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Critical properties of a branched polymer growth model

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We study the branched polymer growth model (BPGM) introduced by Lucena *et al.* [Phys. Rev. Lett. **72**, 230 (1994)] in two dimensions. First the BPGM was simulated in very large lattices with concentrations of impurities $q=0$ and $q=0.2$. The scaling of the mass in chemical space gives accurate estimates of the critical branching probabilities b_c and of the chemical dimensions D_c at criticality, improving previous results. Estimates of the fractal dimension D_F at criticality are consistent with a universal value along the critical line. Our results for $q=0$ suggest small deviations of D_c and D_F from the percolation values. We also simulated the BPGM in finite lattices of lengths between $L=32$ and $L=512$ for the same concentrations q. Using finite-size scaling techniques, we confirm the previous estimates of D_F and the universality along the critical line, and obtain the correlation exponent $\nu=1.43\pm 0.06$. It proves that the BPGM is not in the same universality class of percolation in two dimensions. Finally, we simulate random walks on the critical polymers grown in very large lattices with $q=0$ and $q=0.2$, and obtain the random walk dimension D_w and the spectral dimension D_s. D_w is larger and D_s is smaller than the corresponding values in critical percolation clusters, due to the lower connectivity of the polymers. The scaling relation $D_s=2D_F/D_w$ is not satisfied, as observed in other tree-like structures.

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I. INTRODUCTION

Various statistical models of polymers' growth in free space and in disordered media have been studied in the last decades [1–4]. The simplest model to represent the conformational properties of linear polymers in diluted solutions and good solvent conditions is the self-avoiding walk (SAW) [1], where the interactions between the monomers are represented only by the excluded volume condition. The kinetic growth walk (KGW) has also been introduced to describe the growth of a linear polymer under the excluded volume condition [5,6]. In the KGW, at each time step, a monomer is added to the end of the (rigid) chain in a randomly chosen direction. These problems are not identical because different SAW configurations with the same number of monomers have the same statistical weight, but this is not true in the KGW model, although the ensemble elements are the same. Despite this important difference, it was shown that the SAW and the KGW have the same critical exponents [7].

In 1994, Lucena *et al.* [8] introduced the branched polymer growth model (BPGM), which is a generalization of the KGW that includes branching. It was originally proposed as a realistic model for the kinetic growth of ramified polymers in dirty media. Since then, the BPGM was studied in free and diluted two- and three-dimensional lattices by several authors [8–11], and also in the Cayley tree [10]. In this model, the center of the lattice at $t=0$ is occupied by a seed. At $t=1$, one monomer is added in a random direction with probability 1, and a second monomer is added in another direction with probability b. Thus, b is called the bifurcation probability. This process is continued so that, at time $t+1$, the polymer can grow from each of the sites added at the previous time t, which are named active sites. From each active site, one monomer is added with probability 1 and a second monomer is added with probability b, provided that there are sufficient empty neighbors. If an active site has no empty neighbor, it stops growing and a dangling end is created. If there are no active sites with empty neighbors, the polymer stops growing.

In two dimensions, it was shown [8] that, at a certain critical branching probability b_c, there is a transition between a finite growth regime ($b<b_c$), where the polymer structure is asymptotically equivalent to the KGW (or the SAW), and an infinite growth regime ($b>b_c$), where the polymer is compact. In a diluted lattice, the value of b_c depends on the concentration of impurities q, which is the fraction of nonavailable lattice sites.

