Linear and nonlinear experimental regimes of stochastic resonance

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We investigate the stochastic resonance phenomenon in a physical system based on a tunnel diode. The experimental control parameters are set to allow the control of the frequency and amplitude of the deterministic modulating signal over an interval of values spanning several orders of magnitude. We observe both a regime described by the linear-response theory and the nonlinear deviation from it. In the nonlinear regime we detect saturation of the power spectral density of the output signal detected at the frequency of the modulating signal and a dip in the noise level of the same spectral density. When these effects are observed we detect a phase and frequency synchronization between the stochastic output and the deterministic input.

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I. INTRODUCTION

The renewed interest of the last two decades on stochastic processes modeling different phenomena of physics, chemistry, and engineering sciences has led to the discovery of noise-induced phenomena in nonlinear systems away from equilibrium. In these systems a variation of the level of external noise can qualitatively change the response of the system. The paradigmatic example of these noise-induced phenomena is stochastic resonance [1] (for a recent review see Ref. [2]). Other examples of noise-induced phenomena comprise resonant activation [3], noise-induced transitions [4], and noise enhanced stability [5,6].

Stochastic resonance (SR) manifests itself as an enhancement of the system response for certain finite values of the noise strength. In particular the signal-to-noise ratio (SNR) shows a maximum as a function of the noise intensity. In other words, a statistical synchronization of the random transitions between the two metastable states of the nonlinear system takes place in the presence of an external weak periodic force and noise. Such a physical system presents a time-scale matching condition, which can be observed by tuning the noise level to such a value that the period of the driving force approximately equals twice the noise-induced escape time. The SR phenomenon appears in a large variety of physical systems and has been observed in different systems, ranging from sets of neurons [7], to lasers [8] and to solid-state devices, like superconducting quantum interference devices and tunnel diodes [9].

The SR phenomenon is a well investigated phenomenon both from a theoretical and an experimental point of view [2]. However, few studies systematically analyze the SR phenomenon for different values of the frequency and amplitude of the modulating signal [8–12]. In this paper we systematically study the SR phenomenon as a function of the frequency and amplitude of the modulating signal in a physical bistable system based on a tunnel diode. Our experimental setup allows us to investigate this phenomenon in a range of amplitude and frequency spanning several orders of magnitude. By varying the amplitude and the frequency of the modulating signal we detect both the regime of the SR phenomenon described by the linear-response theory and the nonlinear deviation from it. In the linear regime we observe the customary behavior of stochastic resonance whereas in the nonlinear regime we detect a saturation of the power spectral density measured at the frequency of the modulating signal and a depletion of the power spectral density of the noise at the same frequency. When the noise depletion takes place we observe phase and frequency synchronization between the stochastic output and the deterministic input.

The paper is organized as follows. In Sec. II we describe the experimental apparatus and we discuss the stochastic differential equation associated to the electronic circuit based on a tunnel diode. In Sec. III we present our experimental results of the power amplification and SNR as a function of the noise intensity for different values of the amplitude and frequency of the modulating signal. In this section we discuss the detected deviations from the predictions of the linear-response theory and we present an evidence of phase and frequency synchronization detected in the nonlinear regime of high values of the amplitude of the modulating signal. In Sec. IV we briefly draw our conclusions.

II. EXPERIMENTAL APPARATUS AND THE TUNNEL DIODE

The experimental setup used for investigating the SR phenomenon is a bistable electronic system based on a tunnel diode. The physical system is a series of a resistor (tunable to a desired value) and a tunnel diode in parallel to a capacitor. The tunnel diode is a highly doped semiconductor device with a typical current-voltage characteristic showing a region of negative differential resistance, which is due to a tunneling current from the valence band of the *n*-doped region to the conducting band of the *p*-doped region. There are two stable states and one unstable state [13–15]. For details about this experimental setup see Ref. [16].

A network of general purpose very low-noise wideband operational amplifier is used to sum the driving periodic signal and the noise signal. The noise signal is the output of a

