Multisoliton solutions and integrability aspects of coupled higher-order nonlinear Schrödinger equations

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Using Painlevé singularity structure analysis, we show that coupled higher-order nonlinear Schrödinger (CHNLS) equations admit the Painlevé property. Using the results of the Painlevé analysis, we succeed in Hirota bilinearizing the CHNLS equations for the integrable cases. Solving the Hirota bilinear equations, one soliton and two soliton solutions are explicitly obtained. Lax pairs are explicitly constructed.

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I. INTRODUCTION

The last three decades witnessed extensive theoretical and experimental studies on optical solitons because of their potential applications in long-distance communication. The invention of high-intensity lasers helped Mollenauer *et al.* [1] to verify experimentally the pioneering theoretical work on optical solitons initiated by Hasegawa and Tappert [2]. The solitons, localized-in-time optical pulses, evolve from a nonlinear change in the refractive index of the material, known as the Kerr effect, induced by the light intensity distribution. When the combined effect of the intensity-dependent refractive index nonlinearity and the frequency-dependent pulse dispersion exactly compensate each other, the pulse propagates without any change in its shape, being self-trapped by the wave guide nonlinearity. The propagation of optical solitons in a nonlinear dispersive optical fiber is governed by the well-known nonlinear Schrödinger (NLS) equation of the form

$$iu_t+\alpha u_{zz}+\beta|u|^2u=0, \tag{1}$$

where u is the complex amplitude of the pulse envelope, α and β are the group velocity dispersion (GVD) and self-phase modulation parameters, respectively, and subscripts z and t represent the spatial and temporal coordinates, respectively.

When ultrashort pulses (USPs) are transmitted through fibers, higher-order effects such as third-order dispersion, (TOD), Kerr dispersion, and stimulated Raman scattering (SRS) come into play as experimentally reported by Mitschke and Mollenauer [3]. The Kerr dispersion, also known as self-steepening, is caused by the intensity dependence of the group velocity which results in asymmetrical spectral broadening of the pulse since the peak of the pulse travels slower than the wings. The SRS causes a self-frequency shift which is a self-induced redshift in the pulse spectrum as the low-frequency components of the pulse obtain Raman gain at the expense of the high-frequency components. With the inclusion of all these effects, Kodama and Hasegawa [4] have proposed that the dynamics of femtosecond pulse propagation be governed by a higher-order NLS (HNLS) equation. The HNLS equation allows soliton-type propagation only for certain choices of parameters [5,6].

A coupled NLS equation was proposed by Manakov, by taking into account the fact that the total field comprises two fields with left and right polarizations [7]. The coupled equation takes the form

$$\begin{aligned}&iu_t+c_1u_{zz}+(\alpha|u|^2+\beta|v|^2)u=0,\\ &iv_t+c_2v_{zz}+(\beta|u|^2+\gamma|v|^2)v=0.\end{aligned} \tag{2}$$

The above equations are integrable only for the following parametric choices: (i) $c_1=c_2$, $\alpha=\beta=\gamma$ and (ii) $c_1=-c_2$, $\alpha=-\beta=\gamma$. Recently, for USPs, Eq. (2) was generalized to a set of coupled higher-order NLS (CHNLS) equation which can be derived from the Maxwell's equations in order to investigate the effects of birefringence on pulse propagation in the femtosecond regime [8,9]. The general form of CHNLS equations are

$$\begin{aligned}&iu_t+u_{zz}+2(|u|^2+|v|^2)u-i\lambda[\beta_1u_{zzz}\\ &\quad+\beta_2(|u|^2+|v|^2)u_z+\beta_3(|u|^2+|v|^2)_zu]=0,\\ &iv_t+v_{zz}+2(|u|^2+|v|^2)v-i\lambda[\beta_1v_{zzz}\\ &\quad+\beta_2(|u|^2+|v|^2)v_z+\beta_3(|u|^2+|v|^2)_zv]=0.\end{aligned} \tag{3}$$

