Bragg-grating solitons in a semilinear dual-core system

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We investigate the existence and stability of gap solitons in a double-core optical fiber, where one core has the Kerr nonlinearity and the other one is linear, with the Bragg grating (BG) written on the nonlinear core, while the linear one may or may not have a BG. The model considerably extends the previously studied families of BG solitons. For zero-velocity solitons, we find exact solutions in a limiting case when the group-velocity terms are absent in the equation for the linear core. In the general case, solitons are found numerically. Stability borders for the solitons are found in terms of an internal parameter of the soliton family. Depending on the frequency ω, the solitons may remain stable for large values of the group velocity in the linear core. Stable moving solitons are also found. They are produced by interaction of initially separated solitons, which shows a considerable spontaneous symmetry breaking in the case when the solitons attract each other.

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I. INTRODUCTION AND FORMULATION OF THE MODEL

It is well known that the combination of the Kerr nonlinearity with a strong effective dispersion induced by the resonant reflection of light on the Bragg grating (BG) gives rise to a vast family of gap solitons, frequently called BG solitons [1] (in this work, we use the term ''soliton'' in the loose sense, without implying integrability of the model where it appears; in particular, it will be shown that interactions between ''solitons'' in a model to be introduced below may be essentially inelastic). A generally accepted mathematical model of the nonlinear fiber equipped with a BG is the so-called generalized massive Thirring model (GMTM) [2]. Thorough theoretical investigation of BG solitons, an important step in which was the discovery of a class of exact single-soliton solutions to the GMTM [2], was followed by observation of BG solitons created by a very strong laser pulse launched into a short segment ($\sim$6 cm) of a nonlinear optical fiber with the resonant BG written on it [3]. Experimental studies of BG solitons were further developed (including, in particular, formation of multiple BG solitons) in Refs. [4].

Observation of solitons in such a short fiber paves the way for many potential applications, as well as for further experiments aimed at the study of fundamental properties of optical solitons. This also makes it relevant to consider more sophisticated nonlinear systems based on fiber gratings, where the properties of solitons might be still more promising. In particular, one can look for solitons in a *dual-core* system with linear coupling between the cores, a BG being written on both cores or a single one. The case of two identical BG-carrying cores was considered in Ref. [5], where it was found that the model gave rise to a *bifurcation* at a critical value of the soliton's energy. The bifurcation destabilizes a symmetric two-component solution, simultaneously generating a nontrivial asymmetric soliton. A dual-fiber system with unlike cores is easier to fabricate and may offer other possibilities. One of the most interesting dual systems with different cores is a *semilinear* one, where one core is linear. Semilinear dual-core models without BG's were introduced earlier; both continuous-wave and soliton states in them have been studied in various contexts [6,7].

The objective of this work is to introduce a semilinear dual-core model in which the BG is written either on the nonlinear core only or on both cores, and to search for solitons in it (which makes it necessary, first of all, to explore the system's linear spectrum). Following the derivation of the GMTM [1] and of the standard equations for a dual-core fiber (see, e.g., Ref. [8]) from Maxwell's equations, a general model for the semilinear dual-core BG-equipped system can be written as the following set of normalized equations:

$$iu_t+iu_x+[|v|^2+(1/2)|u|^2]u+v+\kappa\phi=0, \tag{1}$$

$$iv_t-iv_x+[|u|^2+(1/2)|v|^2]v+u+\kappa\psi=0, \tag{2}$$

$$i\phi_t+ic\phi_x+\kappa u+(\lambda+i\mu)\psi=0, \tag{3}$$

$$i\psi_t-ic\psi_x+\kappa v+(\lambda-i\mu)\phi=0. \tag{4}$$

Here, u and v represent the forward- and backward-propagating waves in the nonlinear core, ϕ and ψ are their counterparts in the linear one, κ is the coefficient of linear coupling between the cores, while λ and μ are the real and imaginary parts of the BG coupling coefficient in the linear core [which is, generally, complex if its counterpart in the nonlinear core is normalized to be 1, as is the case in Eqs. (1) and (2)]. Lastly, the group velocity in the nonlinear core is set equal to 1, and c is the relative group velocity in the linear core.

The simplest case is $\lambda=\mu=0$ (corresponding to the linear core without BG), while cross-core coupling κ is nonzero. Below, we will always set $\mu=0$; in most cases, λ will also be zero, but effects of $\lambda\neq0$ on the solitons' stability will be investigated too. Note that, although the present model finds its most natural formulation in the temporal domain, it can also be readily interpreted in terms of the *spatial-domain*

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evolution of the fields in a two-core planar waveguide, the BG being realized as a system of parallel scores written on the waveguide(s) [9].