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commercial digital noise source, whose spectral density is approximately flat up to 20 MHz and whose root-mean-square voltage V_{rms} may be selected, at the output of the network of operational amplifiers, within the range from 0.133 to 5.5 V with a 27-mV resolution. At the output of the operational amplifier the statistical properties of the noise are altered by the filtering of the operational amplifier. We measure the noise $v_n(t)$ at the output of the operational amplifier and we observe that it is a Gaussian noise characterized by a spectral density, which is flat at low frequency ($f < 1.25$ MHz). By defining the correlation time of the Gaussian noise as the time at which the normalized autocorrelation function assumes the value $1/e$, we measure $\tau_n = 120$ ns. In our measurements we vary the amplitude and the frequency of the driving periodic signal $v_s(t) = V_s \cos(2\pi f_s t)$. Specifically, we vary the amplitude V_s from 0.0067 to 1.00 V, and the frequency f_s from 1 Hz to 1 MHz. The output voltage across the diode v_d is detected by a digital oscilloscope and transferred to a PC. A typical time series has 4096 records. The digitized time series are analyzed on line by using a home developed computer code that uses a fast Fourier transform (FFT) routine. The values of the power level of the signal and noise component of $v_d(t)$ localized around the frequency of the modulating signal are obtained by recording a number k of different time series for each set of the control parameters. The measured power levels are the average values of the k-detected FFT. In our experiments, the typical value of k is $k = 10$ but measurements with $k = 100$ are also performed when necessary. We perform experiments by varying the modulating frequency from 1 Hz to 1 MHz. The experiment that needs the longest acquisition time is, of course, the experiment performed at the lowest frequency. For such an experiment, our computer controlled experimental setup needs 340 seconds for the recording and processing of a single realization approximately. This implies that the measure of a complete curve of the signal and/or of the SNR as a function of the noise amplitude in the various cases considered here (in the present study we detect for each experimental curve from 36 to 176 different values of the measured variable by setting a frequency value as low as $f_s = 1$ or $f_s = 10$ Hz) can take as long as 40 h (when $k = 10$). This implies that some of the measurements summarized in the figures of the present paper require an acquisition time of 180 h. Hence the duration of the measurements sets a limit to the realistic number of different realizations that can be averaged to minimize the experimental uncertainty.

We model our electronic circuit by writing down its differential equation. This equation is

$$\frac{dv_d}{dt} = -\frac{dU(v_d,t)}{dv_d} + \frac{1}{RC} v_n(t), \tag{1}$$

which is formally equivalent to a stochastic differential equation describing the position of an overdamped random particle moving in a generalized potential. In this equation R is the biasing resistor, C is the parallel capacitor (in our case 45 pF) of the circuit, and $v_n(t)$ is a noise voltage mimicking the presence of a finite temperature in the corresponding overdamped system with a physical particle. The generalized potential $U(v_d,t)$ associated to our physical system is [16]

$$U(v_d,t) = -\frac{V_b v_d}{RC} + \frac{v_d^2}{2RC} + \frac{1}{C}\int_0^{v_d} I(v)\,dv - \frac{V_s v_d}{RC}\sin(\omega_s t), \tag{2}$$

here V_B is the biasing voltage of the electronic bistable network. In principle, Eq. (2) allows us to describe in a precise way the shape of the generalized potential under the assumption that $I(v_d)$ is known for any value of v_d. However, in practice this is unfortunately not true for part of the region of negative resistance of the tunnel diode. In fact the detailed measurement of $I(v_d)$ in the region of negative resistance is a major experimental problem that cannot be solved without changing the matching impedances of the experimental setup. This prevent us from a precise quantitative theoretical determination of the barrier height of the generalized potential $U(v_d)$.

Our circuit presents two control parameters affecting the shape of the associated generalized potential. They are the biasing resistance R and the biasing voltage V_b. We control both of them independently. We perform our experiments by ensuring a symmetric escape from one potential well to the other. This is done by selecting the values of the two control parameters ($R = 770$ Ω and $V_b = 6.76$ V) in such a way that we do not detect power spectral density of the output voltage v_d above the noise level at even harmonics of the frequency of modulating signal. We also verify that for this choice of control parameters the experimentally measured residence times have approximately the same value in the two potential wells.

One aim of this paper is to investigate the SR phenomenon over a wide interval of the frequency f_s of the modulating signal. To perform such a task, we have to overcome two experimental conflicting constraints: (i) we are forced to set the time constant of our system $\tau \equiv RC$ to a low value satisfying the inequality $f_s \ll 1/2\pi\tau$, and (ii) we need to use a high value of τ to maintain the ratio τ_n/τ as low as possible to conduct our experiments in the "white-noise" limit of $v_n(t)$. The best compromise we find is to set $\tau \equiv RC = 34.6$ ns. With this choice $1/2\pi\tau \approx 4 f_s^{max}$ and $\tau_n/\tau \approx 3.47$. In other words we guarantee the investigation of the SR phenomenon over a rather wide range of f_s by performing our experiments in a regime of moderately colored noise. This choice allows us to investigate the phenomenon over a range of the frequency, which extends from 1 to 10^6 Hz, but prevents us from an experimental determination of the barrier height based on experiments verifying Kramers theory of the thermal activation between the two wells. In fact, in the presence of colored noise, Kramers theory is no more valid and needs to be extended by theories whose results are dependent on the exact shape of the noise power spectrum.

III. STOCHASTIC RESONANCE FOR DIFFERENT VALUES OF FREQUENCY AND AMPLITUDE OF THE MODULATING SIGNAL

The SR phenomenon occurs when a *weak* signal acting in a noisy bistable environment induces a larger signal at the output of the nonlinear system. In the SR phenomenon the synchronization is mediated by the presence of a *finite* amount of noise. By considering these basic aspects, it is then natural to expect that linear-response theory together with more general concepts of perturbation theory for spectral quantities such as Floquet theory are useful methods in the description of such phenomenon.

