{i=1}^{S}\int d\xi_{S+1}\{\hat{V}(\xi_i,\xi_{S+1})f_{S+1}(\xi_1\ldots\xi_{S+1};t)-[W^{(1)}(\xi_i,\xi_{S+1})f_{S+1}(\xi_1\ldots\xi_{S+1};t)\\&\quad-W^{(2)}(\xi_i-\Delta_{iS+1},\xi_{S+1}+\Delta'_{iS+1})f_{S+1}(\xi_1\ldots\xi_i-\Delta_{iS+1}\ldots\xi_{S+1}+\Delta'_{iS+1};t)]\},\end{aligned} \tag{37}$$

which represents the entire BBGKY hierarchy. Here

$$\Delta_{iS+1}\equiv\Delta_{\xi_i\xi_{S+1}}, \quad \Delta'_{iS+1}\equiv\Delta'_{\xi_i\xi_{S+1}}.$$

For systems with pure Coulomb interaction ($W^{(1)}=W^{(2)}=0$), Eq. (37) reduces to the BBGKY hierarchy for ordinary plasmas. In the case of neutral particles interacting by means of elastic collisions, the hierarchy for the hard-sphere gas [10] is recovered.

Thus, we have basic equations for the study of statistical properties of dusty plasmas with regard for both elastic and inelastic collisions and for grain charging dynamics. These are the microscopic equations (11) and (24) and the BBGKY hierarchy (37) for the sequence of distribution functions. They provide the natural basis for a rigorous derivation of kinetic equations for dusty plasmas. Depending on the problem under consideration it is more convenient to apply either microscopic equations, or the BBGKY hierarchy, as has been done in the theory of ''ordinary'' plasmas. In the latter case the microscopic description is highly efficient for the study of electromagnetic fluctuations and calculations of the collision integrals in terms of the fluctuation approach. The reason for this high efficiency is that fluctuations in ordinary weakly coupled plasmas can be studied by means of simple perturbation methods with the solution of the linearized evolution equation being taken for the zero-order approximation. In the case of dusty plasmas with strong coupling between plasma particles and grains such perturbation approach, however, requires considerable improvement. A possible way of appropriate modification of the fluctuation approach for the case of dominant influence of charging collisions is discussed in the next section.

IV. KINETIC EQUATIONS FOR DUSTY PLASMAS IN THE APPROXIMATION OF DOMINANT CHARGING COLLISIONS

We have already mentioned that kinetic equations for dusty plasmas can be introduced in two ways. The first possibility is to calculate the binary distribution functions, which determine the collision terms, from the reduced BBGKY hierarchy. In this approach the problem of the hierarchy closure arises that requires postulated relations between the distribution functions of different multiplicity. In the case of a dusty plasma with strong coupling between plasma particles and grains, the traditional assumption that influence of triple correlations is negligible is obviously inapplicable. Moreover, the equation for the binary correlation function itself is much more complicated than in the case of ordinary plasmas.

The second possibility to derive kinetic equations is associated with the so-called fluctuation approach. The basic idea of this approach is to calculate binary correlation functions in terms of the correlation functions of microscopic phase densities which can be found by averaging the products of relevant microscopic quantities governed by the fluctuation evolution equations. The advantage of the fluctuation approach is that appropriate choice of the physically infinitesimal time τ_{ph} associated with the level of statistical description makes it possible to linearize the fluctuation evolution equations. Of course, the explicit form of such equations depends on the time scales of the fluctuations under consideration. For example, in the case of weakly coupled plasmas the evolution equation for fluctuations with the characteristic time $\tau_{\mathrm{cor}} \ll \tau_{\mathrm{col}}$ (τ_{col} is the mean free time) is the linearized Vlasov equation. For $\tau_{\mathrm{cor}} > \tau_{\mathrm{col}}$, however, the evolution equations contain linearized Balescu-Lenard or Landau collision terms. The physical reason is that for $\tau > \tau_{\mathrm{col}}$ the linear collisionless approximation is no longer valid. This means that the collision terms in the fluctuation evolution equation could be interpreted as the result of renormalization of the linearized equation with respect to higher perturbation orders.

The application of the fluctuation approach to the derivation of kinetic equations is especially simple in the case of weakly coupled ordinary plasmas since, as we have mentioned above, the fluctuation evolution equations are reduced to the linearized Vlasov equations. However, this approximation cannot be applied to dusty plasmas when the problem of charge grain dynamics is studied. Highly charged grains which absorb plasma particles induce strong correlations which cannot be described by perturbation methods. So, the first step in the derivation of fluctuation evolution equations is to find the lowest-order approximation for the collision integrals in order to introduce these integrals in the evolution equation. In this paper we do this using physical arguments.

Let us consider in more detail the first equation of the hierarchy. In terms of notation (30), it is given by

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{1}{m_\sigma}\mathbf{F}_\sigma^{\mathrm{ext}}\frac{\partial}{\partial \mathbf{v}}\right\} f_\sigma(\mathcal{X},t)=\sum_{\sigma'=e,i,g}\int d\mathcal{X}'\,\hat{V}_{\sigma\sigma'}(\mathcal{X},\mathcal{X}')\,f_{\sigma\sigma'}(\mathcal{X},\mathcal{X}',t)-\sum_{\sigma'=e,i,g}\int d\mathcal{X}'\{W^{(1)}_{\sigma\sigma'}(\mathbf{r}-\mathbf{r}',\mathbf{v}-\mathbf{v}')$$
$$\times f_{\sigma\sigma'}(\mathcal{X},\mathcal{X}',t)-W^{(2)}_{\sigma\sigma'}(\mathbf{r}-\mathbf{r}',\mathbf{v}-\delta\mathbf{v}_{\sigma\sigma'}-\mathbf{v}'-\delta\mathbf{v}'_{\sigma\sigma'})$$
$$\times f_{\sigma\sigma'}(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_{\sigma\sigma'},q-\delta q_{\sigma\sigma'},\mathbf{r}',\mathbf{v}+\delta\mathbf{v}'_{\sigma\sigma'},q',t)\}. \tag{38}$$

Introducing the binary correlation function

$$G_{\sigma\sigma'}(\mathcal{X},\mathcal{X}',t)=f_{\sigma\sigma'}(\mathcal{X},\mathcal{X}',t)-f_\sigma(\mathcal{X},t)f_{\sigma'}(\mathcal{X}',t)$$

and the self-consistent electric field

$$\langle\mathbf{E}(\mathbf{r},t)\rangle=-\frac{\partial}{\partial \mathbf{r}}\sum_{\sigma=e,i,g}\int d\mathcal{X}'\,q'\frac{f_\sigma(\mathcal{X}',t)}{|\mathbf{r}-\mathbf{r}'|},$$

we can rewrite Eq. (38) in the form

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{1}{m_\sigma}[\mathbf{F}_\sigma^{\mathrm{ext}}+\langle\mathbf{F}_\sigma\rangle]\frac{\partial}{\partial \mathbf{v}}\right\} f_\sigma(\mathcal{X},t)$$
$$=-\sum_{\sigma'=e,i,g}\int d\mathcal{X}'\{W^{(1)}_{\sigma\sigma'}(\mathbf{r}-\mathbf{r}',\mathbf{v}-\mathbf{v}')f_\sigma(\mathcal{X},t)f_{\sigma'}(\mathcal{X}',t)-W^{(2)}_{\sigma\sigma'}(\mathbf{r}-\mathbf{r}',\mathbf{v}-\delta\mathbf{v}_{\sigma\sigma'}-\mathbf{v}'-\delta\mathbf{v}'_{\sigma\sigma'})$$
$$\times f_\sigma(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_{\sigma\sigma'},q-\delta q_{\sigma\sigma'},t)f_{\sigma'}(\mathbf{r}',\mathbf{v}'+\delta\mathbf{v}'_{\sigma\sigma'},q',t)\}-\sum_{\sigma'=e,i,g}\int d\mathcal{X}'\{W^{(1)}_{\sigma\sigma'}(\mathbf{r}-\mathbf{r}',\mathbf{v}-\mathbf{v}')G_{\sigma\sigma'}(\mathcal{X},\mathcal{X}',t)$$
$$-W^{(2)}_{\sigma\sigma'}(\mathbf{r}-\mathbf{r}',\mathbf{v}-\delta\mathbf{v}_{\sigma\sigma'}-\mathbf{v}'-\delta\mathbf{v}'_{\sigma\sigma'})G_{\sigma\sigma'}(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_{\sigma\sigma'},q-\delta q_{\sigma\sigma'};\mathbf{r}',\mathbf{v}'+\delta\mathbf{v}'_{\sigma\sigma'},q';t)\}$$
$$+\sum_{\sigma'=e,i,g}\int d\mathcal{X}'\,\hat{V}_{\sigma\sigma'}(\mathcal{X},\mathcal{X}')G_{\sigma\sigma'}(\mathcal{X},\mathcal{X}',t). \tag{39}$$