It may also be quite interesting to consider a system where the Kerr nonlinearity and BG are separated, i.e., with the grating written only on the *linear* core. The corresponding model is obtained from the above equations, dropping the linear terms v and u in Eqs. (1) and (2) and setting $\lambda=1$ and $\mu=0$ in Eqs. (3) and (4). This model, which also seems quite promising, will be considered elsewhere.

Before looking for solitons, it is necessary to analyze the spectrum of the linearized system, in order to identify a spectral *gap* in which BG solitons may reside [1]. For a linear wave $\sim\exp(ikx-i\omega t)$ and setting $\mu=0$, a dispersion equation for $\omega(k)$ can be obtained:

$$\omega^4-[1+2\kappa^2+\lambda^2+(1+c^2)k^2]\omega^2+(\lambda-\kappa^2)^2 +(c^2-2c\kappa^2+\lambda^2)k^2+c^2k^4 =0.$$

Analyzing this equation, it is easy to conclude that the gap does not exist in the present model if $\lambda<\kappa^2$ and $c^2-\lambda+\lambda^2<(2c-1)\kappa^2$, or if $(1+2c)^{-1}(c+c^2+\lambda^2)<\kappa^2<\lambda$. In all other cases, a finite gap is present, and BG solitons may exist. In the particular case when the linear properties of the two cores are identical, i.e., $c=1$ and $\lambda=1$, which physically corresponds to having identical BGs written on them, the gap existence condition takes a very simple form, $\kappa^2<1$ [5].

A remarkable property of the above-mentioned GMTM equations, to which Eqs. (1)–(4) reduce if the additional core is dropped, is the availability of exact single-soliton solutions, both quiescent and moving with an arbitrary velocity v, limited by $|v|<1$, despite the fact that the model is not integrable (except for the unphysical case when the self-phase-modulation terms are omitted) [2]. Here, we aim to find soliton solutions to the full system (1)–(4) and investigate their stability and interactions. Solitons with zero velocity will be studied in detail, and moving solitons will also be presented. In fact, the existence of solitons with zero velocity (which have not yet been observed experimentally in single-core fiber gratings) is a most intriguing possibility, as this implies a possibility of ''full stoppage of light'' through its dynamical trapping, which is especially interesting in view of the recent discovery of ''ultraslow light'' in ultracold gases [10].

As for the physical parameters of the system and its soliton solutions, a crucial factor is the ratio of the length z_{coupl} of the coupling between the cores and a characteristic propagation distance (the soliton's *dispersion length*) z_{sol} necessary for the formation of a soliton in a single-core fiber with BG. As is well known, the former length in available dual-core fibers is, normally, $\sim$1 cm, and, according to the experimental data [3,4], z_{sol} is on the same order of magnitude (it is so short, despite the fact that the solitons are relatively broad in the temporal domain, because a BG gives rise to an extremely strong effective dispersion). This circumstance, $z_{\mathrm{coupl}}\sim z_{\mathrm{sol}}$, is quite favorable, as it suggests that the interplay between the resonant light reflection on the BG, the Kerr nonlinearity, and the linear coupling between the cores may give rise to solitons with fairly unusual properties, in comparison with both the usual (single-core) BG solitons [1] and solitons in dual-core fibers without a BG [7].

In line with the above arguments, other basic characteristics of these solitons are expected to be of the same order of magnitude as those for the recently observed BG solitons in a single-core fiber. In particular, the soliton can be generated by a laser pulse of duration $\sim$100 ps, having a fairly high peak power $\sim$5 W (which is, however, still sufficiently far from the optical-breakdown threshold in silica glass), and, accordingly, energy $\sim$500 pJ. The soliton to be created will keep essentially all this energy, self-compressing to the temporal width $\lesssim$50 ps [3,4].

Another crucial ingredient of a possible experiment is the necessary length of the BG-equipped dual-core fiber. As mentioned above, for successful generation and detection of the gap soliton in a single-core BG fiber, a 6 cm fiber was sufficient. In fact, present-day techniques make it quite easy to fabricate a homogeneous dual-core fiber of length $\sim$1 m, as well as to write a uniform BG on it. Therefore, an experiment may be quite feasible in a fiber whose length is of the order of 100 characteristic soliton and coupling lengths (both being $\sim$1 cm; see above), which will be more than enough for the most precise experiments.