In the present paper we aim to detect experimentally the limits of validity of the theoretical predictions obtained by using the linear-response theory in a physical system based on a tunnel diode. Specifically we investigate the SR phenomenon occurring in our physical system over wide ranges of the amplitude and frequency of the modulating signal. The investigated ranges are chosen so wide that they allow us to detect experimentally the crossover between the experimental region well described by the linear-response theory and the region of nonlinear deviation from it. In the regime of validity of the linear-response theory we verify the accuracy of the results predicted by this theory whereas in the nonlinear regime we point out the kind of deviations observed from the theory and we observe the new phenomena of phase and frequency synchronization in the presence of large input signals.

A bistable system based on a tunnel diode provides a versatile physical system in which SR can be investigated [9]. In this paper we perform an investigation of the SR phenomenon as a function of a wide range of amplitude and frequency of the modulating signal. The first investigation concerns the power P_1 of the output signal $v_d(t)$ localized around the frequency of the modulating signal. This quantity is obtained by integrating the spectral density $S(\omega)$ over the deltalike peak observed at angular frequency $2\pi f_s$. The signal ''power'' P_1 used in the theory of linear signal processing obtained by integrating the spectral density over the peak located at f_s is

$$P_1 = 2\pi |M_1|^2, \tag{3}$$

where $|M_1|$ is the magnitude of coefficient of the Fourier series $\langle v_d(t)\rangle = \Sigma_{-\infty}^{\infty} M_n \exp(in2\pi f_s t)$ taken at the frequency of the modulating signal. The linear-response theory for stochastic resonance predicts that $|M_1|$ is proportional to V_s at a fixed value of V_n [17]. We test this dependence on a large interval of values of the amplitude signal V_s. In Fig. 1 we show the measured values of $|M_1|$ as a function of V_s varying in the interval from 0.0067 to 1.00 V. The measures are done by setting $f_s = 10$ Hz and $V_{rms} = 1.89$ V. From the figure it is evident that the prediction $|M_1| \propto V_s$ obtained by using the linear-response theory is valid only within the amplitude interval $0.017 < V_s < 0.067$ V. The deviation observed for the lowest investigated value of the modulating amplitude signal ($V_s = 0.0067$ V) is probably due to experimental detection problems related with the low value of the signal whereas the deviation observed when $V_s > 0.067$ is entirely ascribed to a deviation of the physical system from the behavior predicted by the linear-response theory. In particular a saturation of the power localized at f_s is detected for large values of the amplitude of the modulating signal.

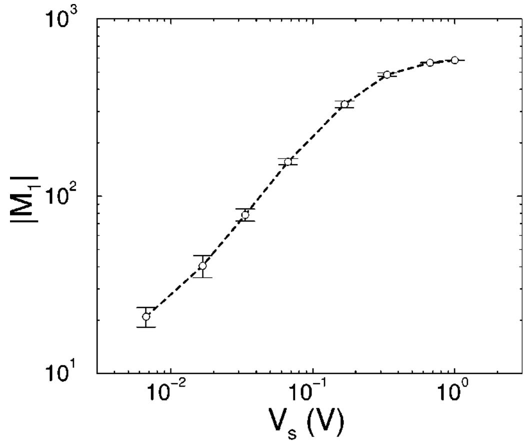

FIG. 1. The amplitude $|M_1|$ of the output signal $v_d(t)$ at the frequency of the modulating signal as a function of the amplitude of the modulating signal V_s. The noise level V_{rms} and the frequency of the modulating signal f_s are kept constant at the values $V_{rms} = 1.89$ V and $f_s = 10$ Hz. A linear relation between $|M_1|$ and V_s is detected within the interval $0.017 \leqslant V_s \leqslant 0.067$.

The second investigation concerns the frequency dependence of $|M_1|$. Within the framework of the linear-response theory, at fixed values of V_n, $|M_1|$ is related to V_s, f_s, and V_n through the relation

$$|M_1| \propto \langle v_d^2 \rangle \frac{V_s}{V_{rms}^2} \frac{\lambda_{min}}{[\lambda_{min}^2 + (2\pi f_s)^2]^{1/2}}, \tag{4}$$

where $\langle v_d^2 \rangle$ denotes a stationary mean value of the unperturbed system and λ_{min} is the smallest nonvanishing eigenvalue of the Fokker-Plank operator of the system without periodic driving [17]. It is equal to $\lambda_{min} = 2r_K = r_+ + r_-$, the sum of the Kramers rate of the two wells. This quantity is an exponential function of the noise amplitude V_{rms} under the hypothesis of white noise. We set $V_s = 0.067$ V to ensure that we are in a region of parameters where the linear-response theory may apply and we perform our experiments as a function of f_s for various values of V_{rms}. In Fig. 2 we show the results obtained. The general trend predicted by Eq. (4) is observed. Specifically an almost constant region of $|M_1|$ is observed for low values of f_s whereas an approximately power-law decrease of $|M_1|$ is observed for high values of f_s. The crossover between the two regimes provides a rough estimate of the experimental value of λ_{min} which, as expected, turns out to be dependent on the value of V_{rms}.