In general, the above equations are not completely integrable. However, if some restrictions are imposed on the parametric values, one can obtain several integrable, soliton-possessing NLS-type equations (i) $\lambda=0$, NLS; (ii) $\beta_1:\ \beta_2:\ \beta_3=0:1:1$, derivative NLS [10]; (iii) $\beta_1:\ \beta_2:\ \beta_3=0:1:0$, derivative mixed NLS [10]; (iv) $\beta_1:\ \beta_2:\ \beta_3=1:6:0$, the Hirota equation [11]; and (v) $\beta_1:\ \beta_2:\ \beta_3=1:6:3$, Sasa-Satsuma equation [12]. Along the lines of Refs. [12,13] we choose $\beta_1=1$, $\beta_2=6$, $\beta_3=3$. For this choice of parameters, Eqs. (3) become

$$\begin{aligned}&iu_t+u_{zz}+2(|u|^2+|v|^2)u-i\lambda[u_{zzz}+6(|u|^2+|v|^2)u_z\\ &\quad+3(|u|^2+|v|^2)_zu]=0,\\ &iv_t+v_{zz}+2(|u|^2+|v|^2)v-i\lambda[v_{zzz}+6(|u|^2+|v|^2)v_z\\ &\quad+3(|u|^2+|v|^2)_zv]=0.\end{aligned} \tag{4}$$

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The plan of this paper is as follows. In Sec. II, we establish the Painlevé property of the above system of equations. In Sec. III, we rewrite Eqs. (7) in a Hirota bilinear form. Section IV is devoted to the construction of exact one soliton and two soliton solutions. In Sec. V, Lax pair for CHNLS equations are obtained. A conclusion and discussion of the present calculation are presented in Sec. VI.

II. PAINLEVÉ ANALYSIS OF CHNLS EQUATIONS

In this section we study the Painlevé analysis of Eqs. (4). The motivation behind this exercise is that the Painlevé condition is necessary one for studying the integrability of nonlinear partial differential equations [14–19] and helps construct solutions. The method for applying the Painlevé test to partial differential equations were introduced by Weiss, Tabor, and Carnavale [15] with simplification due to Kruskal [16] involves seeking a solution of a given partial differential equation in the form

$$u(z,t)=\phi^{\alpha}\sum_{j=0}^{\infty} u_j(t)\phi^j(z,t), u_0\neq 0,$$
$$v(z,t)=\phi^{\beta}\sum_{j=0}^{\infty} v_j(t)\phi^j(z,t), v_0\neq 0 \tag{5}$$

with

$$\phi(z,t)=z+\psi(t)=0, \tag{6}$$

where $\psi(t)$ is an arbitrary analytic function of t, $u_j(t)$, and $v_j(t)$ $(j=0,1,2,\ldots,)$, in the neighborhood of a noncharacteristic movable singularity manifold defined by $\phi=0$.

Apart from providing the integrability property of a given nonlinear partial differential equations, the Painlevé analysis also provides information about Bäcklund transformation (BT), Lax pair, Hirota's bilinear representation, special and rational solutions, etc. [15–17]. Many of these results are obtained by truncating the Laurent series at a constant level term [18,19].

In order to investigate the integrability properties of Eqs. (4), we rewrite it in terms of four complex functions *a*, *b*, *c*, and d by defining $u=a$, $u^*=b$, $v=c$, $v^*=d$. Consequently, we have the following equations:

$$\begin{aligned}
&ia_t+a_{zz}+2(ab+cd)a-i\lambda[a_{zzz}+6(ab+cd)\\
&\quad\times a_z+3(ab+cd)_z a]=0,\\
&-ib_t+b_{zz}+2(ab+cd)b+i\lambda[b_{zzz}+6(ab+cd)\\
&\quad\times b_z+3(ab+cd)_z b]=0,\\
&ic_t+c_{zz}+2(ab+cd)c-i\lambda[c_{zzz}+6(ab+cd)\\
&\quad\times c_z+3(ab+cd)_z c]=0,\\
&-id_t+d_{zz}+2(ab+cd)d+i\lambda[d_{zzz}+6(ab+cd)\\
&\quad\times d_z+3(ab+cd)_z d]=0.
\end{aligned} \tag{7}$$

The Painlevé analysis essentially consists of four main stages: (i) determination of leading-order behavior, (ii) identifying the resonance values, (iii) verifying that at resonance values sufficient number of arbitrary functions exist without the introduction of movable critical manifold, and (iv) identifies connection with the integrability properties such as Lax pair and BT.