It is important for further calculation to understand the physical meaning of the integral terms in the right-hand part of Eq. (39). The terms with the Coulomb operator $\hat{V}_{\sigma\sigma'}$ describe the average change of the particle phase density per unit time due to the deviations of the microscopic field from the self consistent field. Since these deviations are associated with particle discreteness, such terms can be treated as Coulomb collision terms. Within the approximation of weakly coupled plasmas these terms reduce to the Balescu-Lenard collision integrals.

The terms with the kernels $W^{(1)}_{\sigma\sigma'}$ and $W^{(2)}_{\sigma\sigma'}$ are responsible for the contact collisions (both elastic and inelastic) between the grains (elastic collisions) or between the grains and plasma particles (inelastic collisions). The first term describes such collisions disregarding the direct interaction between the particles and the second term is given rise by the correlations effects. In the case of strong coupling between plasma particles and grains, such correlations could be very important. In particular, they result in a modification of the contact collision cross sections which are determined not only by the grain sizes but also by their charges. Such cross sections naturally appear in course of simplification of Eq. (39) within the approximation of the dominant influence of binary contact collisions. This approximation can be used as the zero-order approximation in deriving the kinetic equations in terms of the fluctuation approach. The idea is to present the binary distribution function as

$$f_{\sigma\sigma'}(\mathcal{X},\mathcal{X}',t)=f^{(0)}_{\sigma\sigma'}(\mathcal{X},\mathcal{X}',t)+[f_{\sigma\sigma'}(\mathcal{X},\mathcal{X}',t)-f^{(0)}_{\sigma\sigma'}(\mathcal{X},\mathcal{X}',t)] \quad (40)$$

under the assumption that the second term is much smaller than the first one. Here, $f^{(0)}_{\sigma\sigma'}(\mathcal{X},\mathcal{X}',t)$ is the binary distribution function in the approximation of dominant contact collisions, i.e., the main contribution to the binary correlations is determined by the contact collisions with grains. This provides a possibility to reduce the asymptotics of $f^{(0)}_{g\sigma}(\mathcal{X},\mathcal{X}',t)(\sigma=e,i)$ to the form

$$f^{(0)}_{\sigma g}(\mathcal{X},\mathcal{X}',t)_{|\mathbf{r}-\mathbf{r}'|\gg a}=f_{\sigma}(\mathcal{X},t)f_{g}(\mathcal{X}',t)\,\theta(\vartheta-\vartheta^{\sigma}_{\min}), \quad (41)$$

where ϑ is the angle between $\mathbf{r}-\mathbf{r}'$ and $\mathbf{v}-\mathbf{v}'$. Equation (41) means that if a grain with velocity $\mathbf{v}'$ occurs at the point $\mathbf{r}'$, such that $\vartheta<\vartheta^{\sigma}_{\min}$ there are no plasma particles with velocity $\mathbf{v}$ at the point $\mathbf{r}$, or, in other words, no plasma particle trajectories have intersected the grain surface in the past. The elementary treatment on the basis of energy and angular momentum conservation yields an estimate for $\vartheta^{\sigma}_{\min}$ given by

$$\sin^2\vartheta^{\sigma}_{\min}=\frac{a^2}{|\mathbf{r}-\mathbf{r}'|^2}\left(1-\frac{2e_{\sigma}(\varphi(a)-\varphi(\mathbf{r}-\mathbf{r}'))}{m_{\sigma}(\mathbf{v}-\mathbf{v}')^2}\right)\times\theta\left(1-\frac{2e_{\sigma}(\varphi(a)-\varphi(\mathbf{r}-\mathbf{r}'))}{m_{\sigma}(\mathbf{v}-\mathbf{v}')^2}\right),$$
$$0\leqslant\vartheta^{\sigma}_{\min}\leqslant\pi/2, \quad (42)$$

where φ is the electric potential in plasmas.

With the assumptions

$$\varphi(a)\simeq\frac{q}{a},$$
$$|\varphi(|\mathbf{r}-\mathbf{r}'|)|\ll|\varphi(a)|, \quad (43)$$

Eq. (43) reduces to

$$\sin^2\vartheta^{\sigma}_{\min}=\frac{a^2}{|\mathbf{r}-\mathbf{r}'|^2}\left(1-\frac{2e_{\sigma}q}{am_{\sigma}(\mathbf{v}-\mathbf{v}')^2}\right).$$

We see that in the case under consideration the asymptotics of the binary distribution function $f_{\sigma g}(\mathcal{X},\mathcal{X}',t)$ cannot be reduced to a product of one-particle distribution functions. Thus, the plasma particles absorption by grains is one of the mechanisms that produce strong correlations.

It seems that the asymptotics (41) cannot be substituted in Eq. (38) since integration over $\mathbf{r}'$ should be performed for $|\mathbf{r}-\mathbf{r}'|=a$ (i.e., over the grain surface). Nevertheless, it is sufficient to estimate the absorption terms in the equations for plasma particles. The reason is that these terms are nothing but fluxes of incoming plasma particles through the grain surface. Since according to the above approximation (absorption of plasma particles by a grain is not directly influenced by another grain) only one grain is the absorption center for particles in its vicinity, the integral flux should be conserved and thus we can calculate these fluxes with the use of asymptotics (41). With the inhomogeneity of the distribution functions at distances of the order of the grain size being disregarded, we obtain

$$-\int d\mathcal{X}'\,\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')f^{(0)}_{\sigma g}(\mathcal{X},\mathcal{X}',t)\,\delta(|\mathbf{r}-\mathbf{r}'|-a)\simeq\int d\mathbf{v}'\int dq'\,\sigma_{\sigma g}(q',\mathbf{v}-\mathbf{v}')\times|\mathbf{v}-\mathbf{v}'|f_{\sigma}(\mathbf{r},\mathbf{v}',q';t)f_{\sigma}(\mathcal{X},t), \quad (44)$$

where

$$\sigma_{\sigma g}(q,v)=\pi a^2\left(1-\frac{2e_{\sigma}q}{m_{\sigma}v^2}\right)\theta\left(1-\frac{2e_{\sigma}q}{m_{\sigma}v^2}\right). \quad (45)$$

Similar estimates can be made for the case of grain-grain collisions. Obviously, in this case the angle ϑ is not limited and thus the asymptotics for $f_{gg}(\mathcal{X},\mathcal{X}',t)$ has the standard form

$$f^{(0)}_{gg}(\mathcal{X},\mathcal{X}',t)_{|\mathbf{r}-\mathbf{r}'|\gg a}=f_{g}(\mathcal{X},t)f_{g}(\mathcal{X}',t). \quad (46)$$