The measurements performed by setting $V_{rms} = 1.73$ V allow us to quantitatively estimate the power-law behavior observed for high values of f_s. In our experiments, we observe in this regime that $|M_1| \propto f_s^{-0.72}$. This is close but not coincident with the behavior expected from the linear-response theory $|M_1| \propto f_s^{-1}$. We do not have an explanation

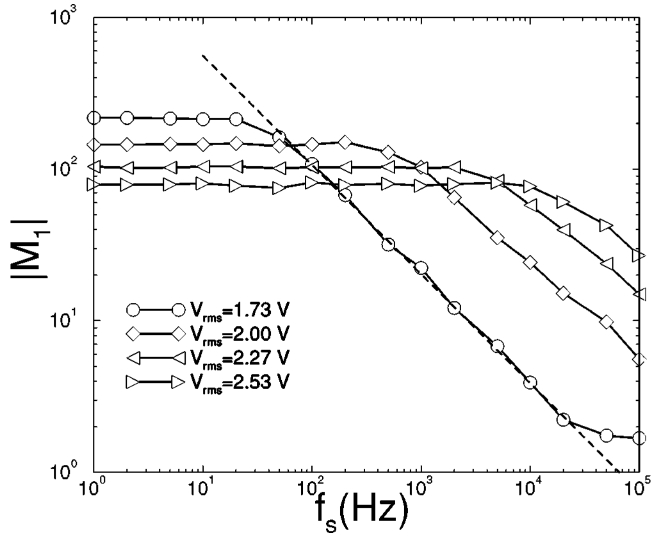

FIG. 2. The amplitude $|M_1|$ of the output signal $v_d(t)$ at the frequency of the modulating signal as a function of the frequency of the modulating signal f_s. The amplitude of the modulating signal V_s is kept constant at the value $V_s = 0.067$ V, whereas the noise amplitude takes four different values: 1.73, 2.00, 2.27, and 2.53 V. For each value of the noise amplitude $|M_1|$ presents two regimes. An almost constant regime for low values of f_s and a power-law regime for high values of f_s. The dashed line is the best power-law fit performed for the case $V_{rms} = 1.73$ V in the fitting interval $10^2 \leqslant f_s \leqslant 10^4$. The power-law exponent is -0.72. Experiments clearly show that the crossover between the two regimes is controlled by the value of V_{rms}.

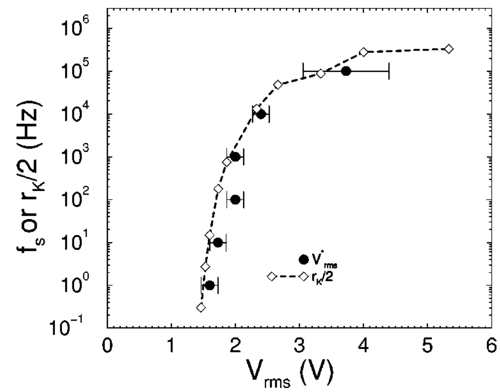

FIG. 3. Comparison of (i) the occurrence of the maximal value of the SNR (black circles) as a function of V_{rms} and f_s (error bars indicate the experimental uncertainty in the determination of the maximal SNR with (ii) the experimental values of half of the Kramers rate $r_k/2$ observed in the same system in the absence of the modulating signal. In all the SR measurements the amplitude of the modulating signal is set to 0.067 V. The stochastic synchronization at the SR between half of the Kramers rate and the frequency of the modulating signal is experimentally observed in a frequency interval spanning six orders of magnitude.

for this experimental discrepancy. Hereafter we list and briefly discuss some possible explanations for the observed deviation:

(i) the deviation is induced by a distortion of the $|M_1|$ measured value introduced by the noise background present in our measurements. In fact, in the experimental investigations, one cannot separate the signal $|M_1|$ from the noise present at the same frequency. For example the ''contamination'' of the two signals is evident in the last two experimental points of the curve of Fig. 2 obtained by setting $V_{rms} = 1.73$ V. For those points the presence of a noise floor alters the measured values. We tried to take into account this problem by limiting the best fitting of the power-law regime within the frequency interval $10^2 \leqslant f_s \leqslant 10^4$ Hz. However a weaker effect of the noise floor on the measurements done in this interval cannot be excluded.

(ii) Equation (4) is obtained for a quartic bistable model potential in the presence of white noise whereas the experimental conditions are slightly different. Deviation from the linear-response theory may be then ascribed to the different experimental conditions. In fact we already discussed in the previous section that we are doing our measurement in a bistable physical potential with a shape differing from the model bistable potential and in the presence of a moderately colored noise.