Looking at the leading order behavior, we substitute $a\simeq a_0\phi^{\alpha_1}$, $b\simeq b_0\phi^{\alpha_2}$, $c\simeq c_0\phi^{\alpha_3}$, $d\simeq d_0\phi^{\alpha_4}$ in Eqs. (7) and balancing the different terms, we obtain the following results:

$$\alpha_1=\alpha_2=\alpha_3=\alpha_4=-1,$$
$$a_0b_0+c_0d_0=-\frac{1}{2}. \tag{8}$$

For finding the powers at which the arbitrary functions can enter into the series, we substitute the expressions

$$a=a_0\phi^{-1}+a_j\phi^{j-1}, b=b_0\phi^{-1}+b_j\phi^{j-1},$$
$$c=c_0\phi^{-1}+c_j\phi^{j-1}, d=d_0\phi^{-1}+d_j\phi^{j-1} \tag{9}$$

into Eqs. (7), and comparing the lowest-order terms we obtain a system of four linear algebraic equations in (a_j, b_j, c_j, d_j). In matrix form it may be conveniently written as

$$[A(j)][X]=0, \tag{10}$$

where $[X]=(a_jb_jc_jd_j)^T$ and

$$[A(j)]=\begin{bmatrix}
B & -6a_0^2+3a_0^2(j-2) & -6a_0d_0+3a_0d_0(j-2) & -6a_0c_0+3a_0c_0(j-2)\\
-6b_0^2+3b_0^2(j-2) & B & -6b_0d_0+3b_0d_0(j-2) & -6b_0c_0+3b_0c_0(j-2)\\
-6b_0c_0+3b_0c_0(j-2) & -6a_0c_0+3a_0c_0(j-2) & C & -6c_0^2+3c_0^2(j-2)\\
-6b_0d_0+3b_0d_0(j-2) & -6a_0d_0+3a_0d_0(j-2) & -6d_0^2+3d_0^2(j-2) & C
\end{bmatrix}$$

where

$$B=(j-1)(j-2)(j-3)-3(j-1)-6a_0b_0+3(j-2)+3$$

and

$$C=(j-1)(j-2)(j-3)-3(j-1)-6c_0d_0+3(j-2)+3.$$

To have a nontrivial solution for a_j, b_j, c_j, and d_j, we demand that

$$\det A(j)=0. \tag{11}$$

On solving Eq. (11), we get the resonance values as $j=-1,0,0,0,2,2,2,3,4,4,4,4$. The resonance at $j=-1$ corresponds to the arbitrariness of $\psi(z,t)$. On equating the coefficients of ψ^{-4}, we get a unique equation defining a_0, b_0, c_0, and d_0 which is given by

$$a_0b_0+c_0d_0=-\frac{1}{2}. \tag{12}$$

This shows that any three of the four functions a_0, b_0, c_0, and d_0 are arbitrary which corresponds to $j=0,0,0$.

Proceeding further and equating the coefficients of $(\psi^{-3},\psi^{-3},\psi^{-3},\psi^{-3})$, we obtain

$$\begin{aligned} a_1&=\frac{a_0}{3i\lambda},\\ b_1&=-\frac{b_0}{3i\lambda},\\ c_1&=\frac{c_0}{3i\lambda},\\ d_1&=-\frac{d_0}{3i\lambda}. \end{aligned} \tag{13}$$

On the other hand, the coefficients of $(\psi^{-2},\psi^{-2},\psi^{-2},\psi^{-2})$ in Eqs. (7) reduce to a single equation

$$b_0a_2+a_0b_2+d_0c_2+c_0d_2=\frac{\psi_t}{6\lambda} \tag{14}$$

so that three of the four functions a_2, b_2, c_2, and d_2 are arbitrary which corresponds to $j=2,2,2$. Similarly from the powers of $(\psi^{-1},\psi^{-1},\psi^{-1},\psi^{-1})$ and $(\psi^{-0},\psi^{-0},\psi^{-0},\psi^{-0})$, we find that Eqs. (4) admit the sufficient number of arbitrary functions and hence Eqs. (7) possess the Painlevé property and hence they are expected to be integrable.