However, it is useful to divide this asymptotics into two parts

$$f^{(0)}_{gg}(\mathcal{X},\mathcal{X}',t)=f_{g}(\mathcal{X},t)f_{g}(\mathcal{X}',t)\{\theta(\vartheta-\vartheta_{\min})+\theta(\vartheta_{\min}-\vartheta)\} \quad (47)$$

associated with grain trajectories which do not touch the grain at the point $\mathbf{r}'$ [the first term in Eq. (47)] and with reflected grains (the second term). Such division separates the contributions to the distribution function $f_{\sigma\sigma}(\mathcal{X},\mathcal{X}',t)$ of reflected particles and particles that arrive at the point $\mathbf{r}$ without collisions with the grain at the point $\mathbf{r}'$. This makes it possible to calculate the fluxes of approaching and reflected particles. Of course, in the case of elastic collisions the radial fluxes of bombarding and reflected particles at the grain surface are equal. In view of the observation that the terms with $W^{(1)}_{\sigma\sigma'}$ and $W^{(2)}_{\sigma\sigma'}$ are related to the distributions before and after collisions, respectively, we can calculate the incoming and outcoming fluxes making use of the separate parts of the asymptotics (47). For example,

$$-\int d\mathcal{X}'' \mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')\delta(|\mathbf{r}-\mathbf{r}'|-2a)[f_{gg}^{(0)}(\mathcal{X},\mathcal{X}'';t)-f_{gg}^{(0)}(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_g,q-\delta q_g;\mathbf{r}',\mathbf{v}'+\delta\mathbf{v}_g,q';t)]$$

$$\simeq\int\frac{d\Omega}{2\pi}\int d\mathbf{v}'\int dq'|\mathbf{n}(\mathbf{v}-\mathbf{v}')|\{\sigma_{gg}(q,q';\mathbf{v}-\mathbf{v}')f_g(\mathcal{X},t)f_g(\mathbf{r},\mathbf{v}',q';t)$$

$$-\sigma_{gg}(q-\delta q_g,q',\mathbf{v}-\delta\mathbf{v}_g)f_g(\mathbf{r},\mathbf{v}+\delta\mathbf{v}_g,q-\delta q_g;t)f_g(\mathbf{r},\mathbf{v},q';t)\}, \tag{48}$$

where

$$\sigma_{gg}(q,q';v)=4\pi a^2\left(1-\frac{2qq'}{m_g v^2}\right)\theta\left(1-\frac{2qq'}{m_g v^2}\right);$$

$\mathbf{n}$ is the unit vector that specifies the direction of the spatial angle $d\Omega$.

The terms responsible for the grain-plasma-particle collisions are given by

$$-\int d\mathcal{X}''[\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')\delta(|\mathbf{r}-\mathbf{r}'|-a)f_{g\sigma'}^{(0)}(\mathcal{X},\mathcal{X}'',t)-\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}'-\delta\mathbf{v}_{\sigma'})f_{g\sigma'}^{(0)}(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_{\sigma'},q-e_{\sigma'};\mathbf{r}',\mathbf{v}',q';t)]$$

$$\simeq\int d\mathbf{v}'[\sigma_{\sigma' g}(q,\mathbf{v}-\mathbf{v}')f_g(\mathcal{X},t)f_{\sigma'}(\mathbf{r},\mathbf{v}',q';t)|\mathbf{v}-\mathbf{v}'|\quad-\sigma_{\sigma' g}(q-e_{\sigma'},\mathbf{v}-\mathbf{v}'-\delta\mathbf{v}_{\sigma'})$$

$$\times f_g(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_{\sigma'},q-e_{\sigma'};t)f_{\sigma'}(\mathbf{r},\mathbf{v}',q';t)(|\mathbf{v}-\mathbf{v}'-\delta\mathbf{v}_{\sigma'}|)]. \tag{49}$$

Thus, with regard for Eqs. (40), (44), (48), and (49), Eq. (38) for the one-particle distribution functions of plasma particles and grains can be written as

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial\mathbf{r}}+\frac{e_\sigma}{m_\sigma}\langle\mathbf{E}_\sigma^{\mathrm{eff}}\rangle\frac{\partial}{\partial\mathbf{v}}\right\}f_\sigma(X,t)=-\int d\mathbf{v}'\int dq'\,\sigma_{\sigma g}(q',\mathbf{v}-\mathbf{v}')|\mathbf{v}-\mathbf{v}'|f_\sigma(X,t)f_g(\mathbf{r},\mathbf{v}',q',t)+\delta I_\sigma, \tag{50}$$

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial\mathbf{r}}+\frac{q}{m_g}\langle\mathbf{E}_g^{\mathrm{eff}}\rangle\frac{\partial}{\partial\mathbf{v}}\right\}f_g(\mathcal{X},t)=-\sum_{\sigma=e,i}\int d\mathbf{v}'[\sigma_{\sigma' g}(q,\mathbf{v}-\mathbf{v}')|\mathbf{v}-\mathbf{v}'|f_g(\mathcal{X},t)-\sigma_{\sigma' g}(q-e_\sigma,\mathbf{v}-\mathbf{v}'-\delta\mathbf{v}_{\sigma'})$$

$$\times|\mathbf{v}-\mathbf{v}'-\delta\mathbf{v}_{\sigma'}|f_g(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_{\sigma'},q-e_{\sigma'},t)]f_{\sigma'}(\mathbf{r},\mathbf{v}',t)$$

$$-\int\frac{d\Omega}{2\pi}\int d\mathbf{v}'\int dq'|\mathbf{n}(\mathbf{v}-\mathbf{v}')|[\sigma_{gg}(q,q',\mathbf{v}-\mathbf{v}')f_g(\mathcal{X},t)f_g(\mathbf{r},\mathbf{v}',q',t)$$

$$-\sigma_{gg}(q-\delta q_g,q';\mathbf{v}-\mathbf{v}')f_g(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_g,q-\delta q_g,t)f_g(\mathbf{r},\mathbf{v}'+\delta\mathbf{v}_g,q',t)]+\delta I_g, \tag{51}$$

where

$$\delta I_\sigma=\delta I_\sigma^{\mathrm{C}}+\delta I_\sigma^{\mathrm{B}}, \tag{52}$$

$$\delta I_\sigma^{\mathrm{C}}=\sum_{\sigma'}\int d\mathcal{X}''\hat{V}_{\sigma\sigma'}(\mathcal{X},\mathcal{X}'')[\langle\delta N_\sigma(\mathcal{X},t)\delta N_{\sigma'}(\mathcal{X}'',t)\rangle-G_{\sigma\sigma'}^{(0)}(\mathcal{X},\mathcal{X}'',t)], \tag{53}$$

$$\delta I_\sigma^{\mathrm{B}}=\sum_{\sigma'}\int d\mathcal{X}''\{W_{\sigma\sigma'}^{(1)}(\mathbf{r}-\mathbf{r}',\mathbf{v}-\mathbf{v}')[\langle\delta N_\sigma(\mathcal{X},t)\delta N_{\sigma'}(\mathcal{X}'',t)\rangle-G_{\sigma\sigma'}^{(0)}(\mathcal{X},\mathcal{X}'',t)]-W_{\sigma\sigma'}^{(2)}(\mathbf{r}-\mathbf{r}',\mathbf{v}-\mathbf{v}'-\delta\mathbf{v}_{\sigma\sigma'}-\delta\mathbf{v}'_{\sigma\sigma'})$$

$$\times[\langle\delta N_\sigma(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_{\sigma\sigma'},q-\delta q_{\sigma\sigma'},t)\delta N_{\sigma'}(\mathbf{r},\mathbf{v}+\delta\mathbf{v}_{\sigma\sigma'},q',t)\rangle-G_{\sigma\sigma'}^{(0)}(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_{\sigma\sigma'},q-\delta q_{\sigma\sigma'};\mathbf{r}',\mathbf{v}'+\delta\mathbf{v}_{\sigma\sigma'},q',t)]\}, \tag{54}$$

