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Manifestation of anisotropy persistence in the hierarchies of magnetohydrodynamical scaling exponents

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An example of a turbulent system where the failure of the hypothesis of small-scale isotropy restoration is detectable both in the ''flattening'' of the inertial-range scaling exponent hierarchy and in the behavior of odd-order dimensionless ratios, e.g., skewness and hyperskewness, is presented. Specifically, within the kinematic approximation in magnetohydrodynamical turbulence, we show that for compressible flows, the isotropic contribution to the scaling of magnetic correlation functions and the first anisotropic ones may become practically indistinguishable. Moreover, the skewness factor now diverges as the Péclet number goes to infinity, a further indication of small-scale anisotropy.

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A wide interest has recently been devoted to the possible occurrence of small-scale isotropy restoration for scalar (see e.g., Ref. [1] and references therein), Navier-Stokes [2,3], and magnetohydrodynamical (MHD) turbulence [4–6]. The scenario can be summarized as follows. In the presence of anisotropic large-scale injection mechanisms, the inertial-range statistics is characterized by an infinite hierarchy of scaling exponents; however, the leading contribution to scaling comes from the isotropic component. From this point of view, one might argue that large-scale anisotropy does not affect inertial-range scaling properties. Actually, focusing on a larger set of observables, small-scale anisotropies become manifest. It turns out that the behavior of odd-order dimensionless ratios (e.g., skewness and hyperskewness) is completely different from the case of small-scale isotropy restoration. Such indicators go to zero down to the inertial range much slower than predicted by dimensional considerations [7] or, more dramatically, they diverge at the smallest scales [6] (see also [8,9]).

The main aim of this Rapid Communication is to present a model of MHD turbulence where, by varying the degree of compressibility of the velocity field, anisotropic persistence is now detectable both from the ''flattening'' of the hierarchy of inertial-range scaling exponents (the isotropic component and the first anisotropic ones may become practically indistinguishable) and the divergence of the skewness factor with the Péclet number. We give the basic ideas and results; longer and more exhaustive technical discussions will be presented elsewhere.

In the presence of a mean component $\mathbf{B}^o$ (actually varying on a very large scale L) and for the compressible velocity field $\mathbf{v}$, the kinematic MHD equations describing the evolution of the fluctuating (divergence-free) part $\mathbf{B}$ of the magnetic field are [10]:

$$\partial_t B_\alpha + v_i\partial_i B_\alpha = -(B_\alpha + B^o_\alpha)\partial_i v_i + B_i\partial_i v_\alpha + B^o_i\partial_i v_\alpha + \kappa_0\partial^2 B_\alpha, \quad \alpha=1,\dots,d, \tag{1}$$

where κ_0 is the magnetic diffusivity. The field $\mathbf{B}^o$ plays the same role as an external forcing driving the system and is also a source of anisotropy for the magnetic field statistics.

Our choice for the velocity statistics generalizes that of the well-known kinematic Kazantsev-Kraichnan model for the compressible case: $\mathbf{v}$ is a Gaussian process of zero average, homogeneous, isotropic, and white in time. It is self-similar and defined by the two-point correlation function

$$\langle v_\alpha(t,\mathbf{x})v_\beta(t',\mathbf{x}')\rangle = \delta(t-t')[d^0_{\alpha\beta} - S_{\alpha\beta}(\mathbf{x}-\mathbf{x}')], \tag{2}$$

where $d^0_{\alpha\beta} = \text{const}\,\delta_{\alpha\beta}$ and $S_{\alpha\beta}(\mathbf{x}-\mathbf{x}')$ is fixed by isotropy and scaling:

$$S_{\alpha\beta}(\mathbf{r}) = r^\xi\left[\mathcal{X}\delta_{\alpha\beta} + \mathcal{Y}\frac{r_\alpha r_\beta}{r^2}\right], \quad r\equiv|\mathbf{x}-\mathbf{x}'|, \tag{3}$$

with the coefficients

$$\mathcal{X} = \frac{\mathcal{S}^2(d+\xi-1)-\xi\mathcal{C}^2}{(d+\xi)(d-1)\xi}, \quad \mathcal{Y} = \frac{d\mathcal{C}^2-\mathcal{S}^2}{(d+\xi)(d-1)}. \tag{4}$$

The degree of compressibility is thus controlled by the ratio $\wp\equiv\mathcal{C}^2/\mathcal{S}^2$, with $\mathcal{S}^2\propto\langle(\partial_i v_k\partial_i v_k)\rangle$ and $\mathcal{C}^2\propto\langle\partial_i v_i)^2\rangle$. It satisfies the inequality $0\leqslant\wp\leqslant 1$; $\wp=0$ and 1 corresponding to the purely solenoidal and potential velocity fields, respectively.