III. HIROTA BILINEARIZATION

Hirota's bilinear method [20] is one of the most direct and elegant methods available to generate multisoliton solutions of nonlinear partial differential equations. To avoid mathematical complexities, it is rather convenient to transform Eqs. (4) to a simpler form, so that we may be able to obtain multisoliton solutions. We make the following transformations to convert CHNLS to a complex modified K-dV (CMK-dV) equation:

$$\begin{aligned} u(z,t)&=Q_1(Z,T)\exp\left[-i\left(\frac{Z}{3\lambda}-\frac{T}{27\lambda^2}\right)\right],\\ v(z,t)&=Q_2(Z,T)\exp\left[-i\left(\frac{Z}{3\lambda}-\frac{T}{27\lambda^2}\right)\right],\\ t&=T,\ Z=z+\frac{t}{3\lambda}. \end{aligned} \tag{15}$$

Using the above transformations in Eqs. (4), the resultant CMK-dV equation is obtained in the form

$$\begin{aligned} &Q_{1T}-\lambda[Q_{1ZZZ}+6(|Q_1|^2+|Q_2|^2)Q_{1Z}\\ &\quad+3Q_1(|Q_1|^2+|Q_2|^2)_Z]=0,\\ &Q_{2T}-\lambda[Q_{2ZZZ}+6(|Q_1|^2+|Q_2|^2)Q_{2Z}\\ &\quad+3Q_2(|Q_1|^2+|Q_2|^2)_Z]=0. \end{aligned} \tag{16}$$

In order to construct Hirota's bilinear form, we consider Hirota bilinear transformations in the form

$$Q_1=\frac{G}{F} \quad Q_2=\frac{H}{F}, \tag{17}$$

where $G(Z,T)$ and $H(Z,T)$ are complex functions and $F(Z,T)$ is a real function. Now using the transformations (17), (16) can be rewritten as

$$\begin{aligned} &F^2[(D_T-\lambda D_Z^3)(G\cdot F)]-\lambda[\{-3D_z^2(F\cdot F)D_Z\\ &\quad+12(|G|^2+|H|^2)D_Z(G\cdot F)+3GFD_Z(G\cdot G^*)\\ &\quad+3H^*FD_Z(H\cdot G)-3HFD_Z(G\cdot H^*)]=0,\\ &F^2[(D_T-D_Z^3)(H\cdot F)]-\lambda[\{-3D_z^2(F\cdot F)D_Z\\ &\quad+12(|G|^2+|H|^2)D_Z(H\cdot F)+3HFD_Z(H\cdot H^*)\\ &\quad-3G^*FD_Z(H\cdot G)+3GFD_Z(H\cdot G^*)=0, \end{aligned} \tag{18}$$

where the Hirota bilinear operators D_z and D_t are defined as

$$\begin{aligned} D_Z^mD_T^nG(Z,T)F(Z',T')&=\left(\frac{\partial}{\partial Z}-\frac{\partial}{\partial Z'}\right)^m\left(\frac{\partial}{\partial T}-\frac{\partial}{\partial T'}\right)^n\\ &\quad\times G(Z,T)F(Z',T')|_{Z=Z',T=T'} \end{aligned} \tag{19}$$

and the centered dot stands for ordered multiplication by the preceding operators.

IV. EXACT SOLITON SOLUTIONS

Equations (18) can be decoupled into a set of bilinear equations as

$$\begin{aligned} &(D_T-\lambda D_Z^3)(G\cdot F)=0, \quad (D_T-\lambda D_Z^3)(H\cdot F)=0,\\ &D_Z^2(F\cdot F)=4(|G|^2+|H|^2), \end{aligned} \tag{20}$$

$$D_Z(G\cdot G^*)=0, D_Z(H\cdot H^*)=0, D_Z(G\cdot H^*)$$
$$=0, D_Z(H\cdot G^*)=0, D_Z(G\cdot H)=0.$$

In order to obtain soliton solutions, we are applying a perturbative technique by writing the variables F, G, H as a series in an arbitrary parameter ε:

$$F=1+\varepsilon^2 f_2+\varepsilon^4 f_4+\cdots,$$
$$G=\varepsilon g_1+\varepsilon^3 g_3+\varepsilon^5 g_5+\cdots, \tag{21}$$
$$H=\varepsilon h_1+\varepsilon^3 h_3+\varepsilon^5 h_5+\cdots.$$

A. One-soliton solutions

For one-soliton solution (1SS), we assume solutions in a series form in ε such that

$$F=1+\varepsilon^2 f_2, \quad G=\varepsilon g_1, \quad H=\varepsilon h_1. \tag{22}$$