Thus, experimental generation of the solitons to be theoretically studied in the present work is not going to be much harder than the recent experiments reported in Refs. [3] and [4]. The only essentially different issue in the experiment may be the question of whether to focus the input laser pulse on the entrance face of one core only, as usual, or it is necessary to split it, in a special fashion, between the two cores. Although it may be premature here to discuss experimental technicalities in such detail, we note that having the fiber length much longer than $z_{\mathrm{coupl}}\sim 1$ cm (see above) will provide enough room for the proper redistribution of power between the cores, so that the experiment will not be critically sensitive to details of launching the input pulse.

The rest of the paper is organized as follows. In Sec. II we display *exact analytical* soliton solutions that can be found in the present model with $c=0$, and results of simulations of their stability, which show that they are stable in a broad parametric region. In the case $c\neq 0$, soliton solutions can be found only numerically, which is done in Sec. III, together with systematic simulations of their stability. It is found that, depending on the value of the frequency ω, the solitons may remain stable up to a large value $c=c_{\max}$. At $c>c_{\max}$, the soliton becomes unstable. This instability, however, does not destroy it; after shedding some radiation, it evolves into another member of the soliton family. In Sec. IV we directly simulate interactions between two solitons placed initially at some distance from each other. It is found that the result of the interaction strongly depends on the relative phase of the two solitons. In particular, the interaction can easily generate moving solitons and leads to spontaneous symmetry breaking.

II. EXACT SOLITON SOLUTIONS AND THEIR STABILITY

Exact zero-velocity soliton solutions to Eqs. (1)–(4) can be found only in the particular case $c=0$. Starting with the usual ansatz,

$$u = U(x)\exp(-i\omega t), \quad v = V(x)\exp(-i\omega t), \tag{5}$$

$$\phi = \Phi(x)\exp(-i\omega t), \quad \psi = \Psi(x)\exp(-i\omega t), \tag{6}$$

and following the pattern of the exact GMTM solutions [2], we find

$$U(x) \equiv \left[\frac{(\omega^2-\lambda^2-\mu^2+\lambda\kappa^2)^2+\mu^2\kappa^4}{(\omega^2-\lambda^2-\mu^2)^2}\right]^{1/4} e^{+i\delta/2}A(x),$$

$$V(x) \equiv \left[\frac{(\omega^2-\lambda^2-\mu^2+\lambda\kappa^2)^2+\mu^2\kappa^4}{(\omega^2-\lambda^2-\mu^2)^2}\right]^{1/4} e^{-i\delta/2}B(x), \tag{7}$$

where $\delta = \tan^{-1}[\kappa^2\mu/(\omega^2-\lambda^2-\mu^2+\lambda\kappa^2)]$, and

$$A(x) = \sqrt{2/3}(\sin\theta)\operatorname{sech}(\eta x\sin\theta - i\theta/2),$$

$$B(x) = -\sqrt{2/3}(\sin\theta)\operatorname{sech}(\eta x\sin\theta + i\theta/2), \tag{8}$$

$$\Phi(x) = -\frac{\kappa\omega}{\omega^2-\lambda^2-\mu^2}U + \frac{\kappa(\lambda+i\mu)}{\omega^2-\lambda^2-\mu^2}V,$$

$$\Psi(x) = \frac{\kappa(\lambda-i\mu)}{\omega^2-\lambda^2-\mu^2}U - \frac{\kappa\omega}{\omega^2-\lambda^2-\mu^2}V. \tag{9}$$

Here θ, which takes values between 0 and π, is an arbitrary parameter of the soliton family. The frequency ω and inverse width η of the soliton are determined, in terms of θ, by the equations

$$\frac{\omega(\omega^2-\lambda^2-\mu^2-\kappa^2)}{\sqrt{(\omega^2-\lambda^2-\mu^2+\lambda\kappa^2)^2+\mu^2\kappa^4}}\operatorname{sgn}(\omega^2-\lambda^2-\mu^2) = \cos\theta, \tag{10}$$

$$\eta \equiv \left[\frac{(\omega^2-\lambda^2-\mu^2+\lambda\kappa^2)^2+\mu^2\kappa^4}{(\omega^2-\lambda^2-\mu^2)^2}\right]^{1/4}. \tag{11}$$

It is relevant to note that these exact solutions resemble those found earlier in a linearly coupled system of cubic and linear Ginzburg-Landau (GL) equations [13]; however, the exact solutions to the GL equations exist as isolated ones, rather than in families, i.e., they do not contain any arbitrary parameter.

