Stochastic resonance and noise-enhanced order with spatiotemporal periodic signal

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Stochastic resonance is investigated in a chain of coupled threshold elements driven by independent noises and a plane traveling wave. Both stochastic resonance in an individual element embedded in the chain, characterized by a maximum of the signal-to-noise ratio for nonzero noise intensity, and stochastic resonance with spatiotemporal signal, characterized by a maximum of the spatiotemporal input-output correlation function, are observed. For a wide range of wavelengths of the plane wave an optimum value of coupling exists for which both kinds of stochastic resonance are most pronounced, i.e., the phenomenon of array enhanced stochastic resonance is observed. For large wavelengths the enhancement of stochastic resonance coincides with a maximum of spatiotemporal synchronization among elements with the same phase of the periodic signal at inputs. This synchronization is a manifestation of spatiotemporal order induced in the system by the cooperative influence of noise and periodic signal.

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I. INTRODUCTION

Stochastic resonance (SR) [1] is a phenomenon in which noise plays a constructive role by increasing the degree of periodicity of a properly defined output signal in a system driven by a combination of a periodic signal and noise (for a review, see [2,3]). A commonly used measure of SR is the signal-to-noise ratio (SNR), evaluated from the output power spectral density, which shows a maximum as a function of the input noise intensity. The models of SR most often studied are based on bistable dynamical systems [4–6] and both dynamical [7–9] and nondynamical [10–13] threshold-crossing systems. For a few years SR has been investigated in spatially extended systems also, under the general name of spatiotemporal SR (for a review, see [14]), e.g., in chains of diffusively coupled stochastic bistable oscillators [15,16], coupled map lattices [17], systems with solitons [18–20], reaction-diffusion models [21], pattern-forming systems [22], and the Ising model [23–25]. In the case of coupled oscillators it was found that an optimum value of coupling and optimum noise strength exist such that the maximum of the SNR in every oscillator is most significantly enhanced over that in an uncoupled oscillator. This phenomenon is called array enhanced SR [15] and it occurs because all oscillators then show maximum spatiotemporal synchronization with the input periodic signal and among themselves. A similar enhancement of SR due to proper ferromagnetic coupling was also observed in the Ising model [24].

A common feature of the above-mentioned spatiotemporal models is that the noise can be uncorrelated in both space and time, but the periodic signal oscillates only in time and is uniform in space. Only recently has the spatial counterpart of SR with the signal constant in time and periodic in space been demonstrated in the one-dimensional Ising model [26] and in the one- and two-dimensional ϕ^4 model with advection [27]. In this case the SNR is evaluated from the structure factor and exhibits a maximum for nonzero noise intensity [27]. Moreover, in Ref. [27] it was pointed out that the results for spatial SR can be generalized to the case of a spatiotemporal signal, e.g., a plane traveling wave in a bistable medium. SR with a signal like this was also investigated by us in a small system of two coupled threshold elements fed by periodic signals with identical amplitudes and frequencies, but shifted in phase [28,29]. In this case the enhancement of the SNR due to proper coupling and the presence of a maximum of the spatiotemporal input-output correlation function for nonzero input noise intensity were demonstrated for almost any phase shift between the two signals. The phenomenon was called SR with spatiotemporal signal. These examples revealed an unusual feature of SR, namely, that noise can increase not only temporal order in the output time series of certain systems, but also spatial order in spatially extended systems.

In this paper we extend our previous study of SR with spatiotemporal signal to the case of a chain of coupled threshold elements. Such elements are known to exhibit SR [10–13] and can be used for qualitative simulations of SR in biological neuron models [13]. The spatiotemporal periodic signal is a plane traveling wave, and the elements are also driven by independent noise sources. The study of this kind of SR seems natural since signals at two distant points can be shifted in phase due to the finite velocity of the signal. Our investigations are based on numerical simulations and simple theoretical considerations. First we show, for a wide range of wavelengths of the signal, the effect of array enhanced SR, i.e., the enhancement of SR in an individual element embedded in the chain due to proper coupling. Second, we demonstrate SR with spatiotemporal signal characterized by the maximum of the spatiotemporal input-output correlation function for nonzero input noise intensity. Third, we present evidence for an ordering effect of noise on the spatiotemporal structure of the chain characterized by a maximum of a suitably defined spatial correlation function for nonzero noise intensity. Finally, we also explore the connection between the spatiotemporal order induced by noise and the enhancement of the SNR due to coupling.