Substituting Eq. (22) in Eqs. (20) and then collecting coefficients of terms with same powers in ε, we obtain for ε

$$(D_T-\lambda D_Z^3)(g_1\cdot 1)=0 \quad (D_T-\lambda D_Z^3)(h_1\cdot 1)=0, \tag{23}$$

for ε^2

$$D_Z^2(1\cdot f_2+f_2\cdot 1)=4(g_1\cdot g_1^*+h_1\cdot h_1^*), \tag{24}$$

$$D_Z(g_1\cdot g_1^*)=0, D_Z(h_1\cdot h_1^*)=0, D_Z(g_1\cdot h_1^*)$$
$$=0, D_Z(h_1\cdot g_1^*)=0, D_Z(g_1\cdot h_1)=0,$$

for ε^3

$$(D_T-\lambda D_Z^3)(g_1\cdot f_2)=0, (D_T-\lambda D_Z^3)\,(h_1\cdot f_2), \tag{25}$$

and for ε^4

$$D_Z^2(f_2\cdot f_2)=0. \tag{26}$$

One can easily check that the solution, which is consistent with the system (23)–(26), is

$$g_1=\cos\phi\exp(\eta+i\theta), \quad h_1=\sin\phi\exp(\eta-i\theta), \tag{27}$$
$$f_2=\left(\frac{1}{2k^2}\right)\exp(2\eta),$$

where

$$\eta=kZ+\lambda k^3 T \tag{28}$$

and ϕ and k are real constants. Using Eqs. (27) and (28) in Eq. (22) and then in Eq. (17), after absorbing ε the one-soliton solution can easily be worked out to be

$$Q_1=\left(\frac{k}{\sqrt{2}}\right)\cos\phi\exp(i\theta)\mathrm{sech}(kZ+\lambda k^3 T+\eta_0),$$
$$Q_2=\left(\frac{k}{\sqrt{2}}\right)\sin\phi\exp(-i\theta)\mathrm{sech}(kZ+\lambda k^3 T+\eta_0), \tag{29}$$

where η_0 is a real constant. Using Eqs. (15), one-soliton solutions of Eqs. (4) are found to be

$$u=\left(\frac{k}{\sqrt{2}}\right)\cos\phi\exp\left[-i\left(\frac{Z}{3\lambda}-\frac{T}{27\lambda^2}-\theta\right)\right]$$
$$\times\mathrm{sech}(kZ+\lambda k^3 T+\eta_0), \tag{30}$$
$$v=\left(\frac{k}{\sqrt{2}}\right)\sin\phi\exp\left[-i\left(\frac{Z}{3\lambda}-\frac{T}{27\lambda^2}\right)+\theta\right]$$
$$\times\mathrm{sech}(kZ+\lambda k^3 T+\eta_0).$$

B. Two-soliton solutions

The two-soliton solutions can be obtained by terminating the series expansion for F, G, H as

$$F=1+\varepsilon^2 f_2+\varepsilon^4 f_4, \quad G=\varepsilon g_1+\varepsilon^3 g_2, \quad H=\varepsilon h_1+\varepsilon^3 h_3 \tag{31}$$

and proceeding as before to obtain

$$g_1=\exp(\eta_1)+\exp(\eta_2), \quad h_1=ig_1,$$
$$g_3=(k_2-k_1)^2\left[\frac{\exp(2\eta_1+\eta_2)}{4k_1^2(k_1+k_2)^2}+\frac{\exp(\eta_1+2\eta_2)}{4k_2^2(k_1+k_2)^2}\right],$$
$$h_3=ig_3, \tag{32}$$
$$f_2=4\left[\frac{\exp(2\eta_1)}{4k_1^2}+2\frac{\exp(\eta_1+\eta_2)}{(k_1+k_2)^2}+\frac{\exp(2\eta_2)}{4k_2^2}\right],$$
$$f_4=\frac{4(k_2-k_1)^4\exp(2\eta_1+2\eta_2)}{16k_1^2k_2^2(k_1+k_2)^4},$$