One key aspect of the SR phenomenon is the statistical synchronization that takes place when the Kramers time $T_K(V_{rms})$ between a two noise induced interwell transition is of the order of half-period of the periodic forcing. In other words statistical synchronization occurs when $f_s = 1/2T_K(V_{rms})$. In our measurement, we verify the validity of this description with the following procedure. We set $V_s = 0.067$ V and we measure the SNR of the output signal $v_d(t)$ at the frequency of the modulating signal for six values of f_s ranging from 1 to 10^5 Hz. We use these experimental results to single out for which value of $V_{rms} \equiv V^*_{rms}(f_s)$ a maximum of the SNR is detected. This is of course the state of maximal statistical noise induced synchronization. We then compare $V^*_{rms}(f_s)$ with the function $y(V_{rms}) = r_K(V_{rms})/2$, where the Kramers rate $r_K(V_{rms})$ is measured in the absence of a modulating signal. The results are shown in Fig. 3. From the figure it is clear that statistical synchronization is observed for all the investigated frequencies, supporting the traditional interpretation [1,2] of the stochastic resonance mechanism over a frequency range of the modulating signal spanning six frequency orders of magnitude. It is worth pointing out that synchronization is observed in spite of the fact that our experiments are performed in a regime of moderately colored noise.

We now investigate the SR phenomenon by studying both the signal power amplification

$$\eta = \frac{P_1}{P_{in}} = 2\left[\frac{|M_1|}{V_s}\right]^2 \tag{5}$$

and the signal-to-noise ratio

$$\mathrm{SNR} = 10\log_{10}\left[\frac{P_1}{N_1}\right]. \tag{6}$$

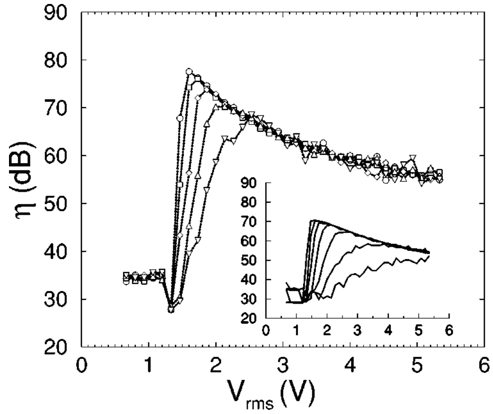

FIG. 4. The signal power amplification η of the output signal $v_d(t)$ at the frequency of the modulating signal as a function of the noise amplitude. The amplitude of the modulating signal V_s is kept constant at the value $V_s=0.067$ V, whereas the frequency takes 5 different values: 1 (circles), 10 (squares), 100 (diamonds), 1000 (triangles up), and 10 000 (triangles down) Hz. The profile of η becomes progressively more sharp as the frequency decreases. In the inset we show the result of the same kind of measurements performed in the nonlinear regime ($V_s=0.33$ V). In this case, higher values of the modulating frequency are investigated. Specifically the curves shown refer to values 1, 10, 100, 1000, 10 000, 100 000, and 1 000 000 Hz, from top to bottom, respectively. Also in this case the profile of η becomes progressively more sharp as the frequency decreases but the low-frequency limit is affected by the saturation of the output signal level and it is less sharp than expected from the linear-response theory.

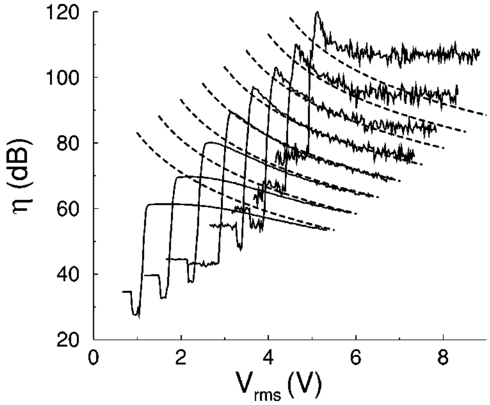

FIG. 5. The signal power amplification η of the output signal $v_d(t)$ at the frequency of the modulating signal as a function of the noise amplitude. The frequency of the modulating signal f_s is kept constant at the value $f_s=10$ Hz, whereas the amplitude takes 8 different values 0.0067, 0.017, 0.033, 0.067, 0.167, 0.333, 0.667, and 1.00 V, which correspond to the lines shown from top to bottom, respectively. For $V_s<1.00$ V, different lines are each shifted of 0.5 V_{rms} along the x axis and of 5 dB along the y axis for the sake of clearness. For example the fifth line from the top ($V_s=0.067$ V) is shifted by 1.5 V_{rms} along x and by 15 dB along y. By diminishing the amplitude V_s the value of η progressively approaches the limit predicted by the linear-response theory (dashed lines shown in the figure) for intermediate values of V_s. The best approximation of the system in terms of the linear-response theory is observed for $V_s=0.067$ V and 0.167 V (fourth and fifth lines starting from top). The convergence breaks down again for lowest values of V_s because for values of $V_s<0.033$ V the presence of a finite noise level makes it difficult to single out the small signal present.