In the present paper our attention will be focused on the inertial-range behavior of magnetic correlation functions, where power laws are expected in their decompositions on a set of orthonormal functions P,

$$\langle B^n_\parallel(t,\mathbf{x})B^q_\parallel(t,\mathbf{x}')\rangle = \sum_{j=0}^{\infty} P_j(\cos\phi)r^{\zeta^{n,q}_j}, \tag{5}$$

$B_\parallel$ being some component of $\mathbf{B}$, e.g., its projection along the direction $\hat{\mathbf{r}}\equiv\mathbf{r}/r$ or $\hat{\mathbf{B}}^o\equiv\mathbf{B}^o/B^o$, and ϕ is the angle between $\mathbf{r}$ and $\mathbf{B}^o$. Due to the anisotropic injection mechanism, the inertial-range statistics is now characterized by an infinite hierarchy of exponents (j denotes the jth anisotropic sector), rather than just one exponent, as in the isotropic case.

If the Kolmogorov 1941 isotropization hypothesis holds, the contribution from the anisotropic sectors (i.e., $j \neq 0$) to the scaling of correlation functions should be negligible with respect to the isotropic component. Such a picture indeed holds for even correlation functions and solenoidal velocity [4,6], but it breaks down for compressibility that is strong enough.

In order to prove this fact, consider first the pair correlation function, $C_{\alpha\beta}(t,\mathbf{r}) \equiv \langle B_\alpha(t,\mathbf{x}) B_\beta(t,\mathbf{x}') \rangle$. Here, exploiting the zero-mode technique [11–14], the complete set of scaling exponents $\zeta_j^{1,1}$ can be found nonperturbatively. We give the basic ideas of the strategy.

A closed equation for $C_{\alpha\beta}$ can be found due to the time decorrelation of the velocity field:

$$\begin{aligned}\partial_t C_{\alpha\beta} = {} & S_{ij}\partial_i\partial_j C_{\alpha\beta} - (\partial_j S_{i\beta})\partial_i C_{\alpha j} - (\partial_j S_{\alpha i})(\partial_i C_{j\beta}) \\ & + (\partial_i\partial_j S_{\alpha\beta})(C_{ij} + B_i^o B_j^o) + 2\kappa_0 \partial^2 C_{\alpha\beta} \\ & - B_\beta^o B_j^o \partial_i\partial_j S_{\alpha i} - B_\alpha^o B_j^o \partial_i\partial_j S_{\beta i} + B_\alpha^o B_\beta^o \partial_i\partial_j S_{ij} \\ & - C_{\alpha j}(\partial_i\partial_j S_{\beta i}) + C_{\beta j}\partial_i\partial_j S_{\alpha\beta} + C_{\alpha\beta}\partial_i\partial_j S_{ij} \\ & + 2(\partial_i C_{\alpha\beta})\partial_j S_{ij} \end{aligned} \tag{6}$$

(see [6] for the derivation in the incompressible case).

Projecting $C_{\alpha\beta}$ on the basis that span the irreducible representations of SO(d) [15], a system of linear algebraic equations for the scaling law coefficients is obtained. The leading solutions are associated with the homogeneous solutions of such system, i.e., with zero modes. Their scaling exponents follow from the imposition that the determinant of the coefficients be zero. Schematically, the solutions can be expressed in the form

$$\zeta_j^{1,1} = \alpha + \sqrt{\beta + \gamma\sqrt{\delta}} \quad (j \text{ even}), \tag{7}$$

where α, β, γ, and δ are cumbersome functions of ξ, d, and j and will not be reported here for the sake of brevity. Contributions to the scaling associated with odd j's vanish due to the symmetry $\phi \mapsto -\phi$. In particular, the expression derived in Ref. [16] in the isotropic case has been recovered here for $j=0$. Following the same arguments reported in Ref. [4], it is possible to show from Eq. (7) that the treshold to the dynamo is the same as in the incompressible case. For $j=0$ and $j=2$, the limit $\xi \to 0$ in Eq. (7) yields

$$\zeta_0^{1,1} = (-1 + 2\wp - d\wp)\xi + O(\xi^2), \tag{8}$$

$$\zeta_2^{1,1} = \frac{2 - \wp[4 + d(d-2)(d+1)]}{(d-1)(d+2)}\xi + O(\xi^2), \tag{9}$$

while for large d we have

$$\zeta_0^{1,1} = -\frac{\wp\xi}{1+\wp\xi}d - \frac{1-2\wp(1-\xi)}{1+\wp\xi}\xi + O(1/d), \tag{10}$$

$$\zeta_2^{1,1} = -\frac{\wp\xi}{1+\wp\xi}d - \frac{\wp\xi(\xi-2)}{1+\wp\xi} + O(1/d). \tag{11}$$