where

$$\eta_j=k_jZ+\lambda k_j^3 T, \quad j=1,2. \tag{33}$$

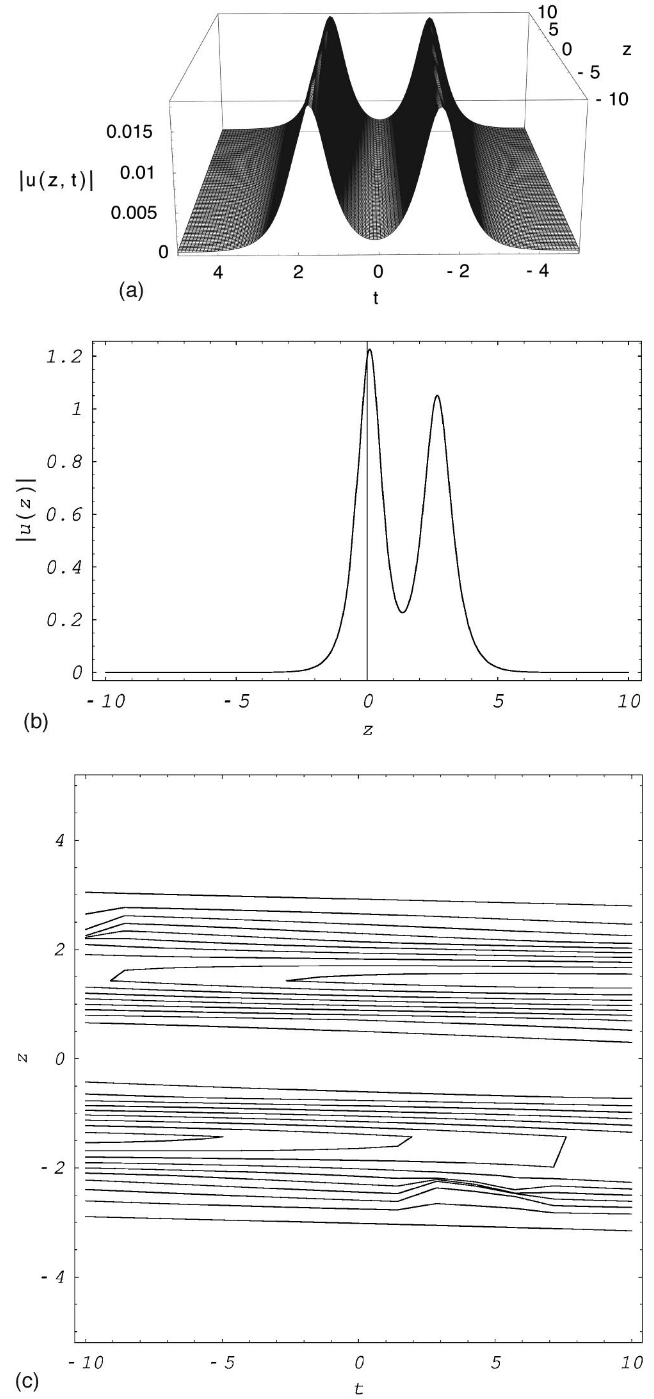

FIG. 1. (a) 3D profile of $|u(z,t)|$ for the two-soliton solution of Eq. (34) with the parameter values $k_1=0.034$, $k_2=0.04$, $\lambda=0.005$. (b) 2D profile of $|u(z,t)|$ for the two-soliton solution of Eq. (34) with the parameter values $k_1=2$, $k_2=3$, $\lambda=0.002$. (c) Contour plot of $|u(z,t)|$ with respect to z and t for the parameter values $k_1=0.034$, $k_2=0.04$, $\lambda=0.005$.