$$G_{\sigma\sigma'}^{(0)}(\mathcal{X},\mathcal{X}'',t)=-f_\sigma(\mathcal{X},t)f_{\sigma'}(\mathcal{X}'',t)\,\theta(\vartheta_{\sigma\sigma'}(|\mathbf{r}-\mathbf{r}'|,|\mathbf{v}-\mathbf{v}'|)-\vartheta),$$

$$\sin^2\vartheta_{\sigma\sigma'}(r,v)=\frac{a_{\sigma\sigma'}^2}{r^2}\left(1-\frac{2qq'}{m_\sigma a_{\sigma\sigma'}v^2}\right)\theta(r-a_{\sigma\sigma'}), \tag{55}$$

$$\langle E_\sigma^{\rm eff}\rangle=-\sum_{\sigma'=e,i} e_{\sigma'}\frac{\partial}{\partial\mathbf{r}}\int dX'\frac{f_{\sigma'}(X',t)}{|\mathbf{r}-\mathbf{r}'|}-\int d\mathcal{X}' q'\frac{\partial}{\partial\mathbf{r}}\frac{f_g(\mathcal{X}',t)}{|\mathbf{r}-\mathbf{r}'|}\theta(\vartheta-\vartheta_{\sigma g}(|\mathbf{r}-\mathbf{r}'|,|\mathbf{v}-\mathbf{v}'|)),$$

$$\langle E_g^{\rm eff}\rangle=-\sum_{\sigma'} e_{\sigma'}\frac{\partial}{\partial\mathbf{r}}\int dX'\frac{f_{\sigma'}(X',t)}{|\mathbf{r}-\mathbf{r}'|}\theta(\vartheta-\vartheta_{\sigma g}(|\mathbf{r}-\mathbf{r}'|,|\mathbf{v}-\mathbf{v}'|))-\int d\mathcal{X}' q'\frac{\partial}{\partial\mathbf{r}}\frac{f_g(\mathcal{X}',t)}{|\mathbf{r}-\mathbf{r}'|}. \tag{56}$$

In deriving Eqs. (51)–(56), use has been made of the relation

$$f_{\sigma\sigma'}(\mathcal{X},\mathcal{X}',t)=f_{\sigma\sigma'}^{(0)}(\mathcal{X},\mathcal{X}',t)+[\langle\delta N_\sigma(\mathcal{X},t)\delta N_{\sigma'}(\mathcal{X}',t)\rangle-G_{\sigma\sigma'}^{(0)}(\mathcal{X},\mathcal{X},t)]-\delta_{\sigma\sigma'}\delta(\mathcal{X}-\mathcal{X}')f_\sigma(\mathcal{X},t).$$

Thus, we represented the collisions integrals as a sum of two parts. Those explicitly expressed in terms of the contact collision cross sections $\sigma_{\sigma\sigma'}(r,v)$ describe the contribution of inelastic plasma-grain collisions [the first terms in the right-hand parts of Eqs. (50) and (51)] and the elastic contact grain-grain collisions [the second term in the right-hand part of Eq. (51)]. The quantities δI_σ give the contribution of the elastic (noncontact) Coulomb collisions between particles of all species. With the accuracy up to the term responsible for the grain-grain contact collisions, Eqs. (50) and (51) reproduce the kinetic equations of Refs. [6,7].

To be rigorous, Eqs. (50) and (51) still cannot be regarded as kinetic equations since correlation functions $\langle\delta N_\sigma(\mathcal{X},t)\delta N_{\sigma'}(\mathcal{X}',t)\rangle$ are not yet specified. However, under the approximation of dominant contact collisions, the terms δI_σ could be treated as small quantities, and thus the above correlation functions could be calculated within the fluctuation approach. This opens the possibility to derive the relations for the collision terms δI_σ and to introduce the kinetic equations for dusty plasmas with regard for both elastic and inelastic collisions. A necessary step in such derivation is to study stationary distributions of grain velocities and charges in the zero-order approximation, i.e., for $\delta I_\sigma=0$.

V. STATIONARY DISTRIBUTIONS OF GRAIN VELOCITIES AND CHARGES IN DUSTY PLASMAS

In the approximation of dominant charging collisions ($\delta I_\sigma=0$), Eq. (51) for the grain distribution function can be written as

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial\mathbf{r}}+\frac{q}{m}\langle E_g^{\rm eff}\rangle\frac{\partial}{\partial\mathbf{v}}\right\}f_g(\mathbf{r},\mathbf{v},q,t)$$
$$=-\sum_{\sigma=e,i}\int d\mathbf{v}'\{\sigma_{g\sigma}(q,\mathbf{v}-\mathbf{v}')|\mathbf{v}-\mathbf{v}'|f_g(\mathbf{r},\mathbf{v},q,t)$$
$$-\sigma_{g\sigma}(q-e_\sigma,\mathbf{v}-\mathbf{v}'-\delta\mathbf{v}_\sigma)|\mathbf{v}-\mathbf{v}'-\delta\mathbf{v}_\sigma|$$
$$\times f_g(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_g,q-e_\sigma,t)\}f_\sigma(\mathbf{r},\mathbf{v},t). \tag{57}$$

Here, we neglect elastic Coulomb collisions and elastic contact collisions between the grains (estimates for the conditions of such approximation are given below).

Inasmuch as e_σ and $\delta\mathbf{v}_\sigma$ are small, we can expand the right-hand part of Eq. (57) in power series of these quantities. With the accuracy up to the second order, Eq. (57) in the stationary isotropic and homogeneous case reduces to

$$\frac{\partial}{\partial\mathbf{v}}\left[\frac{\partial}{\partial\mathbf{v}}(D_\parallel f_g(\mathbf{v},q))+\beta\mathbf{v}f_g(\mathbf{v},q)+\frac{\partial}{\partial q}(q\gamma\mathbf{v}f_g(\mathbf{v},q))\right]$$
$$+\frac{\partial}{\partial q}\left[\frac{\partial}{\partial q}(Qf(\mathbf{v},q))-If_g(\mathbf{v},q)\right]=0, \tag{58}$$

where $D_\parallel$, β, Q, γ and I are the Fokker-Planck kinetic coefficients generated by charging collisions and given by

$$D_\parallel\equiv D_\parallel(q,\mathbf{v})=\sum_\sigma\frac{1}{2}\left(\frac{m_\sigma}{m_g}\right)^2\int d\mathbf{v}'\frac{(\mathbf{v}\cdot\mathbf{v}')^2}{v^2}|\mathbf{v}-\mathbf{v}'|\sigma_{g\sigma}$$
$$\times(q,\mathbf{v}-\mathbf{v}')f_\sigma(\mathbf{r},\mathbf{v}'),$$
$$\beta\equiv\beta(q,v)=-\sum_\sigma\frac{m_\sigma}{m_g}\int d\mathbf{v}'\frac{\mathbf{v}\cdot\mathbf{v}'}{v^2}|\mathbf{v}-\mathbf{v}'|\sigma_{g\sigma}$$
$$\times(q,\mathbf{v}-\mathbf{v}')f_\sigma(\mathbf{r},\mathbf{v}'),$$
$$\gamma\equiv\gamma(q,v)=\sum_\sigma\frac{m_\sigma}{m_g}\frac{e_\sigma}{q}\int d\mathbf{v}'\frac{\mathbf{v}\cdot\mathbf{v}'}{v^2}|\mathbf{v}-\mathbf{v}'|\sigma_{g\sigma}$$
$$\times(q,\mathbf{v}-\mathbf{v}')f_\sigma(\mathbf{r},\mathbf{v}'), \tag{59}$$

$$Q\equiv Q(q,v)=\sum_\sigma\frac{e_\sigma^2}{2}\int d\mathbf{v}'|\mathbf{v}-\mathbf{v}'|\sigma_{g\sigma}(q,\mathbf{v}-\mathbf{v}')f_\sigma(\mathbf{r},\mathbf{v}'),$$

$$I\equiv I(q,v)=\sum_\sigma e_\sigma\int d\mathbf{v}'|\mathbf{v}-\mathbf{v}'|\sigma_{g\sigma}(q,\mathbf{v}-\mathbf{v}')f_\sigma(\mathbf{r},\mathbf{v}').$$

The quantities $D_\parallel(q,v)$ and $Q(q,\mathbf{v})$ characterize the grain diffusion in the velocity and charge spaces, respectively, $\beta(q,\mathbf{v})$ and $\gamma(q,\mathbf{v})$ are the friction coefficients which determine the bombardment force $\mathbf{F}_{\rm b}(q,\mathbf{v})=-m_g\beta(q,\mathbf{v})\mathbf{v}$ associated with charging collisions and the correction to this force $\delta\mathbf{F}_{\rm b}(q,\mathbf{v})=-m_g\gamma(q,\mathbf{v})\mathbf{v}$ due to mutual effect of grain charge and velocity distributions, I is the grain charging current. When deriving the relation for $\beta(q,\mathbf{v})$ we omit the terms of higher order in (m_σ/m_g) associated with the tensor nature of the diffusion coefficient in velocity space.