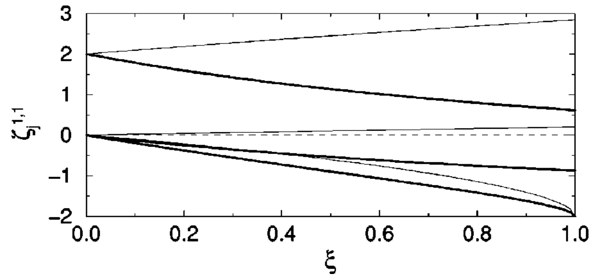

FIG. 1. Behavior of $\zeta_j^{1,1}$ vs ξ for $d=3$ and $j=0$, 2, and 4 (from below to above). Thick lines, $\wp=1$; thin lines, $\wp=0$.

In Fig. 1 we present the behavior of $\zeta_j^{1,1}$ obtained from expression (7) for $\wp=0$ (thin lines) and $\wp=1$ (thick lines). As one can see, for $\wp=0$ we have $\zeta_0^{1,1}<0$ but $\zeta_2^{1,1}>0$ and $\zeta_4^{1,1}>0$ so that, in particular, $(r/L)^{\zeta_0^{1,1}} \gg (r/L)^{\zeta_2^{1,1}}$ in the inertial range of scales.

The situation changes for $\wp > \wp_c$ ($\wp_c \sim 0.1274$ for $j=2$ and $d=3$), when $\zeta_2^{1,1}$ becomes negative for all $0 \leqslant \xi \leqslant 1$. This means that, for compressibility strong enough, $(r/L)^{\zeta_0^{1,1}}$ and $(r/L)^{\zeta_2^{1,1}}$ become very close (still, $\zeta_j^{1,1} < \zeta_k^{1,1}$ for $j<k$). As we can see from Eqs. (10) and (11) the effect becomes dramatic for large dimensions and $\wp \neq 0$ when $\zeta_0^{1,1} \sim \zeta_2^{1,1}$. The contribution to the scaling in Eq. (5) coming from the sector $j=2$ thus becomes less and less subleading as $\wp$ and/or d increase.

Further strong evidence of the crucial role of compressibility in the failure of the small-scale isotropy restoration can be obtained by looking at the higher-order correlation functions [i.e., n and/or $q>1$ in Eq. (5)] and, in particular, at dimensionless ratios of odd-order moments. Were isotropy restored at small scales, such ratios would go to zero for large Péclet number. The latter is defined as Pe $\equiv (L/\eta)^{1/\xi}$, where L is the integral scale and $\eta \propto \kappa_0^{1/\xi}$ is the dissipation scale.

Nonzero values of such indicators are thus the signature of anisotropy persistence. As we are going to show, in contrast to the incompressible case, for $\wp$ large enough the skewness factor now diverges as Pe$\to\infty$. To be more specific, the leading contribution in expression (5) can be written as

$$\langle B_\parallel^n(t,\mathbf{x}) B_\parallel^q(t,\mathbf{x}') \rangle \propto (r/\eta)^{\alpha_0^{n,q}} (r/L)^{\beta_0^{n,q}} \propto r^{\zeta_0^{n,q}}, \tag{12}$$

where we have defined $\zeta_0^{n,q} \equiv \alpha_0^{n,q} + \beta_0^{n,q}$. The expressions for $\zeta_0^{n,q}$ have been obtained up to first order in ξ by means of the field theoretic renormalization group and operator product expansion. A detailed presentation of these techniques for the case $\wp=0$ can be found in [6] (see also Refs. [8,9,17] for the scalar case and [18] for general review); below we confine ourselves to only the necessary information.

The stochastic problem (1) and (2) can be reformulated as a multiplicatively renormalizable field theoretic model; the corresponding RG equations have infrared stable fixed point. This implies existence of scaling behavior for all the correlation functions in the infrared region with certain scaling dimensions, calculated in the form of series in ξ. In this sense, ξ is similar to $\varepsilon = 4-d$ in the models of critical phenomena.