The SNR is customary given in dB and it is obtained by dividing the output signal power level P_1 to the noise level signal N_1. Both quantities are measured at the frequency of the modulating signal.

We investigate both the effect of varying the amplitude and the frequency of the modulating signal on η and SNR. We first consider the role of the frequency of the modulating signal. Specifically we investigate the SR phenomenon as a function of V_{rms} by keeping V_s constant (we choose $V_s=0.067$ V) and by varying f_s from 1 to 10^6 Hz. For each pair of the control parameters V_s and f_s, we vary V_{rms} from 0.67 to 5.33 V. The measured values of the power amplification η are collected in Fig. 4, where we show η as a function of V_{rms} up to 7 different values of f_s, which are 1, 10, 100, 10^3, 10^4, 10^5 and 10^6 Hz. The classical profile of the SR phenomenon [2] is observed for the lowest values of f_s. For higher values of f_s, η deviates from the canonical SR profile by lowering and broadening its maximum. These results are in qualitative agreement with the explicit results theoretically obtained for the signal power amplification in a model bistable system [17].

The next investigation of the power amplification η concerns the study performed by keeping f_s constant, whereas V_s is varied. We set $f_s=10$ Hz and we vary V_s from 0.0067 to 1.00 V. For all the selected values of V_s we check that the amount of the amplitude of the modulating signal is not sufficient to induce deterministic jumps between the two wells. In Fig. 5 we show the experimental values of η versus V_{rms} obtained for 8 different values of V_s. The selected values are 0.0067, 0.017, 0.033, 0.067, 0.167, 0.333, 0.667, and 1.00 V. In the three dimensional figure the top line corresponds to $V_s=0.0067$ V, whereas the bottom line refers to the value $V_s=1.00$ V. The Z shift is kept constant for the sake of simplicity. In the figure we also show the functional prediction $\eta \propto V_{rms}^{-4}$ of the linear-response theory [17] expected for a quartic bistable model potential for large values of V_{rms} as a dashed line. From the figure one notes that the profile of η becomes progressively closer to the asymptotic behavior predicted by the linear-response theory for intermediate values of V_{rms} as $V_{rms}=0.067$ and $V_{rms}=0.167$ V. In the same figure, the curves measured for highest values of V_s show a significant deviation from the behavior predicted by the linear-response theory for large values of V_{rms}. This is due to the presence of a saturation of the output signal observed for large values of the amplitude of the modulating signal. A difference from the theoretical behavior expected in a model system is is also detected when one considers the lowest values of V_s. These deviations are observed in our experiment because for these values of V_s (0.0067, 0.017, and 0.033 V) the signal P_1 becomes of the same level of the noise level N_1 and it is therefore mixed and difficult to distinguish from it. Another difference is observed for low values of V_{rms}. In particular, in this region, η shows a sharper peak for the lowest values of V_s. We do not have a simple

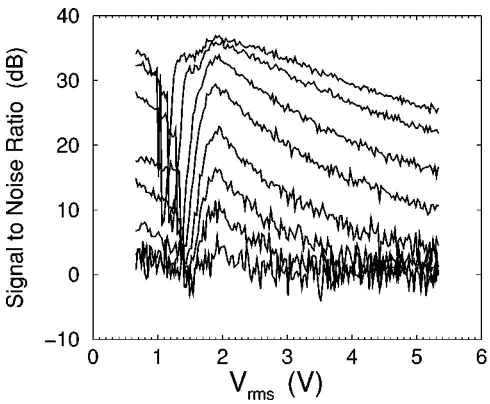

FIG. 6. SNR ratio of the output signal $v_d(t)$ at the frequency of the modulating signal as a function of the noise amplitude. The frequency of the modulating signal f_s is kept constant at the value $f_s = 10$ Hz, whereas the amplitude V_s takes 8 different values 0.0067, 0.017, 0.033, 0.067, 0.167, 0.333, 0.667, and 1.00 V, which correspond to the lines shown from bottom to top, respectively. The customary general profile of the stochastic resonance is observed. However shape differences are observed by investigating the phenomenon at different values of V_s. Specifically, high values of V_s (top curves) are characterized by distortions introduced by the active nonlinearities, whereas at low values of V_s (bottom curves) the SNR becomes negligible. As expressed in the caption of Fig. 5, the experimental conditions better interpreted in terms of the linear-response theory are the one observed for $V_s = 0.067$ and 0.167 V (fourth and fifth lines from bottom).