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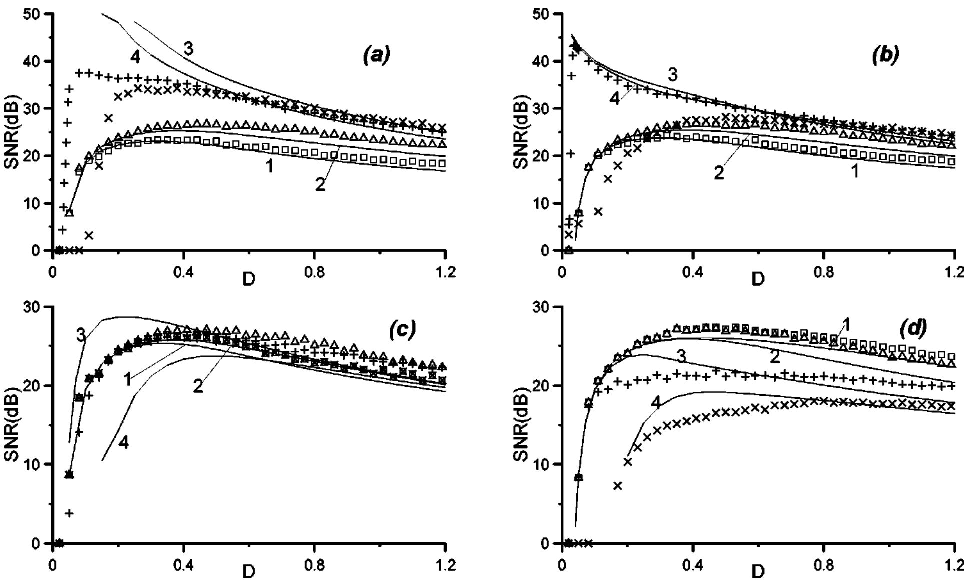

FIG. 1. The SNR vs D for various wave vectors k and coupling constants w, and for the length of the chain $N=128$ and period $T_s=128$: (a) $k=0$, (b) $k=\pi/4$, (c) $k=\pi/2$, (d) $k=\pi$. Numerical results are shown with symbols: (□) $w=-1.5$, (△) $w=-0.1$, (+) $w=1.0$, (×) $w=1.5$. Theoretical results are shown with numbered solid lines: (1) $w=-1.5$, (2) $w=-0.1$, (3) $w=1.0$, (4) $w=1.5$.

II. SYSTEM AND METHODS OF ANALYSIS

We investigate a chain of N coupled threshold elements denoted as i, $i=0,1,2,\ldots,N-1$, with two-state output 0 or 1. The coupling is typical of artificial neural networks, symmetric and constrained to nearest neighbors. The time steps $n=0,1,2,\ldots$ are discrete, which significantly speeds up numerical simulations while retaining intact the basic features of SR [9,17]. The chain is driven by a plane traveling wave with amplitude A, frequency ω_s, period $T_s=2\pi/\omega_s$, wave vector k, and wavelength $\lambda=2\pi/k$. In addition, the elements are driven by independent white Gaussian noises $\eta_n^{(i)}$ with variance D. The system dynamics is given by

$$x_{n+1}^{(i)}=\Theta\bigg[A\sin(\omega_s n-ki+\phi)+\eta_n^{(i)}+\frac{w}{2}(x_n^{(i-1)}+x_n^{(i+1)})-b\bigg], \tag{1}$$

$$x_n^{(0)}=x_n^{(N-1)},$$

where $x_n^{(i)}$ is the output of the element i at time n, $\Theta(\cdot)$ is the Heaviside step function, ϕ is the initial phase, w is the coupling strength, and b is the threshold. The periodic signal is assumed as subthreshold with $A<b$, and the length of the chain N is an integer multiple of the wavelength, i.e., $N=N'\lambda$.

As a measure of SR in an individual element we take the SNR ($\mathcal{R}$) in the middle element of the chain, obtained from the power spectral density $S(\omega)$ of its output signal and defined as $\mathcal{R}=10\log_{10}[S_P(\omega_s)/S_N(\omega_s)]$. Here $S_P(\omega_s)=S(\omega_s)-S_N(\omega_s)$ is the height of the peak at $\omega=\omega_s$ and $S_N(\omega_s)$ is the noise background in the vicinity of ω_s. In our numerical simulations the SNR is normalized to the frequency bandwidth $\Delta f=2^{-12}$ Hz.