The key role is played by the scaling dimensions $\Delta[n,j]$ of the jth rank irreducible tensor composite operators

$$B_{\alpha_1}(x)\cdots B_{\alpha_j}(x)[B_\alpha(x)B_\alpha(x)]^l+\cdots. \tag{13}$$

Here $n\equiv 2l+j$ is the total number of B's and the dots stand for the subtractions (needed for $j\geqslant 2$) which make the operator traceless with respect to any pair of indices, for example, $B_\alpha B_\beta-\delta_{\alpha\beta}\mathbf{B}^2/d$ and so on. In the first order in ξ (one-loop approximation) we have obtained

$$\Delta[n,j]=\frac{\xi}{2(d-1)(d+2)}\{n(n-1)\times[2-\wp(d^3-d^2-2d+4)]-(n-p)\times(n+p+d-2)(d+1-2\wp)\}+O(\xi^2). \tag{14}$$

Note that for any fixed n and any $d>1$, dimension (14) decreases with j and reaches its minimum for the minimal possible value j_n, i.e., $j_n=0$ if n is even and $j_n=1$ if n is odd. Furthermore, this minimal value is negative and it decreases monotonically as n grows, $0>\Delta_{2k,0}>\Delta_{2k+1,1}>\Delta_{2k+2,0}$.

Thus the behavior of correlation functions (5) in the infrared range ($r/\eta\gg 1$ and any fixed r/L) has the form

$$\langle B_\parallel^n(t,\mathbf{x})B_\parallel^q(t,\mathbf{x}')\rangle\propto\left(\frac{r}{\eta}\right)^{-\Delta[n,j_n]-\Delta[q,j_q]}\chi\left(\frac{r}{L}\right) \tag{15}$$

(the leading term is given by the *minimal* dimensions), while the form of the scaling function $\chi(r/L)$ at $r/L\to 0$ (inertial range) is obtained with the aid of the operator product expansion (OPE):

$$\chi\left(\frac{r}{L}\right)=\sum_F C_F\left(\frac{r}{L}\right)^{\Delta_F}. \tag{16}$$

Here the sum runs over all possible composite operators F entering the OPE, Δ_F being their scaling dimensions, and C_F numerical coefficients analytical in $(r/L)^2$. The leading term of the small r/L behavior in the jth shell is given by the jth rank operator with minimal dimension; owing to the linearity in $\mathbf{B}$ of Eq. (1), the number of fields $\mathbf{B}$ in the operators F does not exceed their number on the left-hand side of Eq. (15). Thus the exponents in Eq. (5) are related to the dimensions (14) as follows:

$$\zeta_j^{n,q}=\Delta[n+q,j]-\Delta[n,j_n]-\Delta[q,j_q].$$

We thus conclude that the inequality $\partial\Delta[n,j]/\partial j\geqslant 0$, which follows from Eq. (14) for all $\wp$ and $d\geqslant 2$, generalizes the hierarchy discussed above to the higher-order functions. It becomes flatter and flatter as $\wp$ grows ($\partial^2\Delta[n,j]/\partial j\partial\wp\leqslant 0$), while for $d\to\infty$ (and $\wp\neq 0$) the effect becomes even stronger: in the leading $O(d)$, expressions (14) and (17) are now independent of j.

The leading term of Eq. (5) in the inertial range is given by the contribution with the minimal j,

$$\zeta_0^{n,q}=\begin{cases}-\xi\dfrac{nq(1+\wp d^2-2\wp)+(d+1-2\wp)}{(d+2)}\\[2ex] -\dfrac{\xi nq}{(d+2)}(1+\wp d^2-2\wp),\end{cases} \tag{17}$$

where the first holds if both n and q are odd, and the second otherwise.

For $n=q=1$, expression (8) is recovered. Knowing the exponents $\alpha_0^{n,q}$ and $\beta_0^{n,q}$, dimensionless ratios of the form $R_{2n+1}(r)\equiv\langle B_{||}^{2n}(x)B_{||}(x')\rangle/\langle B_{||}(x)B_{||}(x')\rangle^{(2n+1)/2}$ can be constructed and, as in Ref. [7], evaluated at the dissipative scale [i.e., $r=\eta$ in Eq. (12)]. When doing this, the explicit dependence on Pe appears and the final $O(\xi)$ expressions read

$$R_{2n+1}(\mathrm{Pe})\propto\mathrm{Pe}^{\sigma_{2n+1}},\quad n=1,2,\ldots, \tag{18}$$

$$\sigma_{2n+1}(\wp)\equiv\frac{2n^2(1-2\wp+\wp d^2)}{(d+2)}-\frac{(1-2\wp+\wp d)}{2}. \tag{19}$$