Since the universality class of branched polymers (generalization of the SAW including branching) is believed to be the same of lattice animals [12,13], the question of the universality class of the BPGM at criticality attracted some interest. Estimates of the chemical dimension (D_c) of the critical BPGM in two dimensions [9,10] were consistent with the percolation value and led to the proposal that, at the critical line $b_c(q)$, this model belongs to the same universality class of percolation. On the Cayley tree, this equivalence was rigorously proved [10]. However, previous estimates of the fractal dimension D_F of the critical BPGM in the square lattice differed from the percolation value ($D_F^{\mathrm{perc}}=91/48\approx 1.896$): Lucena *et al.* obtained $D_F\approx 1.8$ at some points of the critical line, while Neves and Onody [11] obtained $D_F=1.963\pm 0.006$ for $q=0$ and $D_F=1.946\pm 0.002$ for $q=0.1$. Moreover, the estimate of the exponent ν for $q=0$ of Neves and Onody [11] differed from the percolation value. Consequently further analysis of the BPGM is important, particularly at criticality, not only for its theoretical relevance but also for its possible applications to real systems [8].

The first aim of this work is to improve previous results on the conformational properties of critical branched poly-

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mers in two dimensions, in order to determine critical exponents and discuss the (possible) equivalence to percolation at criticality. Then, for $q=0$ and $q=0.2$, we will simulate very large polymers near the critical line. The study of the BPGM in chemical space will provide estimates of b_c and D_c. The value of D_c seems to decrease as q increases, and is slightly different from the percolation value for $q=0$. The estimates of D_F are consistent with a universal value along the critical line, but the more accurate result for $q=0$ is slightly larger than D_F^{perc}. Subsequently, we will simulate the polymers in finite lattices and study the scaling of the average radius of gyration and of the correlation length. We will obtain estimates of the exponent ν of the BPGM that are definitely not consistent with the percolation value $\nu=4/3$. Then, our results show that the critical BPGM is not in the same universality class of percolation in two dimensions, although some dimensions that characterize the critical polymers (and are intimately related to its topology, such as D_c) are nearly the same.

We will also study random walks in two-dimensional branched polymers grown at criticality, in order to estimate the random walk dimension D_w and the spectral dimension D_s. We will show that D_w and D_s are different from the corresponding values in critical percolation clusters, and these differences are related to the different topologies of those fractals. This study is also relevant in the context of diffusion in disordered media, since we will show that the scaling relation $D_s=2D_F/D_w$ fails, even when strong corrections to scaling are considered in the calculation of D_s.

This paper is organized as follows. In Sec. II we will present results of simulations of the BPGM in (infinitely large) square lattices near the critical points for $q=0$ and $q=0.2$, and we will estimate b_c, D_c and D_F. In Sec. III, we will simulate the BPGM in finite square lattices near the critical points and obtain the exponent ν using finite-size scaling techniques. In Sec. IV we will present results of simulations of random walks on critical polymers and estimate D_w and D_s. In Sec. IV we will summarize our conclusions.

II. CHEMICAL AND FRACTAL DIMENSIONS OF THE CRITICAL BPGM

We simulated the BPGM in the square lattice for $q=0$ and $q=0.2$. Several values of b around the critical points were considered. At each point (q,b), we generated 1.5×10^5 polymers with a maximum number of monomers ranging from $M_{\text{MAX}}=1\times10^5$ to $M_{\text{MAX}}=2\times10^5$. The largest polymers were generated in free lattices near b_c. During the simulations, many polymers stopped growing due to steric hindrance effects, but their data were not considered in the calculation of average quantities. The sizes of the lattices used in the simulations are sufficiently large so that less than 0.1% of the generated polymers touch its borders (such polymers are also discarded from the calculation of average quantities).

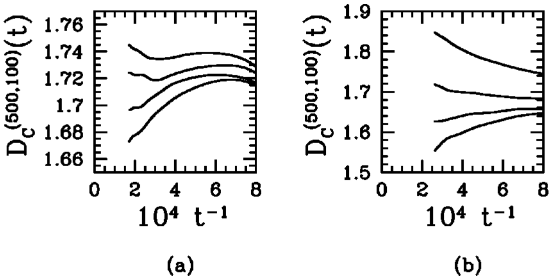


FIG. 1. Estimates of the chemical dimension of the BPGM for (a) $q=0$: from bottom to top, $b=0.0572$, $b=0.0573$, $b=0.0574$, $b=0.0575$; (b) $q=0.2$: from bottom to top, $b=0.164$, $b=0.165$, $b=0.166$, $b=0.167$.

