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# Rules for the distribution of point charges on a conducting disk

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The minimum energy configurations of $N$ equal point charges interacting via the Coulomb potential on an infinitely thin conducting disk are determined and the rules for the distribution of charges on the disk are deduced.

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As Berezin [1] pointed out, the minimum energy configuration for a system of $N$ identical point charges interacting via the $1/r$ Coulomb potential and confined to a circular region in a plane is with all of the charges on the circumference only for $N \leqslant 11$. For $N > 11$, some of the charges are in the interior. Regarding the same problem, Queen [2] pointed out that with increasing $N$, the charges appear to arrange themselves in more and more complex concentric ring patterns with equal spacing between charges on each ring. Rees [3] also pointed out that the charges would group themselves into an $N$-dependent lattice pattern for which it would be interesting to calculate the stable configurations.

We have performed numerical calculations of the minimum energy configurations for this problem and deduced rules for the distribution of $N$ equal point charges interacting Coulombically on an infinitely thin conducting disk or inside a circle. $N$ equal point charges distribute themselves on concentric rings by following the simple relation: $[N^c_{min}, N^c_{max}]_m$, where

$$N^c_{min} = 12 + \sum_{n=1}^{m} (10 + 8n)$$

and (1)

$$N^c_{max} = 16 + \sum_{n=1}^{m} (11 + 10n).$$

Here $N^c_{min}$ and $N^c_{max}$ are the minimum and maximum number of charges respectively for forming concentric rings with the possibility of one charge at the center and $m$ is the number of interior circles. For example, for $m=0$, $N^c_{min}=12$ $N^c_{max}=16$ correspond to the first configuration with one charge at the center, and no interior circle. For the case where $m=1$, $N^c_{min}=30$ and $N^c_{max}=37$ form the second configuration with one charge at the center and one interior circle. Furthermore for $m=2$, $N^c_{min}=56$ and $N^c_{max}=68$ correspond to the third configuration with one charge at the center and two interior

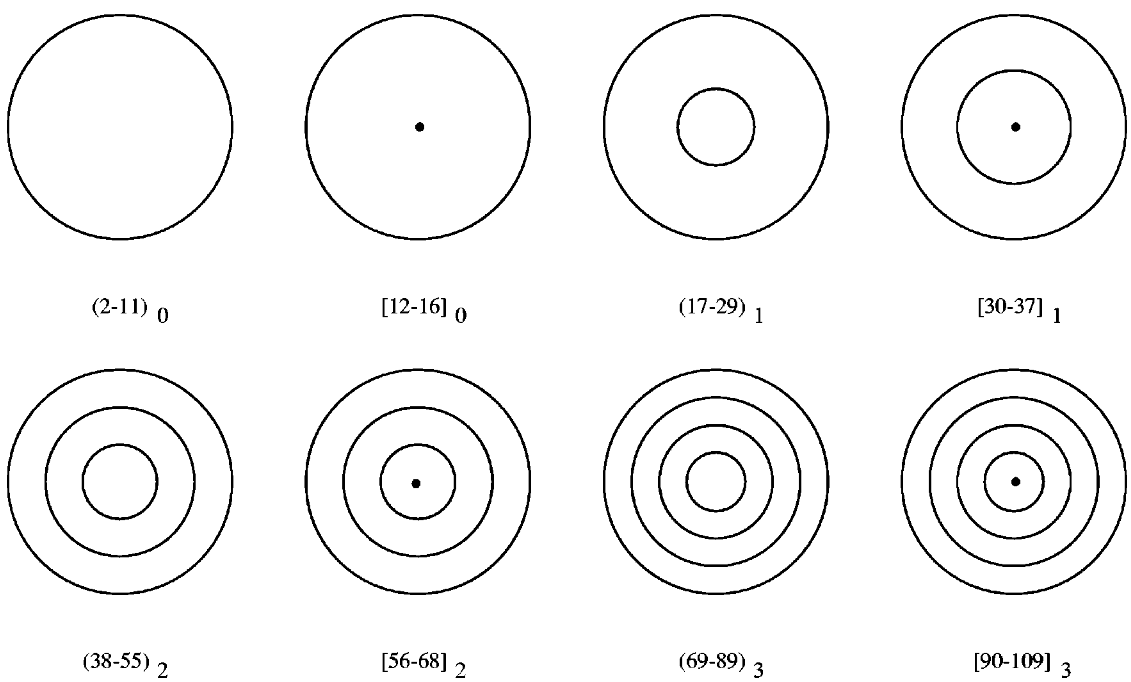


FIG. 1. General patterns of the distribution of $N$ point charges on a disk.

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circles, and so on. On the other hand, the number of charges between these groups of charges form configurations with concentric rings but without charge at the center. Therefore, one may order and classify the number of charges according to the following groupings:

$$(2-11)_0,\ [12-16]_0,\ (17-29)_1,\ [30,37]_1,\ (38-55)_2,\ [56,68]_2,\ (69-89)_3,\ [90-109]_3,\ \dots, \tag{2}$$

where the curved parentheses represent configurations without a charge at the center and the square parentheses represent the configurations with a charge at the center. The subscripts on the parentheses represent the number of interior circles. The charge distributions resulting from these prescriptions, shown in Fig. 1, summarize the rules that we deduced for the distribution of $N$ equal point charges on a conducting disk. The radii of the concentric rings are not equally spaced along the radial direction; the interspace between the successive rings becomes smaller and smaller along the radial direction from center to circumference. The charge distribution is not uniform, charge density increases toward the circumference. This feature is consistent with the continuous charge distribution on a conducting disk.

[1] A.A. Berezin, Nature (London) **315**, 104 (1985).
[2] N.M. Queen, Nature (London) **317**, 208 (1985).
[3] M. Rees, Nature (London) **317**, 208 (1985).