As a measure of SR with spatiotemporal signal we take the correlation function between the spatiotemporal periodic input signal and the output signal,

$$C=\frac{1}{N}\sum_{i=0}^{N-1}C^{(i)},\quad C^{(i)}=\frac{\langle x_n^{(i)}A\sin(\omega_s n-ki+\phi)\rangle}{\sqrt{(A^2/2)[\langle(x_n^{(i)})^2\rangle-\langle x_n^{(i)}\rangle^2]}}, \tag{2}$$

where the angular brackets denote the time average. The functions $C^{(i)}$ are obtained under the assumption that the mean value of the periodic signal at the input of every element is zero and the mean value of the square of this signal is $A^2/2$.

Further, we introduce the idea of spatiotemporal noise-induced order as a concept concerning the varying in space and time of the chain. As a measure of this order we take the mutual correlation function between elements, averaged over all pairs of elements with the same phase of the periodic signal at inputs,

$$C_{\mathrm{mut}}=\frac{1}{NN'}\sum_{\{i,j\}}C_{\mathrm{mut}}^{(i,j)},$$
$$C_{\mathrm{mut}}^{(i,j)}=\frac{\langle x_n^{(i)}x_n^{(j)}\rangle}{\sqrt{[\langle(x_n^{(i)})^2\rangle-\langle x_n^{(i)}\rangle^2][\langle(x_n^{(j)})^2\rangle-\langle x_n^{(j)}\rangle^2]}}, \tag{3}$$

where in the case $k\neq0$ the sum extends over all pairs of elements such that $|i-j|=m\lambda$, $m=0,1,2,\ldots,N'$, and in the case $k=0$ over all pairs. By definition C_{mut} is a measure of spatiotemporal synchronization among elements with the same phase of the periodic signal at inputs. The maximum of the spatiotemporal noise-induced order coincides with the maximum of this correlation function for nonzero noise. This emphasizes that the increase of order is a cooperative effect of noise and the spatiotemporal periodic signal. In the most ordered state, defined in such a way, the character of the plane traveling wave is best reflected in the activity of the elements of the chain.

III. SIMPLIFIED ADIABATIC THEORY

In this section we present a simple extension of the theory of SR in threshold elements with discrete time [12] to the case of a chain of coupled elements. The method of dealing

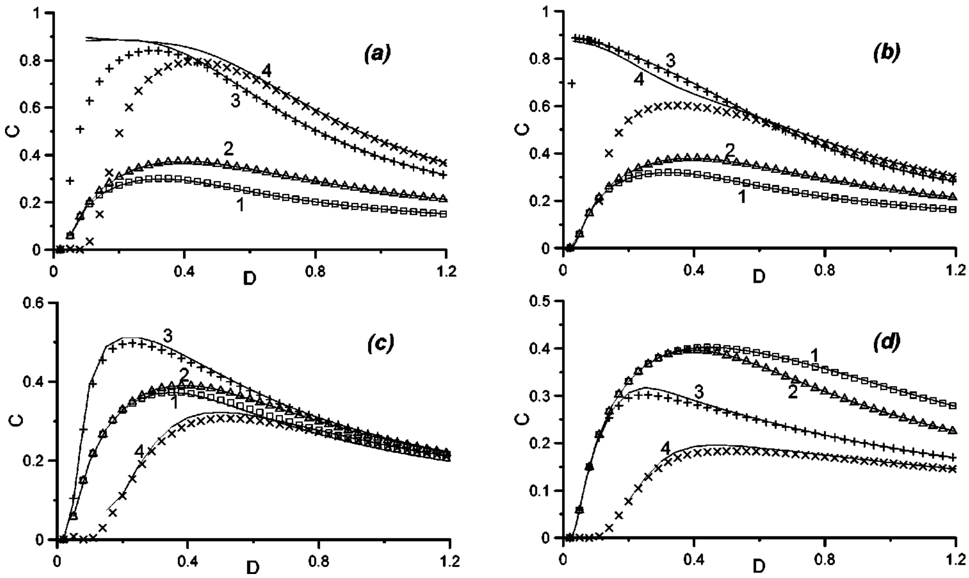

FIG. 2. C vs D for various wave vectors k and coupling constants w, and for the length of the chain $N=128$ and period $T_s=128$: (a) $k=0$, (b) $k=\pi/4$, (c) $k=\pi/2$, (d) $k=\pi$. Numerical results are shown with symbols: (□) $w=-1.5$, (△) $w=-0.1$, (+) $w=1.0$, (×) $w=1.5$. Theoretical results are shown with numbered solid lines: (1) $w=-1.5$, (2) $w=-0.1$, (3) $w=1.0$, (4) $w=1.5$.