The chemical distance l between two given sites of a fractal is defined as the length of the shortest path on the structure that connects those sites. In the case of the BPGM, the t^{th} shell is grown at the time step t from the active sites at time $t-1$. Consequently, that shell is the set of sites with chemical distance $l=t$ from the origin. The chemical dimension D_c relates the mass inside the t^th shell (i.e., the mass of the set with chemical distance $l\leqslant t$ from the origin), $M(t)$, to the chemical distance t:

$$M(t)\sim t^{D_c}. \tag{1}$$

The maximum chemical distance of the polymers in our simulations was $t_{\text{MAX}}=6\times10^3$ for branching probabilities near the critical points.

The fractal dimension D_F relates the mean square radius of gyration of the polymer $\langle R_g^2\rangle$ and its mass M, i.e., the number of monomers:

$$\langle R_g^2\rangle\sim M^{2/D_F}. \tag{2}$$

As observed in previous works [9,10], the convergence of the estimates of D_c to the asymptotic value is more rapid than the convergence of D_F. Consequently, it will be used to estimate b_c.

In order to study the asymptotic behavior of $M(t)$, we analyzed the evolution of the declivities of $\log M(t)\times\log t$ plots. The sequence of estimates

$$D_c^{(i,j)}(t)=\frac{\log(\bar{M}(t,j)/\bar{M}(t-i,j))}{\log(t/(t-i))} \tag{3}$$

was calculated, where the average mass $\bar{M}(t,j)$ is defined as

$$\bar{M}(t,j)=\frac{1}{(2j+1)}\sum_{k=-j}^{j}M(t+k). \tag{4}$$

In Eq. (4), $M(t)$ is the average mass inside the t^{th} chemical shell. As $t\rightarrow\infty$, $D_c^{(i,j)}(t)\rightarrow D_c$, for fixed i and j. The use of the average mass in Eq. (3) (with $j=100$) and large values of i ($i=500$) is suitable to reduce the effect of statistical fluctuations of the data with consecutive values of t.

In Figs. 1(a) and 1(b) we show $D_c^{(500,100)}(t)$ versus $1/t$ for polymers in the square lattice with $q=0$ and $q=0.2$, respectively, considering several values of the bifurcation probability b. For $b<b_c$, we expect that $D_c^{(i,j)}(t)\rightarrow1$ (the SAW

TABLE I. Concentration of impurities (q), estimates of critical branching probabilities b_c and estimates at the critical points of D_c, D_F, D_w and D_s.

q	b_c	D_c	D_F	D_w	D_s
0	0.05735 ± 0.00005	1.71 ± 0.02	1.91 ± 0.01	3.10 ± 0.02	1.27 ± 0.02
0.2	0.1655 ± 0.0005	1.67 ± 0.05	1.91 ± 0.02	3.12 ± 0.03	1.265 ± 0.015

value), while for $b > b_c$ we expect that $D_c^{(i,j)}(t) \rightarrow 2$ (compact clusters). The final estimates of b_c and D_c are shown in Table I.

Those estimates improve previous results [9,10] and agree with them within error bars. However, D_c seems to decrease slowly when q increases along the critical line, although the possibility of a universal value cannot be excluded. For $q = 0$, D_c is slightly larger than the estimate in critical percolation clusters $D_c^{\mathrm{perc}} = 1.675 \pm 0.003$ [14]. The simulation of very large polymers was crucial to notice this difference because it was possible to find the critical point with high accuracy. On the other hand, the (less accurate) estimate of D_c for $q = 0.2$ does not exclude the percolation value.