Before proceeding to a numerical search for solitons in the case $c \neq 0$, it is necessary to address the stability of the exact analytical solutions obtained above. The first nonrigorous stability analysis of GMTM solitons was done using the variational approximation [11]. It was predicted that instability might occur when an internal parameter of the GMTM solitons, θ, similar to that introduced above in Eqs. (8), exceeded a certain critical value, which was close to $\pi/2$. Then a rigorous treatment of the stability problem for the GMTM system, based on the consideration of its linearized version, was developed in Refs. [12]. It was demonstrated there that solitons with θ exceeding a critical value, which is slightly larger than $\pi/2$, are indeed unstable. However, the instability is weak; therefore it was hard to observe it in direct simulations.

In this connection, it should be noted that, while the results for the solitons' stability in various models obtained from the solution of the corresponding eigenvalue problem for the linearized equations are more rigorous (and usually are technically more difficult) than those produced by direct simulations of the nonlinear equations, the latter results may be more appropriate for physical applications. Indeed, if the soliton is, rigorously speaking, unstable but the instability is weak (as is the case for the GMTM), it may happen that neither direct simulations performed for a limited evolution time (or propagation distance, depending on the particular system) nor a real experiment in a finite-size sample will demonstrate the instability, so that, in terms of real physics, the soliton should be regarded as a *stable* object, in accordance with the prediction of the direct simulations, and despite the contradiction with the rigorous results. Solitons in a BG fiber may provide an example of this situation. In this case, experimental results [3,4], while being in good agreement with direct simulations, have not been able to demonstrate the sophisticated instability predicted on the basis of the linearized equations in Refs. [12]. On the other hand, it is necessary to mention that, although the physical value of the soliton's peak power in these experiments was quite high, the BG solitons actually observed may still be low-intensity ones from the viewpoint of the corresponding theoretical model. However, the above-mentioned "sophisticated instability" occurs only for high-intensity solitons. Another example that could be cited regarding the fact that soliton instability may sometimes be formal is provided by "spinning" $(2+1)$-dimensional solitons in media with the cubic-quintic nonlinearity. As the analysis of the corresponding linearized problem shows, the solitons with "spin" $s=1$ are, strictly speaking, always unstable against infinitesimal azimuthal perturbations that destroy the cylindrical symmetry of the solitons. Nevertheless, if the size of the spinning soliton is large enough, the instability may be so weak that the soliton may persist as a fairly robust object over several diffraction lengths [14], thus having a fairly good chance of being observed in an experiment.

To test the stability of the exact soliton solutions given by Eqs. (5)–(11), we simulated their evolution by means of the split-step Fourier algorithm, imposing various asymmetric (sometimes nonsmall) initial perturbations. A typical case is displayed in Fig. 1, showing that after shedding some radiation the perturbed pulse readily evolves into a member of the soliton family (in fact, the final soliton in Fig. 1 acquires a very small velocity, because the asymmetric perturbation has "pushed" it; moving solitons will be specially considered below). In particular, an important finding is that, when $\lambda \neq 0$, Eq. (10) gives rise to three distinct roots for ω, of which only the one with largest $|\omega|$ is found to produce a stable soliton. On the other hand, there are two different roots for ω at $\lambda=0$, *both* leading to stable solitons.

We have also found that, as for GMTM solitons, a fundamental property of the soliton family in our extended model is that the stable part of the family is *limited*, $\theta \leqslant \theta_{\max}$, where $\theta_{\max}$ depends on κ and λ. To analyze this in detail, we set $\lambda=0$, focusing on the simplest and most fundamental case when a BG is present in the nonlinear core only, and the stability is solely controlled by the coefficient of the linear coupling between the nonlinear and linear cores. The stability border inside the soliton family, $\theta_{\max}(\kappa)$, was then sought for gradually increasing θ at a fixed value of κ. We started