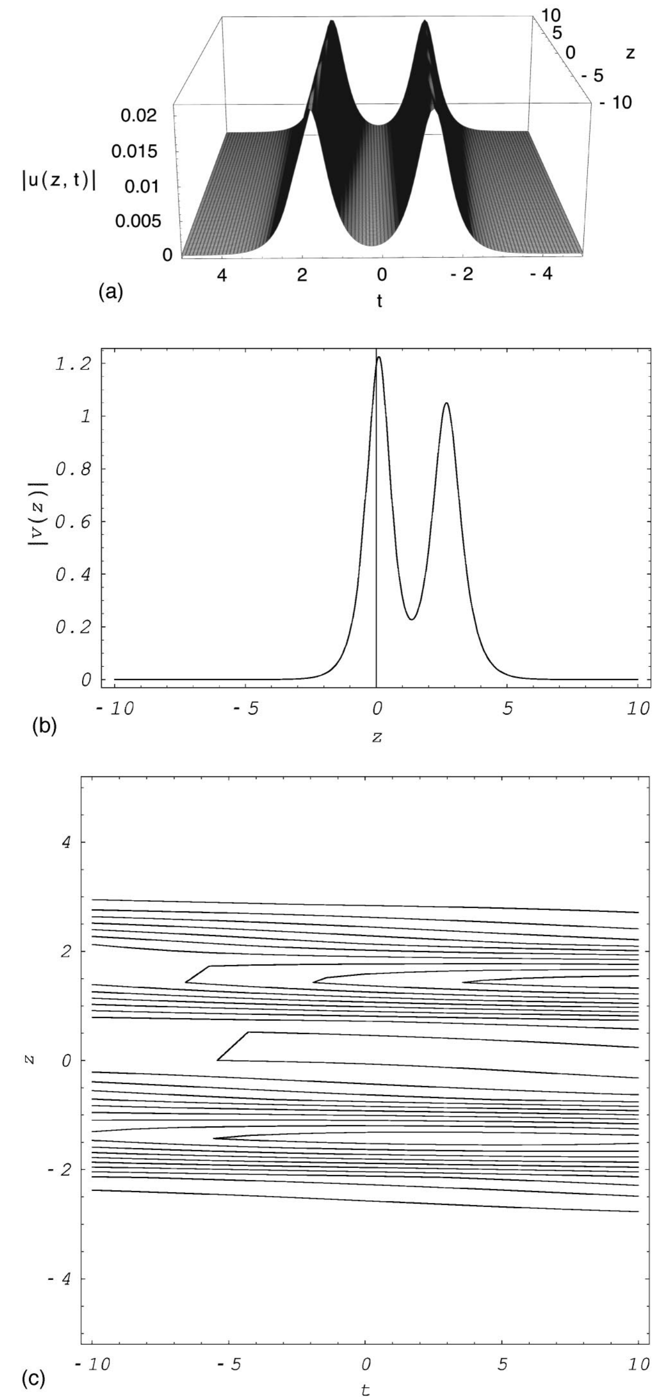

FIG. 2. (a) 3D profile of $|v(z,t)|$ for the two-soliton solution of Eq. (34) with the parameter values $k_1=0.04$, $k_2=0.045$, $\lambda=0.005$. (b) 2D profile of $|v(z,t)|$ for the two-soliton solution of Eq. (34) with the parameter values $k_1=2$, $k_2=3$, $\lambda=0.005$. (c) Contour plot of $|u(z,t)|$ with respect to z and t for the parameter values $k_1=0.04$, $k_2=0.045$, $\lambda=0.005$.

Here k_j is a real constant. Using Eqs. (32) and (33) in Eq. (31) and then in Eq. (17), the two-soliton solutions of Eq. (16) are obtained. Using Eqs. (15), the two-soliton solutions of Eqs. (4) are found to be

$$u=\frac{G}{F}\exp\left[-i\left(\frac{Z}{3\lambda}-\frac{T}{27\lambda^2}\right)\right],$$

$$v=\frac{H}{F}\exp\left[-i\left(\frac{Z}{3\lambda}-\frac{T}{27\lambda^2}\right)\right]. \tag{34}$$

Both 1SS and 2SS are in exact agreement with Eqs. (4). Two-dimensional, three-dimensional, and contour plots of 2SS are given in Figs. 1 and 2.

V. LAX PAIR FOR CHNLS SYSTEM

The linear eigenvalue problem associated with Eqs. (16) are [21,22]

$$\psi_Z = U\psi, \quad \psi_T = V\psi, \quad \psi = (\psi_1 \psi_2)^T, \tag{35}$$

where

$$U = \begin{pmatrix} -i\chi & Q_1 & Q_1^* & Q_2 & Q_2^* \\ -Q_1^* & i\chi & 0 & 0 & 0 \\ -Q_1 & 0 & i\chi & 0 & 0 \\ -Q_2^* & 0 & 0 & i\chi & 0 \\ -Q_2 & 0 & 0 & 0 & i\chi \end{pmatrix}, \tag{36}$$