However, in order to discuss the question of the universality class of the critical BPGM, the calculation of static exponents such as D_F and ν is necessary. At this point it is relevant to stress that any difference in D_c can be related to the different topologies of those structures, while the static exponents may be the same, and vice-versa. For instance, recall that both a random walk and a SAW have $D_c = 1$, while their critical exponents are very different in one, two or three dimensions.

In order to study the asymptotic behavior of $\langle R_g^2 \rangle$ and calculate D_F, we adopted a procedure similar to the previous one. We constructed the sequence of estimates

$$D_F^{(i,j)}(M) = 2\,\frac{\log(M/(M-i))}{\log(\overline{R}_M^{(j)}/\overline{R}_{M-i}^{(j)})}, \qquad (5)$$

where the average (squared) radius $\overline{R}_M^{(j)}$ is defined as

$$\overline{R}_M^{(j)} = \frac{1}{(2j+1)} \sum_{k=-j}^{j} \langle R_g^2 \rangle_{(M+k)}. \qquad (6)$$

In Eq. (6), $\langle R_g^2 \rangle_M$ is the average squared radius of gyration of polymers with M monomers. As $M \rightarrow \infty$, $D_F^{(i,j)}(M) \rightarrow D_F$ for fixed i and j.

In Figs. 2(a) and 2(b) we show $D_F^{(30000,500)}(M)$ versus $1/M$ for polymers in the square lattice with $q = 0$ and $q = 0.2$, respectively, considering some values of the bifurcation probability b around the critical points shown in Table I. Our estimates of D_F for the critical BPGM are also shown in Table I. Within error bars, they are consistent with $D_F^{\mathrm{perc}} = 91/48 = 1.896\ldots$ [15] of two-dimensional critical percolation clusters, although the central estimates are slightly larger than that value. Previous estimates of D_F [8,11], obtained with polymers confined in finite lattices and with different methods, were not universal along the critical line and do not agree with our estimates.

III. FINITE-SIZE SCALING

The correlation function of this problem, $g(r)$, is defined as the probability that a lattice site at distance r from the origin is occupied. For $b > b_c$ only finite polymers must be considered. We expect that

$$g(r) \sim e^{-r/\xi}, \qquad (7)$$

where ξ is the correlation length of the problem. The exponent ν governs the divergence of ξ near the critical point:

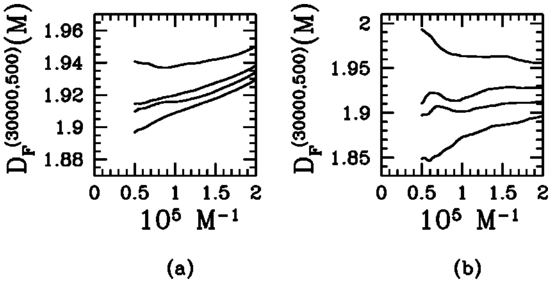
$$\xi \sim |b_c - b|^{-\nu}. \qquad (8)$$

Apart from a constant factor, the average squared correlation length of the BPGM is given by

$$\xi^2 = \frac{\int r^2 g(r) d^D r}{\int g(r) d^D r}, \qquad (9)$$

where the integrals are taken over the D-dimensional space where the polymers are generated. Equation (9) is an average square radius where the statistical weight of each polymer is its number of monomers. At this point, it is important to notice that, in the previous calculation of the average radius of gyration (Sec. II), each polymer was taken with the same weight, since our aim was to study the geometry of infinitely large polymers.

Alternatively, in the averaging process where any polymer has the same statistical weight, the denominator in Eq. (9) is the average number of monomers of finite polymers (average mass), and the numerator is the average moment of inertia $(\int r^2 dm)$. The above averaging process differs from the percolation problem, where the average correlation length is obtained by taking all sites as origins for calculating



(a) (b)

FIG. 2. Estimates of the fractal dimension of the BPGM for (a) $q = 0$: from bottom to top, $b = 0.0570$, $b = 0.0573$, $b = 0.0574$, $b = 0.0580$; (b) $q = 0.2$: from bottom to top, $b = 0.1625$, $b = 0.165$, $b = 0.166$, $b = 0.170$.