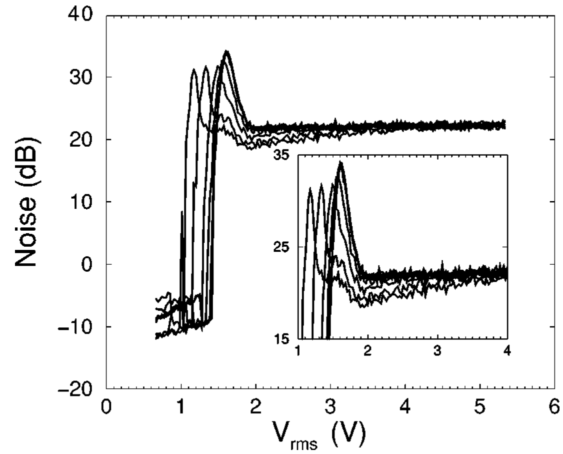

FIG. 7. Noise power level present in the output signal $v_d(t)$ at the frequency of the modulating signal as a function of the noise amplitude. The frequency of the modulating signal f_s is kept constant at the value $f_s = 10$ Hz, whereas the amplitude V_s takes eight different values 0.0067, 0.017, 0.033, 0.067, 0.167, 0.333, 0.667, and 1.00 V, which correspond to the lines shown from top to bottom, respectively. By diminishing the amplitude V_s the noise power level progressively approaches the curve observed in the absence of a modulating signal (indeed the four curves with lowest values of V_s are almost indistinguishable from the $V_s = 0$ V observation). For the highest values of V_s the presence of a dip is observed. The dip is more pronounced for higher values of V_s. When $V_s = 0.667$ and $V_s = 1.00$ V detailed structures emerge in the vicinity of the dip (see the inset for a blow-up of the region). These structures are responsible for the structure observed in the SNR near the maximum for highest values of V_s.

explanation for this behavior observed near the maximum. One possibility is that it could be due to the specific shape of the generalized potential of our system, which is different from the model quartic bistable potential used in theoretical calculation. Another way to assess the role of the presence of noise in the above measurements at low values of V_s is obtained by inspecting Fig. 6, where we present the SNR measured under the same conditions of Fig. 5. In Fig. 6 the bottom curve refers to the case $V_s = 0.0067$ V, whereas the top curve is obtained by setting $V_s = 1.00$ V. From the figure it is evident that for V_s equal to 0.0067, 0.017, and 0.033 V, the SNR becomes zero within the experimental errors for a wide range of values of V_{rms}. This effect is the counterpart in the SNR plot of the deviation of η from the asymptotic theoretical curve observed in Fig. 5 for large values of V_{rms}.

A simultaneous inspection of Figs. 5 and 6 shows that the results obtained with the highest values of V_s are associated with high values of the SNR but are at the same time seriously affected by nonlinear distortion. This nonlinear distortion manifests itself in the broadening of the SNR curve. In other words, by using high values of V_s it is possible to detect a wide interval of V_{rms} where the SR phenomenon occurs, however this interval is not well described in terms of linear-response theory. On the other hand by using low values of V_s one observe experimentally SR on a more limited interval of V_{rms} but the experimental results are in this interval well described by a linear-response theory. Hence from an experimental point of view the more straightforward investigation of the SR phenomenon requires the selection of a value of the amplitude of the modulating signal that allows the detection of a large but undistorted signal. In our case this condition is attained when $V_s \approx 0.067$ V.

The experimental investigations previously discussed allow us to detect when we are under experimental conditions that are well described by the predictions of the linear-response theory or conversely when we are in the nonlinear regime. In the nonlinear regime the investigation of the noise power level presents some interesting features. In Fig. 7 we show the output noise power level N_1 measured at the frequency of the modulating signal as a function of V_{rms} for several values of V_s ranging from 0.0067 to 1.00 V. In these investigations f_s is kept constant at the value of 10 Hz. The noise level N_1 sharply increases at the onset of the SR phenomenon, reaches a maximum, and then decreases. Depending on the value of V_s, the noise level may decrease monotonically to the asymptotic value observed for high values of V_{rms} or reach a minimum value and then increases until reaching the same asymptotic value. In other words, we detect a dip in the noise level for a finite value of V_{rms} for high values of V_s of the nonlinear regime. The dip is shown in the inset of Fig. 7 for the measurements done by setting $f_s = 10$ Hz. The noise dip is more pronounced for high values of the signal amplitude. A similar behavior is also observed for values of f_s satisfying the condition $f_s < 1$ kHz. By taking into account the results previously obtained concerning