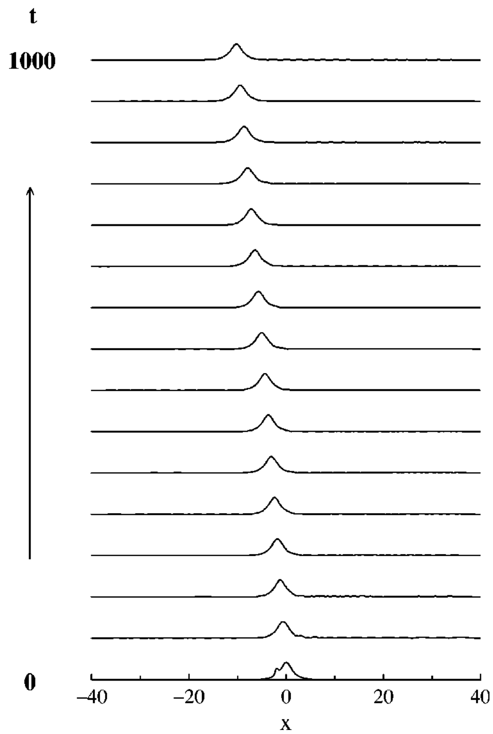

FIG. 1. Evolution of an asymmetrically perturbed soliton when $c=0$ (only the u component is shown). The other parameters are $\lambda=\mu=0$, $\kappa=1$, and the soliton's internal parameter [see Eqs. (8)] $\theta=\pi/3$.

from $\theta=\pi/12$, where the exact soliton is definitely stable, until we hit a value θ_{max} that gave rise to instability. The instability, when it sets in, causes straightforward decay of the soliton into radiation. We have thus found that $\theta_{max}(\kappa=0.01)=\pi/1.7$, $\theta_{max}(\kappa=1)=\pi/2.0$, and $\theta_{max}(\kappa=100)=\pi/1.8$, i.e., the dependence of the stability limit on κ is fairly weak, θ_{max} being close to that in the single-core model, although the shapes of the exact solitons may be quite different.

III. SOLITONS IN THE MODEL WITH $c\neq 0$

The above consideration pertained to the limiting case $c=0$, when the exact solutions are available. The next necessary step is to consider $c\neq 0$, when no exact solution for the zero-velocity solitons could be found. We therefore started by using the known relaxation algorithm [15] in order, first of all, to obtain stationary soliton solutions numerically from the ordinary differential equations produced by the substitution of Eqs. (5) and (6) into Eqs. (1)–(4). By properly setting boundary conditions, it was always possible to obtain a soliton solution for a given ω.

A major objective here is to find out whether at fixed values of all parameters except c there exists a maximum value of c above which the solitons are unstable. It was found that, depending on the value of ω, there indeed exists c_{max} beyond which solitons become unstable. However, the instability at $c>c_{max}$ leads not to the disappearance of solitons, but rather to their self-rearrangement into a slightly different form.

A typical result is displayed in Fig. 2, with $\kappa=1$, $\lambda=\mu=0$, and $\omega=1.6$. This value of ω was chosen since it lies sufficiently deep inside the stability region at $c=0$. Our simulations show that the value c_{max} is very large, ≈ 4.2. As is seen in Fig. 2(b), at $c>c_{max}$ the soliton becomes unstable and, after shedding some radiation, it evolves into another member of the soliton family.

A practically significant consequence of the above result is that values of c close to 1 (recall that 1 is the group ve-