With regard for the fact that $|I(q,v)/Q(q,\mathbf{v})|\to\infty$ as $e_\sigma\to 0$ and $|\beta(q,\mathbf{v})/D_\parallel(q,\mathbf{v})|\to\infty$ as $(m_\sigma/m_g)\to 0$, the asymptotic solution of Eq. (58) can be written as [12]

$$f_g(\mathbf{v},q)=n_{0g}Z^{-1}Q^{-1}(q,\mathbf{v})e^{-W(q,v)+\lambda v^2}$$
$$\times D_\parallel^{-1}(q,\mathbf{v})e^{-V(q,v)+\varepsilon\delta q(v)}, \tag{60}$$

where

$$W(q,v)=-\int_0^q dq\,\frac{I(q',v)}{Q(q',v)},$$

$$V(q,v)=\int_0^v dv'\frac{v'}{D_\parallel(q,v')}\left\{\beta(q,v')+\frac{\partial}{\partial q}(q\gamma(q,v'))\right.$$
$$\left.-q\gamma(q,v')\left[\frac{\partial W(q,v')}{\partial q}+Q^{-1}(q,v')\frac{\partial Q(q,v')}{\partial q}\right]\right\},\tag{61}$$

$$\delta q(v)=q-q(v),$$

$q(v)$ is the stationary charge of the grain moving with the velocity v, given by the equation

$$I(q(v),v)=0,\tag{62}$$

Z is the normalization constant, ε and λ are small functions. Substitution of Eq. (60) into Eq. (58) leads to

$$\varepsilon=\frac{1}{D_\parallel(q,v)}\frac{\partial D_\parallel(q,v)}{\partial q}+\frac{\partial V(q,v)}{\partial q},$$

$$\lambda=\frac{1}{2v}\left\{\frac{1}{Q(q,v)}\frac{\partial Q(q,v)}{\partial v}+\frac{\partial W(q,v)}{\partial v}+\varepsilon\frac{\partial q(v)}{\partial v}\right\}.\tag{63}$$

Equations (60)–(63) give the asymptotically exact solution of Eq. (58) for $(m_g e_\sigma/m_\sigma q)\to\infty$.

The further estimates require the knowledge of the explicit form of the kinetic coefficients. Assuming that plasma particle distributions are Maxwellian, we find the stationary grain distribution with accuracy up to the zeroth order in $(qm_i/e_e m_g)$ to be given by [12]

$$f_g(\mathbf{v},q)=n_{0g}Z^{-1}D_\parallel^{-1}(q,v)e^{-m_g v^2/2T_{\rm eff}(q)}\times Q^{-1}(q,v)e^{-(q-q_0)^2/2a\tilde{T}_{\rm eff}},\tag{64}$$

where

$$T_{\rm eff}(q)=\frac{2T_i(t+z)}{t-z+\frac{(q-q_0)}{q_0}z\left[1+\frac{t-z}{t+z}\left(1+\frac{2Z_i}{1+Z_i}(1+t+z)\right)\right]},\tag{65}$$

$$\tilde{T}_{\rm eff}=\frac{2}{1+Z_i}\frac{1+t+z}{t+z}T_e,\tag{66}$$

and

$$D_\parallel(q,v)\simeq D_0\left[1+\frac{q-q_0}{q_0}\frac{z}{t+z}\right]\left(1+\frac{z}{t}\right),$$

$$Q(q,v)=Q_0\left[1-\frac{q-q_0}{q_0}\frac{z(t+z-Z_i)}{(t+z)(1+Z_i)}\right](t+z)(1+Z_i),\tag{67}$$

$$D_0=\frac{4}{3}\sqrt{2\pi}\left(\frac{m_i}{m_g}\right)\left(\frac{T_i}{m_g}\right)a^2 n_i S_i,$$

$$Q_0=\sqrt{2\pi}\left(\frac{T_e}{T_i}\right)e_i^2 a^2 n_i S_i,$$

n_{0g} is the averaged grain number density. Here, we use the notation

$$z=\frac{e_e^2 Z_g}{aT_e},\quad t=\frac{T_i}{Z_i T_e},\quad S_i^2=\frac{T_i}{m_i},\quad Z_g=\frac{q_0}{e_e},\quad Z_i=\left|\frac{e_i}{e_e}\right|.$$

The quantity q_0 is a stationary equilibrium grain charge that is determined by the equation

$$I(q_0,0)=2\sqrt{2\pi}a^2 e_i^2 n_i S_i\left[1+\frac{z}{t}-\left(\frac{m_i}{m_e}\right)^{1/2}\left(\frac{T_e}{T_i}\right)^{1/2}\frac{n_e}{Z_i n_i}e^{-z}\right]=0,\tag{68}$$

when recovering the well-known equation for dust particle charge obtained within the limiting orbit probe theory [13].

For typical values of plasma parameters in dusty plasma experiments $(t+z>1)$ and $Z_i=1$ we have

$$\tilde{T}_{\rm eff}\simeq T_e.$$

In such case the thermal variation of the grain charge $|q-q_0|^2$ is of the order of aT_e and $|q-q_0|z\sim q_0\sqrt{e_e^2/aT_e}$. This means that for weak plasma coupling defined with the grain size $(e_e^2/aT_e\ll 1)$, the effective temperature of the grain thermal motion $T_{\rm eff}(q)\equiv T_{\rm eff}$ reduces to

$$T_{\rm eff}\simeq 2T_i\frac{t+z}{t-z}\tag{69}$$

and

$$D(q,\mathbf{v})\simeq D_0\left(1+\frac{z}{t}\right),\quad Q(q,v)\simeq Q_0(t+z)(1+Z_i).$$

Thus, in this case we have

$$f_g(\mathbf{v},q)=\frac{n_{0g}}{\sqrt{2\pi a\tilde{T}_{\rm eff}}}e^{-(q-q_0)^2/2a\tilde{T}_{\rm eff}}\left(\frac{m_g}{2\pi T_{\rm eff}}\right)^{3/2}e^{-m_g v^2/2T_{\rm eff}}. \tag{70}$$

This distribution describes the equilibrium Maxwellian velocity distribution and the Gibbs grain charge distribution with the temperatures $T_{\rm eff}$ and $\tilde{T}_{\rm eff}$, respectively. In fact, the electric energy of charge variations of the electric capacity a is equal to $(q-q_0)^2/2a$ and thus, the charge distribution described by Eq. (70) can be interpreted as equilibrium distribution with the effective temperature $\tilde{T}_{\rm eff}$. For $t<1$ and $z<1$, the effective temperature $\tilde{T}_{\rm eff}$ exceeds the electron temperature.