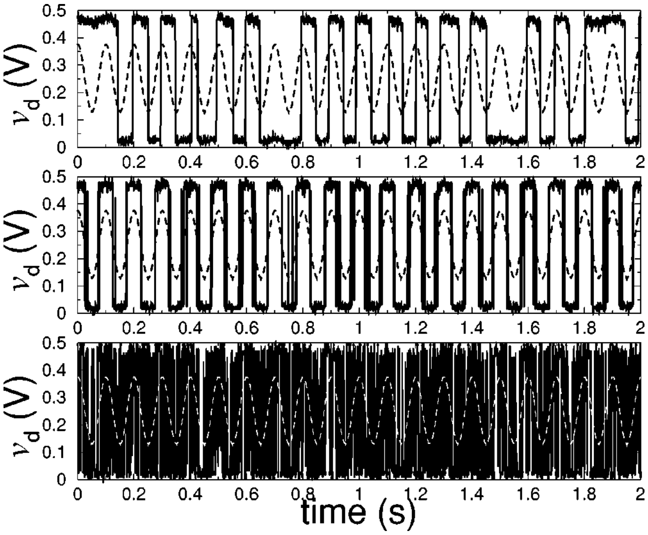

FIG. 8. Time evolution of $v_d(t)$ measured in the nonlinear regime for three different values of the noise amplitude V_{rms}. All the time evolutions (top $V_{rms}=1.33$ V, middle $V_{rms}=2.00$ V, and bottom $V_{rms}=4.67$ V graph) are synchronously recorded with respect to the modulating signal (a sinusoidal time evolution with the same phase and proportional to the modulating signal is shown as a dashed line for reference in each panel too). By setting $t=0$ when the maximum of the modulating signal occurs, phase synchronization is observed for $V_{rms}=1.33$ V (in the top panel jumps occur randomly at times $t=nT/2$) and phase and frequency synchronization is observed for $V_{rms}=2.00$ V (in the middle panel jumps occur almost deterministically at times $t=T/4+nT/2$).

the deviation from the behavior predicted in terms of linear-response theory, we conclude that this behavior belongs to the nonlinear response regime of stochastic resonance. In this regime, sometimes called weak-noise limit, the product of the amplitude of the periodic signal times the maximum value of the output signal is not much less than the noise intensity and the linear-response theory or the perturbation theory is no longer valid [2,18,19].

In the nonlinear-response regime we experimentally detect the phenomenon of phase and frequency locking. Specifically by increasing the value of the amplitude of the modulating signal one observes jumps between the two stable states occurring at phases that are progressively more synchronized with the phases of the modulating signal. Moreover a locking between the period of the output signal and the period of the input signal is also observed for given values of the noise amplitude. We address this last phenomenon as frequency locking. An example of phases and frequency locking is shown in Fig. 8, where we show the digitized time series of $v_d(t)$ recorded by setting $V_{rms}=1.33$, 2.00, and 4.67 V for the top, middle, and bottom time series of Fig. 8, respectively. The parameters of the modulating signal are set in this investigation as $f_s=10$ Hz and $V_s=0.667$ V. A signal proportional to the v_s time series, shifted to a higher average voltage level for the sake of clearness, is also shown in each panel as a dashed line to evaluate the phase differences when jumps occurs. By inspecting Fig. 8 one notes that for low levels of noise amplitude ($V_{rms}=1.33$ V, top time series) the system jumps randomly from one state to the other but the jumps are statistically synchronized in phases. In fact they occur preferentially at times $t=nT/2$, where T is the period of the modulating signal, n is an integer, and at $t=0$ the modulating signal has the maximum value. When the noise amplitude is increased ($V_{rms}=2.00$ V, middle time series) we still observe a phase synchronization, but in this case jumps occurs preferentially at times $t=nT/2+T/4$. Moreover, for the present value of the noise amplitude, jumps occurs with probability almost one at each period. This means that in addition to the phase synchronization we also observe frequency synchronization. A similar effect has been theoretically considered in the literature [20] recently. It is worth pointing out that the noise amplitude at which we observe phase and frequency synchronization is always detected in the region of the dip observed in the noise level of Fig. 7. In other words the dip of the noise may be interpreted as a manifestation of the fact that phases and frequency locking are simultaneously present. By increasing the noise amplitude ($V_{rms}=4.67$ V, bottom time series) jumps becomes very frequent inside a single period of the modulation signal so that phase and frequency synchronization is progressively lost.

IV. CONCLUSIONS

We report an experimental study of stochastic resonance in a physical system. Our physical system, which is characterized by versatility and high stability, allows us to investigate with high precision the SR phenomenon in a wide range of parameters, such as the frequency and the amplitude of the modulating signal and the noise amplitude. In the experiments presented here, the frequency range is spanning up to seven orders of magnitude whereas the amplitude range spans more than two orders of magnitude.

Theoretical and experimental investigations have been mainly focused on the linear regime of SR. However for a complete description of the SR phenomenon it is also important to investigate the nonlinear-response regime of SR. We experimentally investigate the degree of consistence of our experimental results with the results expected in terms of the linear-response theory. We find a range of experimental parameters within which the linear-response theory describes quite well the investigated dynamics. However, outside these intervals, nonlinear deviations from the prediction of the linear-response theory are clearly detected. These deviations primarily manifest themselves (i) in a saturation of the output power spectral density signal and of the signal amplification and (ii) in a nonmonotonic behavior of the output noise level associated with a high degree of phase and frequency synchronization.

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