with this problem is similar to that used previously for two coupled elements [28,29]. The quantities SNR and C can be evaluated provided the time-dependent probability that $x_n^{(i)}=1$, denoted as $\Pr(x_n^{(i)}=1)$, is known. This probability is obtained here under certain simplifying assumptions.

The starting point is the equation for the complete probability that $x_n^{(i)}=1$,

$$\begin{aligned}\Pr(x_{n+1}^{(i)}=1)=&\Pr(x_{n+1}^{(i)}=1\,|\,x_n^{(i-1)}=1,x_n^{(i+1)}=1)\\&\times\Pr(x_n^{(i-1)}=1,x_n^{(i+1)}=1)+\cdots\\&+\Pr(x_{n+1}^{(i)}=1\,|\,x_n^{(i-1)}=0,x_n^{(i+1)}=0)\\&\times\Pr(x_n^{(i-1)}=0,x_n^{(i+1)}=0).\end{aligned}\tag{4}$$

Henceforth, for a given element i, the following notation will be used: $p(n)=\Pr(x_n^{(i)}=1)$ and $\Pi_{\beta,\gamma}(n)=\Pr(x_{n+1}^{(i)}=1\,|\,x_n^{(i-1)}=\beta,x_n^{(i+1)}=\gamma)$, where $\beta,\gamma\in\{0,1\}$. The conditional probabilities can easily be evaluated analytically:

$$\begin{aligned}\Pi_{\beta,\gamma}(n)=&0.5(1-\operatorname{erf}\{[b-(\delta_{\beta,1}+\delta_{\gamma,1})w/2\\&-A\sin(\omega_s n-ki+\phi)]/\sqrt{2D^2}\}),\end{aligned}\tag{5}$$

where $\delta_{\xi,\zeta}$ is the Kronecker delta.

In order to solve Eq. (4) for $p(n)$ the following assumptions are made. First, only the adiabatic limit $\omega_s\to 0$ is considered. Then it is possible to assume on the left-hand side (LHS) of Eq. (4) that $p(n+1)=p(n)$. Since the input signal is periodic in both space and time it is also possible to assume that the probabilities to have 1 as the output for the elements $i-1$ and $i+1$ are given by $p(n+k/\omega_s)$ and $p(n-k/\omega_s)$, respectively. Second, to obtain the joint probabilities on the RHS of Eq. (4) the approximation that the random variables $x_n^{(i-1)}$ and $x_n^{(i+1)}$ are independent is implemented; thus, e.g., $\Pr(x_n^{(i-1)}=1,x_n^{(i+1)}=1)=p(n+k/\omega_s)p(n-k/\omega_s)$. The latter assumption is valid only in the limit of small w. Taking into account that $\Pr(x_n^{(i)}=0)=1-p(n)$, Eq. (4) can be rewritten as

$$\begin{aligned}p(n)=\sum_{\beta,\gamma}\Pi_{\beta,\gamma}(n)&[\delta_{\beta,0}-(-1)^{\beta}p(n+k/\omega_s)]\\&\times[\delta_{\gamma,0}-(-1)^{\gamma}p(n-k/\omega_s)].\end{aligned}\tag{6}$$

Equation (6) is a nonlinear difference equation which, to our knowledge, cannot be solved analytically for $p(n)$. However, numerical solution is possible using the iterative method. At the first iteration an approximate solution for $p(n)$ is assumed as for an uncoupled element,

$$p(n)=0.5(1-\operatorname{erf}\{[b-A\sin(\omega_s n-ki+\phi)]/\sqrt{2D^2}\}).\tag{7}$$

Next, this solution is inserted on the RHS of Eq. (6) and the approximate solution in the second iteration is obtained. This procedure is repeated up to a moment when the consecutive iterated solutions do not change significantly. This usually requires several iterations, apart from the limit of very small noise intensity D in which the convergence of the method is very poor. Thus the results of this procedure are not reliable for $D\to 0$ and they are not discussed in the following.