The resulting velocity distribution is described by the effective temperature $T_{\rm eff}$. Even in the case of neutral grains ($z=0$) this temperature is equal to $2T_i$. The presence of the factor 2 is associated with plasma particle absorption by grains. The charging collisions are inelastic and a part of the kinetic energy of ions is transformed into additional kinetic energy of grains. This makes the difference between the case under consideration and the conventional Brownian motion where the velocity distribution is described by the temperature of the bombarding light particles.

Equation (69) shows that the effective temperature of thermal grain motion could be anomalously high for $z\to t$, or even negative for $z>t$. Physically it can be explained by the decrease of the friction coefficient with increasing grain charge, i.e.,

$$\beta(q,v)\simeq\frac{2}{3}\sqrt{2\pi}\left(\frac{m_\sigma}{m_g}\right)a^2 n_i S_i\left(1-\frac{z}{t}\right)=\beta_0\left(1-\frac{z}{t}\right).$$

The reason is that the difference between the ion fluxes bombarding the grain surface antiparallel and parallel to the grain motion decreases with the charge increase due to the peculiar properties of the ionic charging cross-section whose charge-dependent part is larger for ions moving with smaller relative velocities (i.e., in the parallel direction). The condition $z=t$ corresponds to the zero value of the friction force.

The negative values of the effective temperature $T_{\rm eff}$, if occur, mean that the system is thermodynamically unstable and the approximation of dominant charging collisions is no longer valid, i.e., the Coulomb corrections δI_σ to the collision terms should be involved into consideration. Since calculating such corrections is a special problem, here we restrict ourselves to the qualitative estimates supplementing Eqs. (57) and (58) by the Coulomb collision terms known from the theory of ordinary plasmas. In particular, we use the Balescu-Lenard collision integral in the Fokker-Planck form which in the case under consideration (isotropic spatially homogeneous stationary distribution) is given by

$$\left(\frac{\partial f_g}{\partial t}\right)^C=\frac{\partial}{\partial\mathbf{v}}\cdot\left[\frac{\partial}{\partial\mathbf{v}}(D_{\parallel C}(q,\mathbf{v})f_g(\mathbf{v},q))+\mathbf{v}\beta_C(q,\mathbf{v})f_g(q,v)\right], \tag{71}$$

where $D_{\parallel C}(q,\mathbf{v})$ and $\beta_C(q,\mathbf{v})$ are the Fokker-Planck coefficients associated with Coulomb elastic collisions (see, for example, [11], Chap. 8). With the accuracy up to the dominant logarithmic terms [in this approximation Eq. (71) reduces to the Landau collision term] such coefficients can be reduced to

$$D_{\parallel C}(q,\mathbf{v})\simeq\frac{4}{3}\frac{\sqrt{2\pi}q^2}{m_g^2}\sum_{\sigma=e,i}\frac{n_\sigma e_\sigma^2}{S_\sigma}\ln\Lambda_\sigma\left(1-\frac{v^2}{5S_\sigma^2}\right),$$

$$\beta_C(q,\mathbf{v})\simeq\frac{4}{3}\frac{\sqrt{2\pi}q^2}{m_g}\sum_{\sigma=e,i}\frac{n_\sigma e_\sigma^2}{S_\sigma^3 m_\sigma}\ln\Lambda_\sigma\left(1-\frac{v^2}{5S_\sigma^2}\right),$$

$$S_\sigma=\left(\frac{T_\sigma}{m_\sigma}\right)^{1/2}. \tag{72}$$

In Eqs. (71) and (72) we again neglect the contribution of the transverse part of the diffusion coefficient which gives a correction to $\beta_C(q,v)$ of higher order in (m_σ/m_g) and disregard the grain-grain Coulomb collisions assuming the grain density to be small $[n_g<n_i(Z_i/Z_g)^2(S_g/S_i)^{1/2}(T_g/T_i)]$. We shall introduce the Coulomb logarithms $\ln\Lambda_\sigma$ for each particle species. Usually these quantities are estimated as $\ln\Lambda_\sigma=\ln(k_{\rm max}/k_D)$, where $k_D=r_D^{-1}=\Sigma(4\pi e_\sigma^2 n_\sigma/T_\sigma)^{1/2}$ and $k_{\rm max}$ is the inverse distance of closest approach between colliding particles, i.e.,

$$k_{{\rm max}\sigma}\sim\frac{m_\sigma v^2}{|e_\sigma q|}\sim\frac{8T_\sigma}{\pi|e_\sigma q|}=r_{{\rm L}\sigma}^{-1} \tag{73}$$

($r_{{\rm L}\sigma}$ is the Landau length, here and in what follows we approximate v by the mean velocity $\langle v\rangle=\sqrt{8T/\pi m}$). However, in the case of plasma particle collisions with finite-size grains this estimate could be invalid since for $r_{{\rm L}\sigma}<a$ the Coulomb logarithm includes the contribution of collisions with particles reaching the grain surface, i.e., charging collisions.

An appropriate modification of Λ_σ is achieved by treating $\ln\Lambda_\sigma$ as a logarithmic factor appearing in the momentum transfer cross section for Coulomb collisions. In the case of finite size grains one obtains the following logarithmic factor:

$$\ln\Lambda_\sigma=\ln\left(\sin\frac{\chi_{{\rm max}\sigma}}{2}\Big/\sin\frac{\chi_{{\rm min}\sigma}}{2}\right),$$

where $\chi_{{\rm max}\sigma}$ and $\chi_{{\rm min}\sigma}$ are the scattering angles related to the minimum and maximum impact parameters $b_{{\rm min}\sigma}$ and $b_{{\rm max}\sigma}$ by the Rutherford formula. Following [14] we find $b_{{\rm min}\sigma}$ from the condition that the distance of closest approach is equal to a and thus obtain

$$b_{{\rm min}\sigma}=a\sqrt{1-\frac{2e_\sigma q}{m_\sigma v^2 a}}\theta\left(1-\frac{2e_\sigma q}{m_\sigma v^2 a}\right). \tag{74}$$

Concerning the quantity $b_{{\rm max}\sigma}$, it is reasonable to put $b_{{\rm max}\sigma}=r_D+a$ instead of $b_{{\rm max}\sigma}=r_D$, since in the case of finite size grains its screened potential is given by the Derjaguin-Landau-Verwey-Overbeeck potential

$$\Phi(r)=\frac{q}{r}\left(1+\frac{a}{r_D}\right)^{-1}e^{-(r-a)/r_D}$$

rather than by the Debye potential.

As a result we have

$$\ln \Lambda_i = \frac{1}{2} \ln \frac{(r_D + a)^2 + r_{\mathrm{L}i}^2}{(r_{\mathrm{L}i} + a)^2},$$

$$\ln \Lambda_e = \frac{1}{2} \begin{cases} \ln \dfrac{(r_D + a)^2 + r_{\mathrm{L}e}^2}{(a + r_{\mathrm{L}e})^2}, & a > 2 r_{\mathrm{L}e}, \\ \ln \dfrac{(r_D + a)^2 + r_{\mathrm{L}e}^2}{r_{\mathrm{L}e}^2}, & a < 2 r_{\mathrm{L}e}. \end{cases} \tag{75}$$

In the case $r_D \gg a$, Eq. (75) is in agreement with the results of Ref. [14] provided the surface grain potential is much larger than the plasma potential. We see that for $r_{\mathrm{L}i} \gg r_D$ the ionic Coulomb logarithm can be a small quantity in contrast to the case of ideal plasmas.