$g(r)$ [15]. This difference is related to the presence of a well-defined origin in the BPGM.

Reliable estimates of ν cannot be obtained directly from Eq. (8) due to the error bars of b_c and because Eq. (8) is valid only in a narrow critical region. On the other hand, the exponent ν appears in the finite-size scaling relations of all quantities calculated in finite lattices. Then, the average radius of gyration and the correlation length of confined polymers, as well as their b-derivatives, will be used to estimate that exponent.

It is expected that, in a finite lattice of length L and near the critical point, the average squared radius of gyration $\langle R_g^2 \rangle_L$ (as calculated in Sec. II) of confined polymers scales as [16]

$$\langle R_g^2 \rangle_L \approx L^{D_F} f(|b-b_c| L^{1/\nu}). \tag{10}$$

In Eq. (10), f is a scaling function of the variable $x=|b-b_c|L^{1/\nu}$. The squared correlation length ξ_L^2 of confined polymers scales as

$$\xi_L^2 \approx L^2 g(|b-b_c| L^{1/\nu}), \tag{11}$$

where g is another scaling function of the variable x.

The b-derivative of $\langle R_g^2 \rangle_L$ at $b=b_c$ scales as

$$\rho_L(b_c) \equiv \left(\frac{d\langle R_g^2 \rangle_L}{db} \right)_{b=b_c} \sim L^{\lambda_1}, \quad \lambda_1 \equiv D_F + \frac{1}{\nu}, \tag{12}$$

and the b-derivative of ξ_L^2 at $b=b_c$ scales as

$$\mu_L(b_c) \equiv \left(\frac{d\xi_L^2}{db} \right)_{b=b_c} \sim L^{\lambda_2}, \quad \lambda_2 \equiv 2 + \frac{1}{\nu}. \tag{13}$$

We simulated the BPGM in square lattices of lengths L ranging from $L=32$ to $L=512$, at $b=b_c$ and at two other values of b, typically 0.5% below and above b_c (b_- and b_+). Lattices with $q=0$ and $q=0.2$ were considered. The polymers are grown until there is no growth site with an available empty neighbor (if the polymer touches a border of the lattice, it continues growing). For each L, the total number of polymers grown at b_- and b_+ varied from 10^6 to 10^7, in five different runs. $\rho_L(b_c)$ (Eq. 12) and $\mu_L(b_c)$ (Eq. 13) were calculated numerically from the differences in the average radii of gyration and in the correlation lengths at b_- and b_+. The error bars were estimated from the dispersions of the results in the different runs.

Finite-size estimates of D_F, λ_1 and λ_2 were obtained from

$$D_F(L) = \frac{\log(\langle R_g^2 \rangle_L / \langle R_g^2 \rangle_{L/2})}{\log 2}, \tag{14}$$

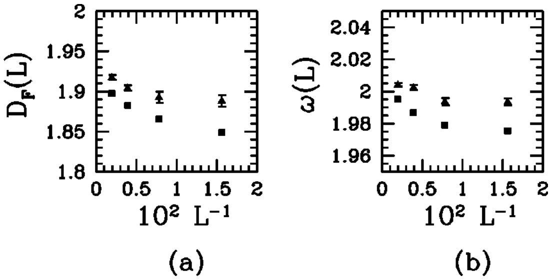
$$\lambda_1(L) = \frac{\log(\rho_L / \rho_{L/2})}{\log 2}, \tag{15}$$

and

$$\lambda_2(L) = \frac{\log(\mu_L / \mu_{L/2})}{\log 2}, \tag{16}$$

with $\langle R_g^2 \rangle_L$, ρ_L and μ_L calculated at $b=b_c$. The scaling of the correlation length [Eq. (11)] suggests the calculation of the finite-size estimates

$$\omega(L) = \frac{\log(\xi_L^2 / \xi_{L/2}^2)}{\log 2}. \tag{17}$$



(a) (b)

FIG. 3. (a) Finite-size scaling estimates of the fractal dimension of the BPGM for $q=0$ (squares) and $q=0.2$ (triangles); (b) finite-size scaling estimates of the squared correlation length dimension ω for $q=0$ (squares) and $q=0.2$ (triangles).