$$V = \frac{-8i\lambda\chi^3}{5}\begin{pmatrix} -4 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} - 4\lambda\chi^2 \begin{pmatrix} 0 & Q_1 & Q_1^* & Q_2 & Q_2^* \\ -Q_1^* & 0 & 0 & 0 & 0 \\ -Q_1 & 0 & 0 & 0 & 0 \\ -Q_2^* & 0 & 0 & 0 & 0 \\ -Q_2 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$+2i\lambda\chi \begin{pmatrix} -2A & -Q_{1Z} & -Q_{1Z}^* & -Q_{2Z} & -Q_{2Z}^* \\ -Q_{1Z}^* & |Q_1|^2 & (Q_1^*)^2 & Q_1^*Q_2 & Q_1^*Q_2^* \\ -Q_{1Z} & Q_1^2 & |Q_1|^2 & Q_1Q_2 & Q_1Q_2^* \\ -Q_{2Z}^* & Q_1Q_2^* & Q_1^*Q_2^* & |Q_2|^2 & (Q_2^*)^2 \\ -Q_{2Z} & Q_1Q_2 & Q_1^*Q_2 & Q_2^2 & |Q_2|^2 \end{pmatrix} \tag{37}$$

$$+\lambda \begin{pmatrix} 0 & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & 0 & a_{24} & a_{25} \\ a_{31} & 0 & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 \\ a_{51} & a_{52} & a_{53} & 0 & a_{55} \end{pmatrix},$$

where $a_{12}=4\ A\ Q_1+Q_{1ZZ}$, $a_{13}=4\ A\ Q_1^*+Q_{1ZZ}^*$, $a_{14}=4\ A\ Q_2+Q_{2ZZ}$, $a_{15}=4\ A\ Q_2^*+Q_{2ZZ}^*$, $a_{21}=-4\ A\ Q_1^*-Q_{1ZZ}^*$, $a_{22}=Q_1Q_{1Z}^*-Q_1^*Q_{1Z}$, $a_{24}=Q_2Q_{1Z}^*-Q_1^*Q_{2Z}$, $a_{25}=Q_2^*Q_{1Z}^*-Q_1^*Q_{2Z}^*$, $a_{31}=-4\ A\ Q_1-Q_{1ZZ}$, $a_{33}=Q_1^*Q_{1Z}-Q_1Q_{1Z}^*$, $a_{34}=Q_2Q_{1Z}-Q_1Q_{2Z}$, $a_{35}=Q_2^*Q_{1Z}-Q_1Q_{2Z}^*$, $a_{41}=-4\ A\ Q_2^*-Q_{2ZZ}^*$, $a_{42}=Q_1Q_{2Z}^*-Q_2^*Q_{1Z}$, $a_{43}=Q_1^*Q_{2Z}^*-Q_2^*Q_{1Z}^*$, $a_{44}=Q_2Q_{2Z}^*-Q_2^*Q_{2Z}$, $a_{51}=-4\ A\ Q_2-Q_{2ZZ}$, $a_{52}=Q_1Q_{2Z}-Q_2Q_{1Z}$, $a_{53}=Q_1^*Q_{2Z}-Q_2Q_{1Z}^*$, $a_{55}=Q_2^*Q_{2Z}-Q_2Q_{2Z}^*$ with $A=|Q_1|^2+|Q_2|^2$.

The compatibility condition $U_T - V_Z + [U,V] = 0$ gives rise to Eqs. (16). The construction of Lax pair confirms that Eqs. (16) and thereby the CHNLS Eqs. (4) are indeed completely integrable.

VI. CONCLUSION

In this paper, we have considered a set of coupled NLS equations with higher-order linear and nonlinear dispersion terms included. Then, by choosing the parameters as in the case of the corresponding integrable uncoupled case, we applied the Painlevé singularity structure analysis and established that for this particular choice of parameters, Eqs. (4) possess the Painlevé property. We have explicitly obtained one-soliton and two-soliton solutions for the integrable cases of CHNLS equations using Hirota bilinearization technique and solutions are plotted. We have also constructed Lax pairs using AKNS formalism. Hence, with these results, we have proved that the CHNLS equations which describe the wave propagation of two fields in fiber systems with all higher-order effects such as TOD, Kerr dispersion, and stimulated Raman effect, will allow soliton-type propagation. From the soliton solutions, one can obtain information about the shape, width, and intensity of the propagation pulse.

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