Comparing Eqs. (58) and (71) it is easy to see that in order to take elastic Coulomb collisions into account it is sufficient to make the following replacements in the obtained solutions

$$D_\parallel(q,v) \to \widetilde{D}_\parallel(q,v) = D_\parallel(q,v) + D_{\parallel C}(q,v),$$

$$\beta(q,v) \to \widetilde{\beta}(q,v) = \beta(q,v) + \beta_C(q,v). \tag{76}$$

In the case of weak plasma coupling ($e_e^2/aT_e \ll 1$), we have

$$\widetilde{D}_\parallel(q,v) \simeq D_0\left(1 + \frac{z}{t} + \frac{z^2}{t^2} \ln \Lambda_i\right),$$

$$\widetilde{\beta}(q,v) \simeq \beta_0\left(1 - \frac{z}{t} + 2\,\frac{z^2}{t^2} \ln \Lambda_i\right), \tag{77}$$

Thus the correction produced by the elastic collisions could be of the same order as that due to charging collisions. The condition for dominant influence of charging collisions is

$$\left|1 - \frac{z}{t}\right| > 2\,\frac{z^2}{t^2} \ln \Lambda_i,$$

which can be realized for small values of z/t or for $z/t \gg r_D^2/a^2 (r_{\mathrm{L}i} \gg r_D^2/a)$.

To be rigorous, Eq. (71) and thus Eq. (77) are definitely valid in the case of weak coupling plasmas ($r_{\mathrm{L}i} \ll r_D$) since this is the condition of the derivation of the Balescu-Lenard (or Landau) collision term. However, one might expect that actually the domain of validity of Eqs. (71) and (77) is not too strongly restricted by such condition. This assumption is in agreement with the direct calculations of the friction coefficient (Coulomb collision frequency) in terms of the binary collision cross-sections. Moreover, it was shown in Ref. [12] that in the case of strong grain-plasma coupling the influence of the Coulomb collisions is also small and the kinetic equation again reduces to the Vlasov equation. This means that fluctuation evolution equations whose solutions determine the explicit form of the Balescu-Lenard collision term are similar to those for the case of weakly coupled plasmas and thus Eq. (71) is still valid.

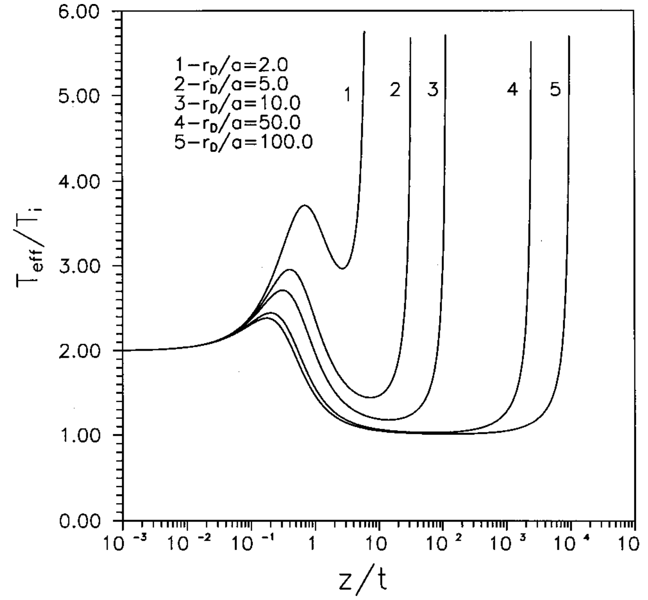

FIG. 1. The normalized grain effective temperature T_{eff}/T_i as a function of the ion-grain coupling constant $\Gamma_{ig} = |z_g e_i|/aT_i = z/t$ for various values of the Debye length to grain size ratio r_D/a in the case of fully ionized plasma.

The new kinetic coefficients yield the effective temperature for thermal grain motion given by

$$T_{\mathrm{eff}} = T_i \frac{2\left(1 + \dfrac{z}{t} + \dfrac{z^2}{t^2} \ln \Lambda_i\right)}{1 - \dfrac{z}{t} + 2\,\dfrac{z^2}{t^2} \ln \Lambda_i}, \tag{78}$$

i.e., elastic collisions can produce saturation of the grain temperature. However, in the case of dominant influence of charging collision T_{eff} can be still anomalously large. Dependences of T_{eff} on the quantity z/t for various values of r_D/a are shown in Fig. 1. The predictions given by Eq. (78) can be used for a qualitative explanation of the experimentally observed grain temperatures which are usually much higher than the ion temperature, $T_g \gg T_i$ (see, for example [15,16], $T_i \sim 0.1$ eV, $T_g \sim 4$–40 eV).

Finally, we point out that the results obtained can also be modified for the case of a plasma with a neutral component. One may introduce an additional collision term along with the term (71). Since the collision integral describing elastic collisions of neutrals with grains also can be represented in the Fokker-Planck form (it follows from the Bol'tzman collision integral) the presence of neutrals results in new additions to $\widetilde{D}_\parallel$ and $\widetilde{\beta}$, namely [12]

$$\widetilde{D}_\parallel(q,v) = D_0\left(1 + \frac{z}{t} + \frac{z^2}{t^2} \ln \Lambda_i + \frac{n_n}{n_i}\left(\frac{m_n}{m_i}\right)^{1/2}\left(\frac{T_n}{T_i}\right)^{3/2}\right),$$

$$\widetilde{\beta}(q,v) = \beta_0\left(1 - \frac{z}{t} + 2\frac{z^2}{t^2} \ln \Lambda_i + 2\frac{n_n}{n_i}\left(\frac{m_n}{m_i}\right)^{1/2}\left(\frac{T_n}{T_i}\right)^{1/2}\right). \tag{79}$$