According to Ref. [12] the SNR can be evaluated from $p(n)$ as

$$\mathcal{R}=10\log_{10}\frac{|P_1|^2}{(\overline{p}-\overline{p^2})\Delta f}\tag{8}$$

where P_1 is the first Fourier coefficient of $p(n)$,

$$P_1=T_s^{-1}\sum_{n=0}^{T_s-1}p(n)\exp(-i\omega_s n),\tag{9}$$

and the overbar denotes the time average over T_s. The resulting SNR is independent of i, which reflects the fact that all elements are equivalent due to periodic boundary conditions and the assumption that an integer number of wavelengths is contained inside the chain. However, it should be pointed out that Eq. (8) is exact only in the case of an uncoupled threshold element driven by a sum of a periodic signal and white noise [12]. Thus in our case Eq. (8) is only

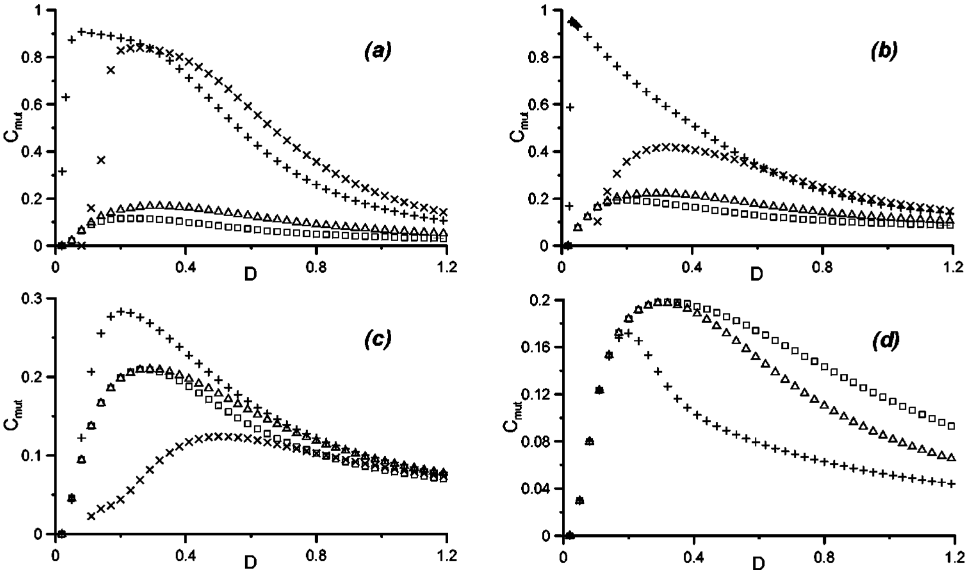

FIG. 3. Numerical curves C_{mut} vs D for various wave vectors k and coupling constants w, and for the length of the chain $N=128$ and period $T_s=128$: (a) $k=0$, (b) $k=\pi/4$, (c) $k=\pi/2$, (d) $k=\pi$; (□) $w=-1.5$, (△) $w=-0.1$, (+) $w=1.0$, (×) $w=1.5$.

approximate since the total random input to element i in Eq. (1) consists of a sum of white noise $\eta_n^{(i)}$ and nonwhite noise $w(x_n^{(i-1)}+x_n^{(i+1)})/2$, which is, moreover, correlated with $x_n^{(i)}$.

The correlation functions $C^{(i)}$ can also be evaluated using $p(n)$ since

$$\langle x_n^{(i)}\rangle=\langle (x_n^{(i)})^2\rangle=\overline{p},$$
$$\langle x_n^{(i)}A\sin(\omega_s n-ki+\phi)\rangle=\overline{A\sin(\omega_s n-ki+\phi)p(n)}. \tag{10}$$

This result also does not depend on i and thus one gets $C=C^{(i)}$.

Equations (8) and (10) enable us to evaluate the SNR and C semianalytically. It should be recollected that due to the assumptions made these equations are exact only in the limit $\omega_s\to 0$, $w\to 0$, and not too small D.

IV. RESULTS AND DISCUSSION

A. Stochastic resonance in an individual element

In this section SR in an individual element embedded in the chain characterized by the SNR is discussed. The numerical and theoretical results obtained for a chain with $N=128$ are summarized in Fig. 1 for various wave vectors k and couplings w. The values of the SNR were obtained for the middle element of the chain with $i=63$.