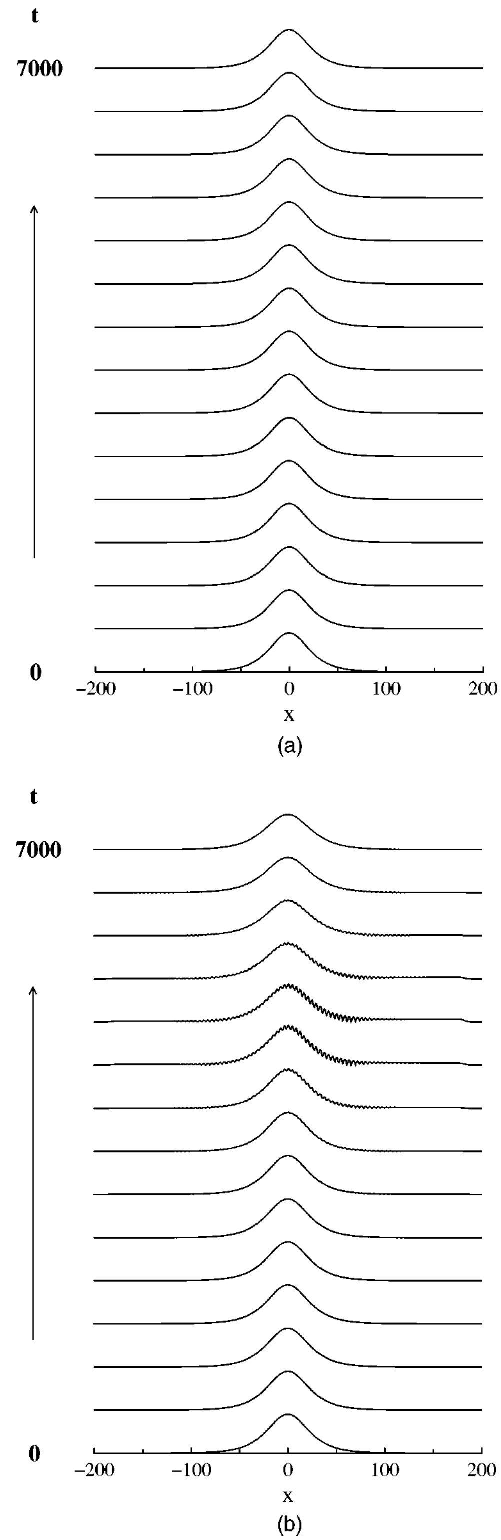

FIG. 2. Evolution of solitons at values of c slightly below and above c_{max}: (a) $c=4.1$; (b) $c=4.3$. The other parameters are $\kappa=1$, $\lambda=\mu=0$, and $\omega=1.6$.

locity in the nonlinear core) definitely give rise to stable solitons. This inference is important for experiments, because, in the most realistic case when both cores are made of the same material, the group velocities in them are necessarily close.

In the case $\lambda \neq 0$ (when a BG is written on the linear core too), the relaxation algorithm also successfully generated stationary solitons. Starting with these, we found that the solitons are stable in a broad parametric region, again including values of c essentially exceeding 1. However, detailed analysis of the interplay of λ with other parameters is very cumbersome and is left aside.

IV. INTERACTIONS BETWEEN SOLITONS AND GENERATION OF MOVING SOLITONS

Since the present model is nonintegrable, interactions between solitons may be quite complex. The simplest approach to simulating these interactions is to start from a superposition of two identical exact solitons placed initially at a distance from each other with some phase difference $\Delta\varphi$. Results of the simulations, typical examples of which are displayed in Fig. 3, are similar for different values of the soliton's internal parameters and initial separation (provided that the solitons overlap weakly), but they strongly depend on $\Delta\varphi$. In the case $\Delta\varphi=\pi$, the solitons, quite naturally, repel each other [cf. the well-known fact that non-linear Schrödinger (NLS) solitons interact repulsively when $\Delta\varphi=\pi$ [16]]. Even if the initial separation between the solitons is relatively large, the repulsion is strong enough to lend the two initially quiescent solitons conspicuous velocities [see Fig. 3(a)]. In this case, the eventual velocities are found to be $W_{\pm}=\pm 0.03$. Thus, these simulations not only shed light on the character of the interaction between the solitons, but also provide a convenient way to generate stable *moving* ones.

It is also interesting to compare the initial *energy* E_i of each soliton, defined as $\int_{-\infty}^{+\infty}(|u|^2+|v|^2+|\phi|^2+|\psi|^2)dx$ (which is a dynamical invariant of the model), and the final values E_f of the energy of the moving solitons. In the case shown in Fig. 3(a), $E_f/E_i=0.986$, i.e., about 1.5% of the initial energy is lost (into emission of radiation) as a result of the interaction process. It should be stressed that moving solitons produced by the interaction exhibit some internal vibrations, i.e., the solitons appear with a weakly excited internal mode (the existence of internal modes in stable GMTM solitons is a known fact [11,12]). It may also happen that they capture some radiation which will be very slowly radiated away in the course of very long evolution (which is not relevant for experiments).

In the opposite case $\Delta\varphi=0$, the solitons attract each other, which is similar to what is known for the NLS solitons. As shown in Fig. 3(b), they temporarily merge into a single pulse, which later splits into two moving solitons with small internal vibrations. In this case, a conspicuous breaking of the initial symmetry between the two solitons is observed (special care has been taken to check that it is not an artifact produced by the numerical scheme). A plausible explanation is that the lump produced by the strong temporary overlapping of the initially attracting solitons [see Fig. 3(b)] is unstable against symmetry-breaking perturbations, the breaking being incomplete since the solitons separate quickly