As a result the effective temperature is modified into

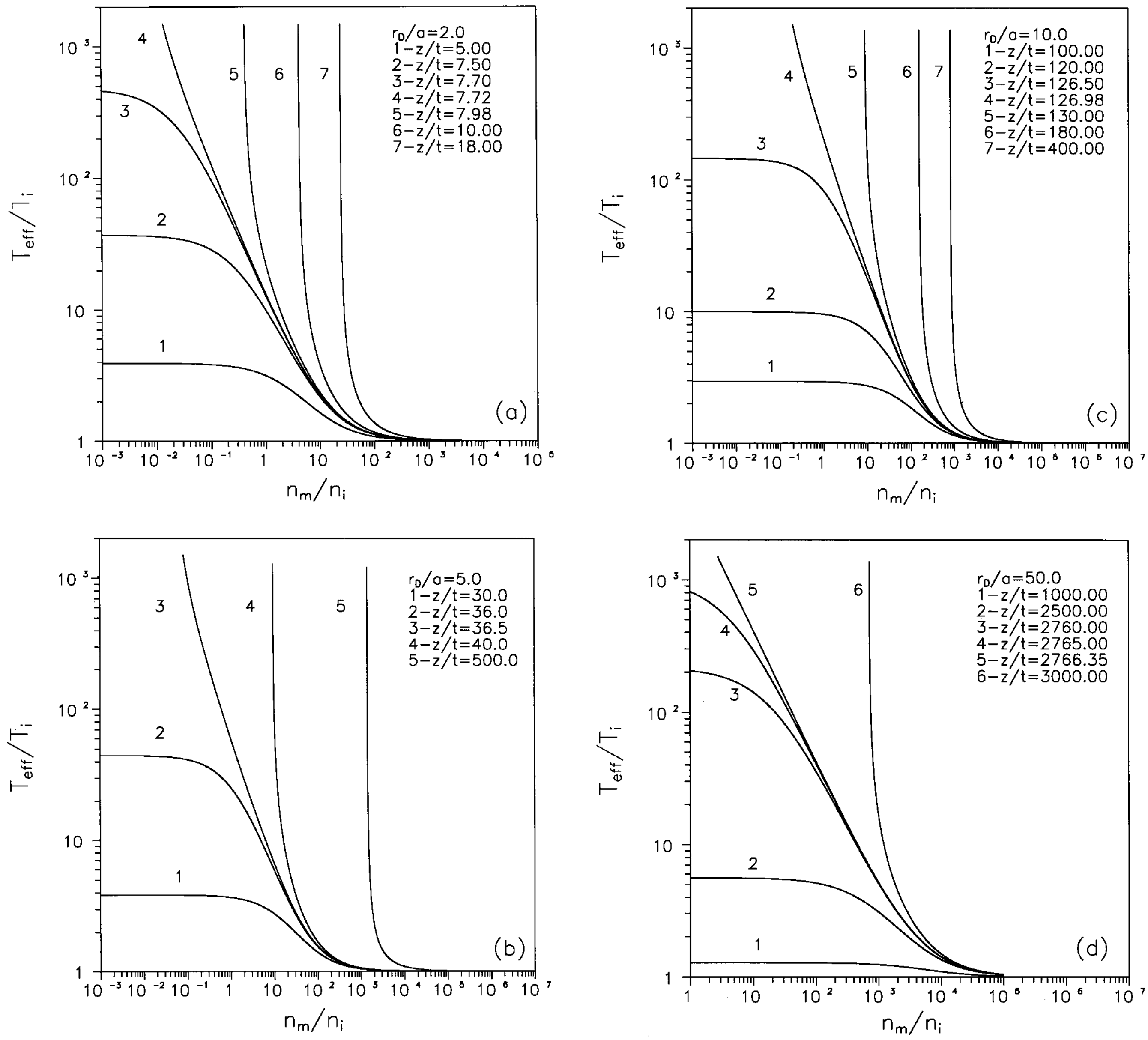

FIG. 2. The normalized grain effective temperature T_{eff}/T_i as a function of the neutral to ion density ratio n_m/n_i for various values of $\Gamma_{ig}=z/t$ and r_D/a in the case of partially ionized plasma [(a) $-r_D/a=2$; (b) $-r_D/a=5$; (c) $-r_d/a=10$; (d) $-r_D/a=50$].

$$T_{\mathrm{eff}}=2T_i\frac{\left(1+\dfrac{z}{t}+\dfrac{z^2}{t^2}\ln\Lambda_i+\dfrac{n_n}{n_i}\left(\dfrac{m_n}{m_i}\right)^{1/2}\left(\dfrac{T_n}{T_i}\right)^{3/2}\right)}{\left(1-\dfrac{z}{t}+2\dfrac{z^2}{t^2}\ln\Lambda_i+2\,\dfrac{n_n}{n_i}\left(\dfrac{m_n}{m_i}\right)^{1/2}\left(\dfrac{T_n}{T_i}\right)^{1/2}\right)}. \tag{80}$$

According to Eq. (80) the effective temperature increases with decreasing neutral density [Figs. 2(a)–2(d)]. The influence of neutral density changes on the effective temperature would be especially important at $1-z/t+2(z^2/t^2)\ln\Lambda_i\lesssim 0$ (the curves 5–7 on Figs. 2). In this case a decrease of the neutral gas pressure can produce anomalous growth of T_{eff}. This conclusion is in qualitative agreement with the experimental observation of dusty crystals melting under the reduction of gas pressure [15,16].

The results obtained show that stationary distributions of grain velocities and charges are described by effective temperatures other than those of the plasma subsystem. These effective temperatures are determined by the competitive mechanics of collisions: grain-neutral collisions and elastic Coulomb collisions result in the equalization of the effective temperature to the temperature of neutrals or ions, respectively, while charging collisions can produce anomalous temperature growth. That could be one of the main mechanisms of grain heating.

VI. SUMMARY AND DISCUSSION

Rigorous evolution equations for microscopic distributions of dusty plasmas are formulated with regard for electron and ion absorption by grains. Abrupt changes of grain momentum and charge due to inelastic collisions between the grains and plasma particles as well as due to elastic collisions between the grains, are taken into account explicitly. The derived equations [Eqs. (11) and (24)] generalize the Klimontovich equation for ordinary plasmas to the case of dusty plasmas. These equations differ from the conventional Klimontovich equations by the presence of ''microscopic'' collision terms which describe contact elastic and inelastic collisions and generate the microscopic bombardment force and the Langevin forces that give rise to the grain diffusion in the charge and velocity spaces. Within the approximation of continuous description of the plasma subsystem (discreteness of electrons and ions is disregarded), the equations derived in Ref. [8] are recovered.

The microscopic equations thus obtained are employed used to derive the BBGKY hierarchy of equations for dusty plasmas. Such hierarchy [Eq. (37)] generalizes th known hierarchies for ordinary plasmas (see, for example, Ref. [11]) and for hard-sphere gas [10].

Various approaches to the derivation of kinetic equations for dusty plasmas on the basis of the derived microscopic equations and the BBGKY hierarchy are discussed. The asymptotics of the binary distribution functions describing the contact collision effects is applied to find the explicit form of kinetic equations in the approximation of dominant charging collisions [Eqs. (50), (51) at $\delta I_\sigma = 0$]. The possibility to calculate the corrections to the collision terms due to the Coulomb particle scattering is also discussed. The obtained kinetic equations are in good agreement with the equations proposed in Refs. [6,7].

The Fokker-Planck approximation for the collision terms in the case of dominant charging collisions is found and used to calculate stationary distributions of grain velocities and charges. Such distributions are shown to be described by the effective temperatures differing from those of the plasma subsystem. The mechanism of anomalous grain heating [15,16] by charging collisions is predicted.

To conclude, it should be noted that within the model under consideration many problems of dusty plasma kinetics still remain open. First of all, this concerns the problems of grain mass dynamics and neutral gas regeneration. Obviously, absorption of electrons and ions by grains should lead to grain mass growth, unless absorbed plasma particles leave the grain in the form of neutral atoms, or molecules, which appear due to surface recombination. If plasma particle absorption results in grain mass changes (bound pairs of electrons and ions do not escape from the grain surface) the problem how to describe the grain kinetics with regard for this effect is especially important since, as it has been shown recently [17], the characteristic time of grain subsystem relaxation could be of the order of the time during which the grain mass is changed considerably. This means that consistent description of grain dynamics requires allowance for the grain mass changes.

Appropriate kinetic equations for grains formulated on the basis of physical arguments (in these equations the grain mass is treated as a new independent variable similar to the grain charge) have already been proposed by one of the authors (S.A.T.) in Refs. [18,19]. The problem of the grain mass dynamics is not so crucial, if bound pairs of electrons and ions escape from the grain surface, i.e., if neutral gas regeneration occurs. In such case the model of a system with fixed grain mass (used in the present paper) looks quite reasonable. We understand that in order to properly describe the effect of neutral gas regeneration on the grain dynamics it is necessary to supplement the microscopic and kinetic equations for grains [Eqs. (24), (51), and (57)] with terms which take into account grain momentum change due to evaporation of neutrals from the grain. Nevertheless, it might be expected that this effect does not lead to considerable qualitative changes of the results obtained in the present paper, at least under the assumption that the distribution of regenerated neutrals at the instant of evaporation is isotropic. The quantitative description of the grain mass dynamics and neutral regeneration effects will be a matter of further studies.

The next problem which is still open is to generalize the kinetic description of dusty plasmas to the case of grains of different sizes (polydispersed grain subsystem). Such generalization could be performed in terms of the same approach as in this paper, but the algebra and final relations in this case become extremely lengthy. In the present state the theory proposed is applicable to the description of dusty plasmas with monodispersed grains. In spite of this restriction, however, the results obtained could be useful for the theoretical treatment of dusty plasma occurring in experiments with specially prepared equal-size grains.

The further generalization is also possible in the direction of more accurate description of plasma particles interaction with grains. In this paper we restrict ourselves to the assumption that any grain absorbs all encountered electrons and ions (i.e., accommodation coefficients are equal to one) that means an essential idealization. Moreover, secondary electronic emission and radiative processes associated with charge particle contact collisions with grains are ignored. Obviously these phenomena could influence grain kinetics and charging dynamics. However, we can expect that the conditions could be realized under which the above assumptions are reasonable (metallic dust particles with the accommodation coefficients close to one, low electron and ion temperature, etc.). The proposed approach makes it possible to extend the results obtained in this direction.

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