First, the numerical results are discussed. In general it can be seen from Fig. 1 that if $0\leqslant k\leqslant\pi/4$ then positive coupling increases the SNR and negative coupling decreases it [Fig. 1(a,b)] and if $k=\pi$ then positive coupling decreases the SNR and negative coupling increases it [Fig. 1(d)]. This is because $w>0$ increases the probability of two coupled elements having the same outputs and $w<0$ increases the probability of having opposite outputs. For example, let us consider the case $0\leqslant k\leqslant\pi/4$ in which the periodic signals at input of neighboring elements has the same sign during most of the period T_s. Then it is clear that if $w>0$ two coupled elements will mutually increase their probabilities to have 1 at the output while the periodic signal at the inputs of both elements is positive. Hence the periodic component of the output signal of an individual element embedded in the chain will be amplified and the SNR will increase due to coupling. Similar arguments apply to the other above-mentioned cases. A limiting case is the one with $k=\pi/2$ for which, in fact, the dependence of the SNR on w is weak [Fig. 1(c)]. It can also be seen that for $0\leqslant k\leqslant\pi/4$ an optimum value of coupling $w_{\mathrm{opt}}>0$ exists for which the maximum of the SNR reaches its highest possible value, i.e., SR in an individual element is enhanced due to proper coupling [Fig. 1(a,b)]. This is in analogy with array enhanced SR in systems with periodic signal uniform in space [15,16,24]. For this optimum coupling the maximum of the SNR occurs for very small D so that the SR effect almost disappears. The fact that the maximum is nevertheless retained, i.e., that the SNR decreases to zero for $D\to 0$, is caused by the deterministic dynamics of the chain, which becomes important for $w\approx 1$ in the limit of negligible noise. For $k=\pi$ the SNR increases for $w\to-\infty$; however, the increase of the maximum of the SNR is very small [Fig. 1(d)].

Comparison between the numerical and theoretical results shows that the theory of Sec. III predicts the dependence of the SNR on D quite well in the following cases: for large D in the whole range of k and w, for $w<0$ in the whole range of k and D, and for $k=\pi$ in the whole range of D and w. Moreover, the increase of the SNR for $0\leqslant k\leqslant\pi/4$ and $w>0$ or $k=\pi$ and $w<0$, and the decrease of the SNR for $0\leqslant k\leqslant\pi/4$ and $w<0$ or $k=\pi$ and $w>0$ in comparison with the SNR in an uncoupled element, are predicted correctly for any k and D. This is so because in Sec. III it was assumed that the probabilities of neighboring elements having 1 at the output are shifted in phase by k; thus the theory takes into account the above-discussed mechanism of amplification of the periodic component of the output signal due to coupling. However, the theoretical curves of the SNR have a tendency to diverge in the limit $D\to 0$ for $0<k\leqslant\pi/4$ and large positive coupling $w\geqslant 1$ [Fig. 1(a,b)] (up to the accuracy of the numerical procedure used to obtain them, discussed in Sec. III). This means that although, e.g., the increase of the SNR for $k=0$ and $w=1$ is predicted correctly, the effect of array enhanced SR is not predicted; instead, monotonic decrease of the SNR and thus disappearance of SR can be expected [Fig.