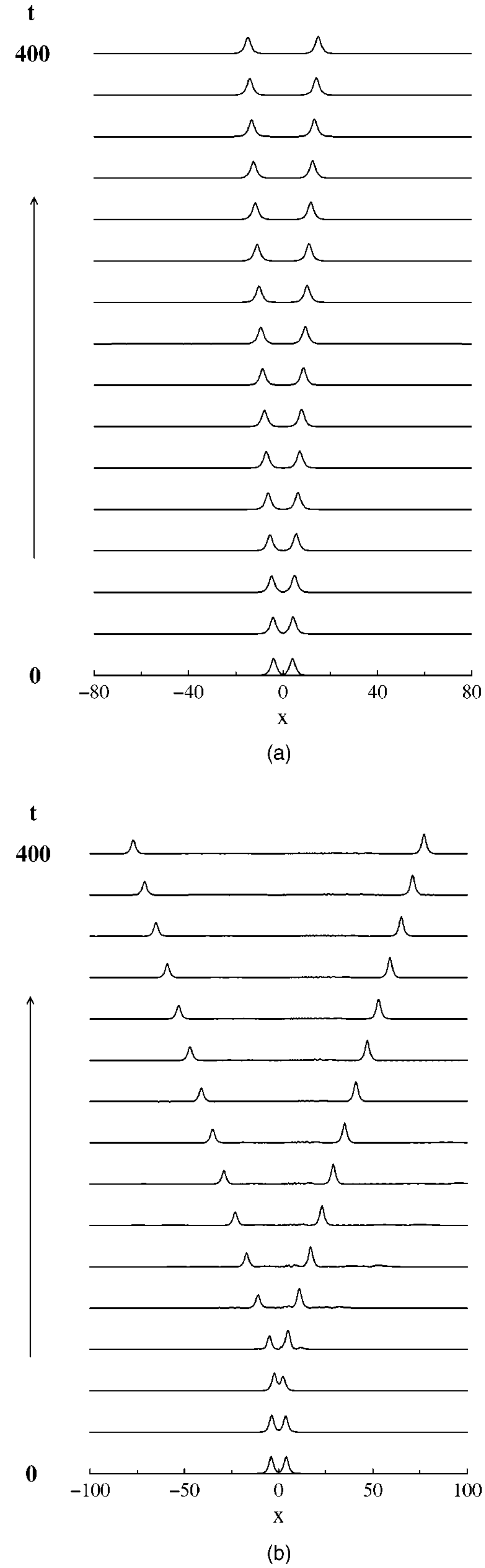

FIG. 3. Interaction of two identical solitons with $\theta=\pi/3$, placed initially at a distance 8 with two different values of the initial phase difference between the solitons: (a) $\Delta\varphi=\pi$ (repulsion); (b) $\Delta\varphi=0$ (attraction). The other parameters are $\kappa=1$, $\lambda=\mu=0$, and the evolution time is 400.

enough. This conjecture seems natural, as it is well known that various multisoliton states in the NLS equation are strongly unstable in the case of attraction [16], but, of course, much more extensive simulations are necessary to check it in detail.

This partial symmetry breaking can be characterized by the final/initial energy ratios for the two solitons shown in Fig. 3(b), which are found to be $E_f/E_i = 0.892$ and 0.864 for the left and right solitons, respectively. In this case, a considerable share of the initial energy, ≈12%, is lost into radiation. The final velocities of the solitons are $W_\pm = \pm 0.22$, i.e., there is no tangible symmetry breaking in terms of the velocities. Note that $|W_\pm|$ are much larger in this case than in the case $\Delta\varphi = \pi$.

In the case $\Delta\varphi = 0$, there is another noteworthy aspect of the symmetry breaking: the final solitons demonstrate an *internal* asymmetry, characterized by the ratios of their partial energies,

$$\varepsilon_- = \int_{-\infty}^{+\infty} |u_-|^2 dx \Bigg/ \int_{-\infty}^{+\infty} |v_-|^2 dx,$$

$$\varepsilon_+ = \int_{-\infty}^{+\infty} |v_+|^2 dx \Bigg/ \int_{-\infty}^{+\infty} |u_+|^2 dx,$$

where the positive (negative) subscript pertains to the right (left) soliton. For the solitons shown in Fig. 3(b), this ratio takes values $\varepsilon_- = 0.470$ and $\varepsilon_+ = 0.465$. In accord with these values, the final solitons, being intrinsically asymmetric, are, to a good approximation, mirror images of each other.

We have also simulated the interaction of solitons when the initial phase difference is $\Delta\varphi = \pi/2$. In this case the solitons repel each other, about 0.7% of the energy is lost into radiation, and the symmetry breaking is much more conspicuous, with the final velocities being $W_- = -0.023$ and $W_+ = 0.019$. The stronger symmetry breaking in this case can be easily understood, as the symmetry of the initial configuration, which was taken as $u_{\rm sol}(x - \frac{1}{2}x_0) + i u_{\rm sol}(x + \frac{1}{2}x_0)$, x_0 being the initial separation between the solitons, is not compatible with Eqs. (1)–(4) and is therefore broken upon propagation in a straightforward way.