Since $\omega(L) \to 2$ as $L \to \infty$, the extrapolation of $\omega(L)$ is a useful test to our methods.

In Fig. 3(a) we show $D_F(L)$ versus $1/L$ and in Fig. 3(b) we show $\omega(L)$ versus $1/L$, for $q=0$ and $q=0.2$. It is clear from Fig. 3(b) that $\omega(L) \to 2$ with an accuracy better than 1%, which also confirms ξ^2 as the (squared) correlation length of the problem. The universality at the critical line is suggested by the convergence of $D_F(L)$ to the same region for $q=0$ and $q=0.2$ in Fig. 3(a). The estimate of D_F from finite-size scaling is shown in Table II and is consistent with the previous one (Table I).

In Fig. 4(a) we show $\lambda_1(L)$ versus $1/L$ and in Fig. 4(b) we show $\lambda_2(L)$ versus $1/L$, for $q=0$ and $q=0.2$. Universality along the critical line is confirmed once again. The final estimates of λ_1 and λ_2 are shown in Table II. The same estimate $\nu=1.43\pm0.06$ is obtained from the exponents relations in Eqs. (12) and (13) (see Table II), which shows the reliability of the method. That estimate is different from the percolation value $\nu^{\mathrm{perc}}=4/3=1.333\ldots$ [15], even if the error bar is considered. It proves that the critical BPGM in two dimensions is not in the universality class of percolation. It is important to mention that our estimate is also very different from the lattice animals exponent $\nu^{LA}=0.6408\pm0.0003$ [17].

TABLE II. Estimates from finite-size scaling of D_F, λ_1 and λ_2, and the estimates of the exponent ν obtained using Eqs. (12) and (13).

D_F	λ_1	λ_2	ν (Eq. 12)	ν (Eq. 13)
1.92 ± 0.02	2.62 ± 0.02	2.70 ± 0.03	1.43 ± 0.06	1.43 ± 0.06

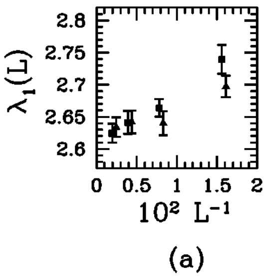


FIG. 4. (a) Finite-size scaling estimates of the exponent λ_1 (Eq. 12) for $q=0$ (squares) and $q=0.2$ (triangles); (b) finite-size scaling estimates of the exponent λ_2 (Eq. 13) for $q=0$ (squares) and $q=0.2$ (triangles).

Our estimates of ν greatly improve the previous ones [11], obtained with different methods and with much larger error bars.

IV. RANDOM WALKS ON POLYMERS AT CRITICALITY

In order to improve the knowledge of the critical polymers' properties, we studied random walks (RW) on those structures. It is interesting for the study of diffusion processes in this system and also for the connection with the problem of lattice vibrations, i.e., for the calculation of the spectral dimension [18].

We generated 3×10^3 polymers with 6×10^5 monomers at ($q=0$, $b_c=0.057\,35$) and at ($q=0.2$, $b_c=0.1655$). On each polymer, we simulated 5×10^3 RW with a maximum number of steps $N_{\text{MAX}}=4\times10^4$. The starting position of each walker was randomly chosen in a square 100×100 window around the origin because the choice of different starting positions for the walker allows it to visit many different microscopic environments on the polymer. At each time step, the walker may move along any monomer that begins or ends at its present position. Less than 0.5% of the RW that were generated have attained the active sites (the border of the polymer network), and these walks were not considered to compute average quantities.