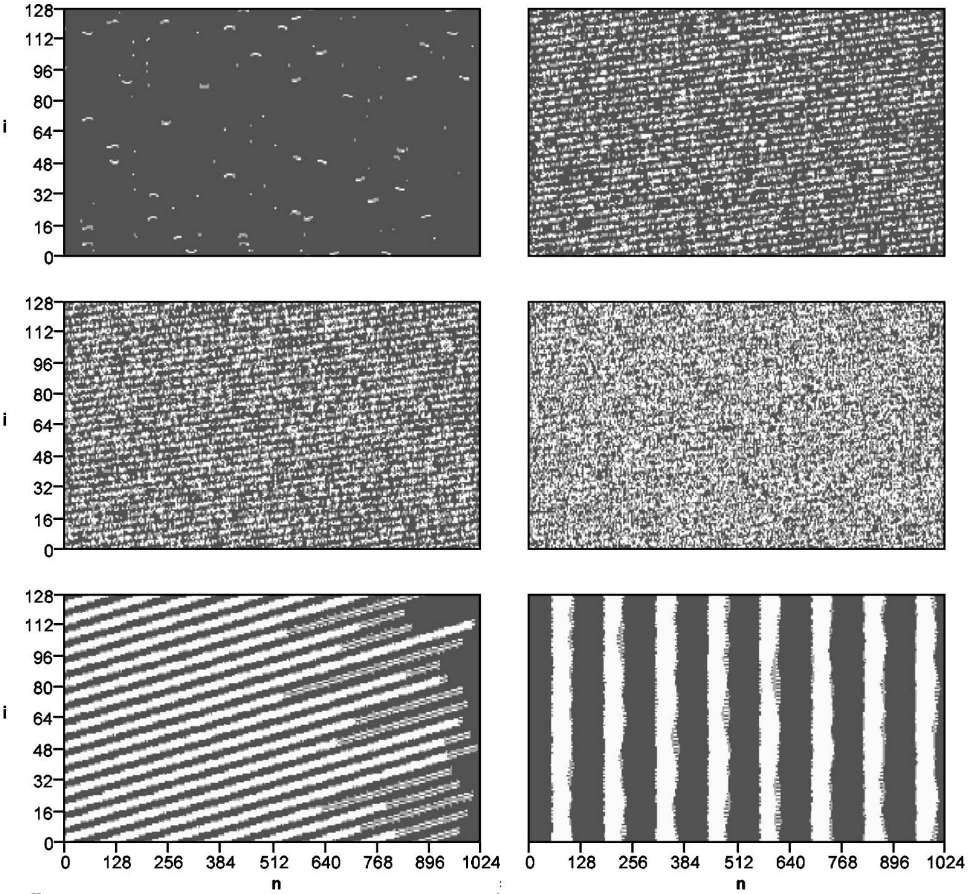

FIG. 4. Spatiotemporal diagrams for a chain of $N=128$ elements driven by a plane traveling wave with $T_s=128$ and for various w, k, and D. From left to right and from top to bottom: $w=1.0$, $k=\pi/2$, $D=0.05$ (small SNR and C), $w=1.0$, $k=\pi/2$, $D=0.23$ (maximum C), $w=1.0$, $k=\pi/2$, $D=0.37$ (maximum SNR), $w=1.0$, $k=\pi/2$, $D=1.20$ (small SNR and C), $w=1.2$, $k=\pi/4$, $D=0.05$ (maximum SNR) $w=1.0$, $k=0$, $D=0.05$ (maximum SNR). White points correspond to elements with output 1, gray points to elements with output 0. If the SNR or C is at a maximum the ordering effect of nonzero noise can be seen, i.e., the shape of the plane traveling wave is best visible.

1(a)]. This discrepancy between the numerical and theoretical results occurs since in the limit of vanishing noise the system dynamics becomes purely deterministic, which is not taken into consideration in the theory of Sec. III (e.g., in the case of deterministic dynamics the variables $x_n^{(i-1)}$ and $x_n^{(i+1)}$ cannot be approximately treated as independent variables).

B. Stochastic resonance with spatiotemporal signal

In this section SR with spatiotemporal signal characterized by the correlation function C is discussed. The numerical and theoretical results are summarized in Fig. 2 in a manner analogous to that in Fig. 1.

The results of numerical simulations show that SR with spatiotemporal signal can be observed for any k, i.e., the curves of C vs D show maxima. Also, for $0\leqslant k\leqslant\pi/2$ an optimum value of coupling exists for which the maximum of the correlation function reaches its highest possible value. Hence SR with spatiotemporal signal can also be enhanced due to proper coupling. The dependence of C on D and w resembles the dependence of the SNR, e.g., for $0\leqslant k<\pi/2$ the values of w_{opt} are the same for both kinds of SR. There are, however, several notable differences. First, SR with spatiotemporal signal can be enhanced due to coupling although SR in an individual element is not enhanced [cf. Fig. 1(c) and Fig. 2(c) for $k=\pi/2$]. Second, the location of the maxima of the curves of SNR vs D and C vs D for optimum coupling need not coincide [cf. Fig. 1(a) and Fig. 2(a)]. This is because the SNR and the correlation function (2) are sensitive to different properties of the output signal.