V. CONCLUSION

In this paper, we have introduced a model consisting of two linearly coupled cores, one having the Kerr nonlinearity and the other being linear. A Bragg grating is written on the nonlinear core, while the linear one may or may not be equipped with a grating. The model allows us to considerably extend the previously studied family of Bragg-grating solitons. Exact solutions were found for zero-velocity solitons in a limiting case when the group-velocity terms are absent in the equations for the linear core, while in the general case solitons were found numerically. The main issue is their stability. We have found a nontrivial stability limit for them in terms of an internal parameter of the soliton family. Depending on the frequency ω, the solitons may remain stable up to quite large values of the group velocity in the linear core. This strongly suggests that stable solitons can indeed be generated experimentally in dual-core systems, with the cores made of the same material. The vast stability region for the zero-velocity solitons in the dual-core model, found in this work, suggests the possibility of looking for the corresponding localized states experimentally with fully trapped light. Interactions of initially separated solitons were investigated also, showing a considerable spontaneous symmetry breaking in the case when the solitons attract each other, which may be the result of a natural instability against symmetry-breaking perturbations. The interaction always results in the appearance of stable moving solitons.

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[1] C. M. de Sterke and J. E. Sipe, in *Progress in Optics*, edited by E. Wolf, (Elsevier, Amsterdam, 1994), Vol. 33, Chap. III.

[2] D. Christodoulides and R. I. Joseph, Phys. Rev. Lett. **62**, 1746 (1989); A. Aceves and S. Wabnitz, Phys. Lett. A **141**, 37 (1989).

[3] B. J. Eggleton, R. E. Slusher, C. M. de Sterke, P. A. Krug, and J. E. Sipe, Phys. Rev. Lett. **76**, 1627 (1996).

[4] B. J. Eggleton, C. M. de Sterke, and R. E. Slusher, J. Opt. Soc. Am. B **14**, 2980 (1997); D. Taverner, N. G. R. Broderick, D. J. Richardson, R. I. Laming, and M. Ibsen, Opt. Lett. **23**, 328 (1998).

[5] W. C. K. Mak, B. A. Malomed, and P. L. Chu, J. Opt. Soc. Am. B **15**, 1685 (1998).

[6] J. Atai and Y. Chen, J. Appl. Phys. **72**, 24 (1992); J. Atai and Y. Chen, IEEE J. Quantum Electron. **QE29**, 242 (1993); G. Assanto, A. Laureti-Palma, C. Sibilia, and M. Bertolotti, Opt. Commun. **119**, 599 (1994).

[7] B. A. Malomed, I. Skinner, P. L. Chu, and G. D. Peng, Phys. Rev. E **53**, 4084 (1996); W. C. K. Mak, B. A. Malomed, and P. L. Chu, Opt. Commun. **154**, 145 (1998); D. J. Kaup and B. A. Malomed, J. Opt. Soc. Am. B **15**, 2838 (1998).

[8] M. Romagnoli, S. Trillo, and S. Wabnitz, Opt. Quantum Electron. **24**, S1237 (1992).

[9] S. Trillo, Opt. Lett. **21**, 1732 (1996); W. C. K. Mak, B. A. Malomed, and P. L. Chu, Phys. Rev. E **58**, 6708 (1998).

[10] L. V. Hau, S. E. Harris, Z. Dutton, and C. H. Behroozi, Nature (London) **397**, 594 (1999).

[11] B. A. Malomed and R. S. Tasgal, Phys. Rev. E **49**, 5787 (1994).

[12] I. V. Barashenkov, D. E. Pelinovsky, and E. V. Zemlyanaya, Phys. Rev. Lett. **80**, 5117 (1998); A. De Rossi, C. Conti, and S. Trillo, *ibid.* **81**, 85 (1998).

[13] J. Atai and B. A. Malomed, Phys. Lett. A **246**, 412 (1998).

[14] M. Quiroga-Teixeiro and H. Michinel, J. Opt. Soc. Am. B **14**, 2004 (1997); D. Mihalache, D. Mazilu, L.-C. Crasovan, B. A. Malomed, and F. Lederer, Phys. Rev. E **61**, 7142 (2000).

[15] W. S. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes in Fortran* (Cambridge University Press, New York, 1996).

[16] Y. S. Kivshar and B. A. Malomed, Rev. Mod. Phys. **61**, 763 (1989).