The first important quantity to be measured is the mean square displacement $\langle R^2\rangle_N$ of N-step RW. It scales as

$$\langle R^2\rangle_N \sim N^{2/D_w}, \tag{18}$$

where D_w is the RW dimension. In Euclidean lattices and weakly disordered lattices (e.g., disordered lattices above the percolation threshold), $D_w=2$, indicating normal diffusion. In highly disordered media, such as most fractal lattices, $D_w>2$, which represents an anomalous diffusion [19–21].

We calculated a sequence of approximants $D_w^{(i,j)}(N)$ defined analogously to Eqs. (5) and (6), by replacing the average squared radius of the polymer ($\langle R_g^2\rangle_M$), by the rms displacement of the RW ($\langle R^2\rangle_N^{1/2}$), and replacing the number of monomers M by the number of RW steps N. In Figs. 5(a) and 5(b) we show $D_w^{(4000,100)}(N)$ versus $1/N$ for RW on critical polymers at ($q=0$, $b_c=0.057\,35$) and ($q=0.2$, $b_c=0.1655$), respectively. We obtain very accurate estimates of D_w, as shown in Table I. We also simulated random walks on polymers grown in other points of the critical regions, i.e., with values of b within the error bars of b_c (Table I). They provide results statistically identical to the ones presented in Figs. 5(a) and 5(b), confirming the reliability of the estimates of D_w.

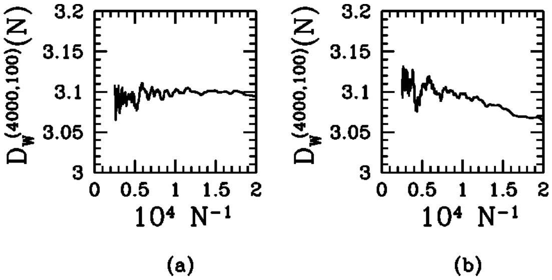


FIG. 5. Estimates of the random walk dimensions on critical polymers grown in lattices with (a) $q=0$ and (b) $q=0.2$.

Our results suggest a universal exponent D_w along the critical line $b_c(q)$, although a very small variation cannot be completely discarded. However, notice that they are remarkably different from the value $D_w^{\text{perc}}=2.871\pm0.001$ [14] for RW in two-dimensional critical percolation clusters. This difference follows a general trend for the variation of D_w in fractals with the same D_F: D_w increases in fractals with a higher density of dangling ends, since they contribute significantly to the delay of the diffusion. This is exactly the case of the polymers grown in the BPGM when compared to the percolation clusters. This trend was previously found in many other fractals where D_F and D_w were exactly known or were calculated with high accuracy [22].

The other quantity that we measured was the mean number of distinct sites visited by N-step walks, $\langle S\rangle_N$. It scales as

$$\langle S\rangle_N \sim N^{D_s/2}, \tag{19}$$

where D_s is the spectral dimension of lattice's vibrations [18]. Equation (19) is valid only if $D_s<2$, but this is the case of the most frequently studied fractal lattices [19–21].

We calculated the sequence of approximants $D_s^{(i,j)}(N)$, defined analogously to Eq. (5), but now replacing $\langle R_g^2\rangle_M$ by $\langle S\rangle_N$. The convergence of those approximants to D_s is usually slow due to strong corrections to scaling; for instance, in a two-dimensional lattice, Eq. (19) has a logarithmic correction [23]. In Figs. 6(a) and 6(b) we show $D_s^{(2000,100)}(N)$ ver-