The theoretical results for C in the whole range of D, w, and k usually fit the numerical ones better than in the case of the SNR. This is because in the evaluation of C from Eq. (10) only the approximations necessary for the derivation of Eq. (6) are important, while in the case of the SNR Eq. (8) requires additional assumptions. Nevertheless, the theoretical curves C again do not show maxima for $0<k\leqslant\pi/4$ and $w\approx1$. Hence, as in the case of SR in an individual element, the increase of C is predicted correctly for such values of coupling, but the enhancement of SR (with maximum of C for nonzero D) is not predicted.

C. Spatiotemporal noise-induced order

In this section the spatiotemporal noise-induced order characterized by the correlation function C_{mut} is discussed. The numerical curves C_{mut} vs D are shown in Fig. 3 for the same values of k and w as in Fig. 1 and Fig. 2. It can be seen that these curves for all k exhibit maxima for nonzero noise intensity. The presence of these maxima provides evidence for the noise-induced order that emerges in the system due to the cooperative influence of the spatiotemporal subthreshold periodic signal and noise. This order results in the maximum spatiotemporal synchronization among elements with the same phase of the periodic signal at inputs. For $0\leqslant k\leqslant\pi/2$ an optimum value of coupling exists for which the maximum of C_{mut} reaches the highest possible value, while for $k=\pi$ the values of C_{mut} increase for $w\to0$, but with no visible increase of the maximum. For $0\leqslant k\leqslant\pi/4$ the value of this optimum coupling and the location of the maximum of C_{mut} coincide with those for which SR in an individual element is most pronounced [cf. Fig. 3(a,b) and Fig. 1(a,b)]. Thus, for long-wave periodic signals, the situation is analogous to that in the case of array enhanced SR with a periodic signal uniform in space: the maximum spatiotemporal synchronization corresponds to the maximum enhancement of SR in an individual element. However, for short-wave periodic signals, e.g., with $k=\pi/2$, a maximum spatiotemporal synchronization occurs [Fig. 3(c)] although there is no enhancement of SR in an individual element [Fig. 1(c)]; thus, these two phenomena are independent. In this case the maximum spatiotemporal synchronization corresponds rather to the best enhancement of SR with spatiotemporal signal due to coupling [Fig. 2(c)].

In a more spectacular way the noise-induced order can be viewed using spatiotemporal diagrams (Fig. 4). Even in the case $k=\pi/2$, in which SR is rather weak, for moderate noise that maximizes the SNR or C the shape of the traveling wave is easily visible while for small and large noise it is distorted. In the case $k \leqslant \pi/4$ the ordering effect of noise is much more pronounced. It follows from Fig. 4 that in the state of maximum spatiotemporal order induced by nonzero noise the character of the plane traveling wave is best reflected in the activity of the elements of the chain.

V. SUMMARY AND CONCLUSIONS

In this paper we investigated SR in the case of a spatiotemporal periodic signal, i.e., a signal that is periodic in both time and space. As a model for this phenomenon we studied a chain of coupled threshold elements driven by a plane traveling wave and independent noises. Two kinds of SR were studied: SR in an individual element embedded in the chain and SR with a spatiotemporal signal, characterized by a local and a global measure of periodicity of the output signal, respectively. It was shown that both kinds of SR can be enhanced due to proper coupling for a wide range of wavelengths of the periodic signal. This effect is a counterpart of the array enhanced SR effect for spatiotemporal periodic signals. It was also shown that for long-wave signals the enhancement of SR in an individual element is related to maximum spatiotemporal noise-induced order in the system, i.e., maximum synchronization among elements with the same phase of the periodic signal at input. This relationship disappears for short-wave periodic signals, i.e., these two phenomena occur independently and for different noise intensities.

It is known that noise can increase the periodicity of the temporal response of spatiotemporal systems to perturbations spatially uniform and periodic in time [15,16], or the periodicity of spatial response to perturbations constant in time and periodic in space [26,27]. In this paper it has been demonstrated that this is also the case for signals periodic in both space and time, in accordance with suggestions in Refs. [27–29]. It was also shown that noise can play a constructive role by increasing spatiotemporal order in spatially extended systems driven by weak periodic spatiotemporal signals. This order means that the character of the plane traveling wave is best reflected in the activity of the elements of the chain.

Spatiotemporal SR is usually investigated in spatially extended bistable systems [15–17,23–27]. Our results show that chains of threshold elements can be used for this purpose also. It follows from the results of this paper and of Ref. [27] that SR with a spatiotemporal signal should be a ubiquitous phenomenon and thus investigations in, e.g., bistable or chaotic spatiotemporal systems should further clarify its properties.

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