It is easy to verify from expression (19) that for $d\geqslant 2$ and $n\geqslant 1$ we have $\partial\sigma_{2n+1}(\wp)/\partial\wp>0$. Negative values of $\sigma_{2n+1}(0)$ may thus become positive due to $\wp$. In particular, for $d=3$ we obtain $\sigma_3(\wp)=(23\wp-1)/10$, which becomes positive for $\wp>1/23$. It then follows that $R_3\to\infty$ as Pe $\to\infty$, the footprint of the persistence of the small-scale anisotropy.

Some remarks on the limit of large space dimensions are worth noting. One immediately realizes from Eq. (17) that, for $d\to\infty$ the scaling exponents reduce to $\zeta_0^{n,q}=-\xi nqd\wp$; that means the vanishing of intermittency. Note that the result $\zeta_0^{1,1}=\zeta_2^{1,1}$ for $d\to\infty$ is nonperturbative. At the dissipative scale η, $\sigma_{2n+1}(\wp)=\wp d(2n^2-1/2)>0$, two clear signatures of small-scale anisotropy persistence. The latter may thus occur also in the absence of intermittency.

Let us now examine the possible mechanisms at the origin of the small-scale anisotropy persistence in our problem. They can be easily grasped in two dimensions; our previous results being valid for all $d\geqslant 2$, it is reasonable to expect that the mechanisms we are going to show hold also for higher d's. Let us start from the incompressible case assuming, without loss of generality, that $\hat{B}^o$ is oriented along the y axis (i.e., $\hat{\mathbf{B}}^o\equiv\mathbf{e}_y$). As we shall see, all our considerations will hold, *a fortiori*, in the compressible case.

It can be shown that the magnetic field can be represented by the (scalar) magnetic flux function in the form $\mathbf{B}=\boldsymbol{\partial}\psi\times\mathbf{e}_z$, where ψ satisfies the passive scalar equation forced by a large scale gradient $\mathbf{G}\equiv\mathbf{B}^o\times e_z$. One finds $\partial_t\Psi+v_i\partial_i\Psi)=\kappa_0\partial^2\Psi$ where $\Psi(t,\mathbf{x})\equiv\psi+G_ix_i$. An interesting feature recently recognized for the passive scalar turbulence (see, e.g., [19] and [20–23]) is related to the formation of "cliffs," i.e., very steep scalar gradients within very short distances separated by "plateaus" where the scalar depends smoothly on the position.

The emergence of this ubiquitous pattern is explained by considering the action of the velocity derivative matrix

$\partial_\alpha v_\beta$. As emphasized in Refs. [20,21], scalar gradients are weak in the elliptic regions of the velocity field where the ''rotational'' character inhibits the formation of strong scalar gradients. On the contrary, in the hyperbolic regions the flow is almost ''irrotational'': the alignment of scalar gradients with the direction of the eigenvector corresponding to the most negative eigenvalue of the velocity derivative matrix (roughly, the direction of compression along one direction) is not discouraged, and actually observed [20], and strong scalar gradients develop.

When the scalar is forced by isotropic injection mechanisms, no preferential direction arises and the intense gradients are randomly oriented. The final result is a small-scale isotropic statistics. The situation changes in the case of nonisotropic injection mechanisms like the one encountered here. In this case strong gradients are oriented along **G** and, as very recently shown in Refs. [22,23], small-scale isotropy is consequently not restored for the scalar field. Exploiting the relation between the magnetic field and the magnetic flux function, we can conclude that extreme magnetic fluctuations have a tendency to occur preferentially along the direction $\hat{\mathbf{B}}^o$, the origin of the observed small-scale anisotropy.

In the compressible case both eigenvalues can be negative. Compression may thus occur in both directions, enhancing the formation of fronts in the magnetic flux function.

In conclusion, we presented a simple model of MHD turbulence where, by varying the degree of compressibility of the velocity field, the persistence of anisotropy is detectable both from the hierarchy of inertial range exponents and from the divergence of skewness factor with the Péclet number. Although our results were obtained on the base of a specific model, they seem to be rather general: a similar behavior is also observed for a passive scalar advected by a compressible velocity field [9] (however, the flattening is less pronounced for the latter). The results presented here give evidence of a turbulent system displaying such twofold behavior.

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