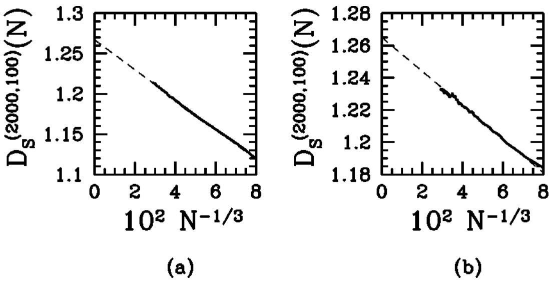


FIG. 6. Estimates of the spectral dimensions of critical polymers grown in lattices with (a) $q=0$ and (b) $q=0.2$. Dashed lines are least squares fits of the data.

sus $1/N^{1/3}$ for random walks on critical polymers at ($q=0$, $b_c=0.057\,35$) and at ($q=0.2$, $b_c=0.1655$), respectively. The $N^{-1/3}$ variable in Figs. 6(a) and 6(b) is the one that best fits the sequences $D_s^{(i,j)}(N)$ to a straight line, for various i and j.

The final estimates of D_s are shown in Table I. They are also consistent with a universal value along the critical line and are different from the estimate $D_s^{\mathrm{perc}}=1.325\pm 0.006$ [24] in two-dimensional critical percolation clusters. This difference is also related to the very different topological structures of the critical polymers and the critical percolation clusters, in particular for the lower connectivity of the polymers. Also notice that the relative difference in D_F is much smaller than the difference in D_s.

We also notice that the above estimates of D_s, D_w and D_F do not satisfy the scaling relation [18]

$$D_s=2\frac{D_F}{D_w}, \tag{20}$$

even if the error bars of those quantities are considered. For instance, for $q=0.2$ we obtain $2D_F/D_w=1.224\pm 0.012$ from the data in Table I, and this value is certainly smaller than the asymptotic value of D_s [see Fig. 6(b)].

Previous works have suggested the failure of Eq. (20) in other tree-like structures [25,26]. In some cases it was recovered with the introduction of strong corrections to scaling [27], but the trends of $D_s^{(i,j)}$ in Figs. 6(a) and 6(b) do not suggest this possibility. On the other hand, it is important to notice that the deviations from Eq. (20) are nearly 3%, indicating that this relation, although not exact, still gives reasonable qualitative information.

V. CONCLUSION

We studied conformational properties of the two-dimensional BPGM in the critical region and diffusion in these polymers.

The critical bifurcation probabilities b_c were estimated from the scaling of the polymer mass in chemical space, which also gives the chemical dimension D_c at criticality. Polymers in free lattices and lattices with concentration $q=0.2$ of impurities were considered. The estimate of D_c for $q=0$ is slightly different from the value in critical percolation clusters, which may be related to the different topologies of these fractals. A small difference from the percolation value is also observed in the fractal dimension D_F for $q=0$. However, within error bars our results are consistent with universality along the critical line. The finite-size scaling study of the problem also suggests this universal behavior, and gives estimates of the correlation length exponent ν that are different from ν^{perc}, indicating that the critical BPGM is not in the universality class of percolation.

Simulations of random walks in the critical polymers provide estimates of the random walk dimension D_w and the spectral dimension D_s. D_w is larger than the value in critical percolation clusters due to a higher density of dangling ends in the polymers. D_s is also different from the value in critical percolation clusters, and the scaling relation (20) is not satisfied, even if strong corrections to scaling are introduced.

The main conclusion from our work is that the BPGM is not in the same universality class of percolation, although the differences in critical exponents and in D_c (which is an intrinsic dimension) are small. On the other hand, the other intrinsic dimension that was calculated here, D_s, is clearly different from the percolation value, as expected from their very different topological structures. It is also relevant to stress that, together with D_F, the calculation of D_s is important for possible applications of the BPGM to real systems.

Note: in a recent paper [28], the two-dimensional BPGM was studied using exact enumeration techniques up to $t=14$. Ergodicity violation in some Monte Carlo implementations of the problem was discussed, which may explain some differences from our results.

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