On-off intermittency at the onset of the ion-acoustic instability in a laboratory plasma

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(Received 12 April 2000)

The ion-ion beam instability is experimentally studied just above threshold in a laboratory double-plasma device. Intermittent bursts of unstable waves are recorded in the target plasma and the distribution of the recurrence times of the bursts is estimated. At the onset of instability, the measurements are in agreement with the expected evolution deduced from a theoretical model that combines the normal form of a supercritical Hopf-Andronov bifurcation and the parametric noisy deviations from threshold typical of on-off intermittency. In both the experiment and the model, the distribution of the recurrence times of the bursts decays as an inverse power law and the evolution of the mean laminar length when the control parameter is increased beyond threshold exhibits a power law of exponent -1.

PACS number(s): 52.35.Py, 52.35.Ra

I. INTRODUCTION

In this paper, we focus on the intermittent regime of the high frequency instability triggered by the injection of an ion beam into a low density laboratory plasma. Bursts of unstable waves are recorded at the onset of the instability. A statistical analysis of the recurrence time of these bursts is performed.

The noise level of this laboratory plasma has a standard value. Up to now, the existing studies in this domain have not taken this parameter into account. Our point here is that the existence of broadband fluctuations of the plasma density is of major importance for the nonlinear evolution of the dynamical system, and in particular for the properties of recurrence times of the bursts. In this specific case, the control parameter is driven by noise near a bifurcation, which is a typical feature of on-off intermittency. This mechanism for intermittent behavior is significantly different from the Pomeau-Manneville and crisis-induced types [1,2].

The purpose of this investigation is also to shed light on the dynamical origin of the statistical properties of the physical system under study. That is why we propose a simple model, which correctly reproduces the observed statistical properties of the experimental system.

The outline of the paper is as follows. In Sec. II, we present the physical system under consideration. In Sec. III, the various types of intermittency studied in the literature are reviewed with special emphasis on on-off intermittency. After describing the experimental setup and presenting the results of the statistical analysis in Secs. IV and V, a dynamical model for the supercritical Hopf bifurcation in the presence of noise is introduced in Sec. VI. Finally, in Sec. VI some conclusions are summarized.

II. ION-ACOUSTIC INSTABILITY

The natural low frequency mode of propagation in a nonmagnetized plasma is the ion-acoustic mode. The phase velocity of this electrostatic mode is close to the ion sound speed $c_s = \sqrt{k_B T_e / M}$, where k_B is Boltzmann's constant, T_e the electron temperature, and M the ion mass. The system is drastically changed when an ion beam is injected into the plasma. Due to the Doppler effect in the laboratory frame, two different modes propagate. A slow ion beam mode and a fast ion beam mode are present. When the ion beam velocity is close to the ion-acoustic velocity, the slow ion beam mode merges with the ion-acoustic mode and the resulting branch is unstable. A convective ion-acoustic instability is excited in the plasma [3].

The main parameters of the instability are the velocity and the relative density of the ion beam. The spatial and temporal growth rates strongly depend on these parameters [4]. This system has been extensively studied and it is known to exhibit turbulent dynamics [4–7]. However, no detailed study of the transition to the turbulent regime has been conducted up to now.

Recent results on nonlinear dynamics make it possible to describe the destabilization of the whole spatiotemporal system by bifurcation theory. When the control parameter is set just above the threshold of the instability, the system undergoes a supercritical Hopf bifurcation. In the standard situation, the plasma is considered to be quiet: a very low fluctuation level of every parameter is assumed. In particular, thermal noise superimposed on the local electron density and plasma potential is not taken into account. At the threshold of the instability, the system exhibits a sudden transition to the unstable—oscillatory—state. The whole plasma volume supports a propagating unstable wave whose saturation is determined by the nonlinear damping and by the modification of the ion velocity distribution.

Previous experimental investigations of the dynamics of the current-induced ion-acoustic instability have been reported [8]: the intermittent signals were interpreted as type-I intermittency. In a subsequent paper, Franck *et al.* proposed a different interpretation involving the background noise of the plasma, and concluded that a Hopf bifurcation exists subject to parametric noise driving [9].

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Indeed, experimental considerations lead to clear evidence that fluctuations are intrinsic for most real plasma systems, and the noisy fluctuations of at least one control parameter can have a dramatic influence on the dynamics at the onset of the instability. This situation deserves a detailed study because the existence of noise is very common in real systems. The implications of the specific dynamics induced in different systems—including other disciplines—are of particular importance.

III. ON-OFF INTERMITTENCY

The occurrence of irregular signals in fluid dynamics has long been labeled ''intermittency.'' These signals correspond to transient irregular relaxation of the turbulent probe signals. Different models of intermittency have been proposed during the last 20 years. All these models are built to describe fast irregular switching between very different regimes of the observed fluctuating variable. Most often, the apparently random switching between distinct chaotic regimes is studied using discrete time models. In particular, intermittency of types I, II, and III was established by Pomeau and Manneville [1]. In those cases, intermittency is associated either with instability or with the death of a fixed point. These bifurcations occur for a fixed value of the control parameter. A statistical study of the distribution of the lengths of the laminar phases was achieved in that classical case and led to either exponential laws or power laws. Another typical signature is the evolution of the mean laminar length as a function of the control parameter.

A different type of intermittency, called on-off intermittency, has been proposed by several authors recently [10,11], and can be consistently described in the framework of the ergodic theory of random transformations [12,13]. The system exhibits a ''random'' switching between a constant state (off) and a bursting state (on). This intermittency has attracted considerable attention recently. In such systems intermittency is mainly due to the competition between the stochastic deviations induced by the driving of the control parameter and the deterministic constraints of the system. Indeed, fluctuations such as thermal or quantum noise are intrinsic in real systems, and their influence on the dynamics of the whole system near a bifurcation point has to be considered. In such a situation, long episodes of laminar dynamics are interrupted by bursts during which the dynamic variable is expelled from the synchronous state. Clearly, the fraction of time spent in the laminar regime decreases as the control parameter departs from threshold. The number of laminar phases (which is of course equal to the number of turbulent ones) first increases and then decreases in such a way that the mean duration of a laminar zone is proportional to the inverse of the distance to threshold. As a consequence, the bursts are more and more frequent as the threshold parameter is increased after the bifurcation point. Notice that such behavior will also be present in the model we propose in Sec. VI.

An important quantity characterizing the intermittency, which is clearly related to the ergodic properties of the system, is the distribution of the duration of laminar phases $P(n)$. In most of the cases studied in the literature, $P(n)$ decays as an inverse power law with exponent γ. The value of the exponent depends in general on the nonlinearity characteristic of the dynamical system considered [14]. However, when a linear approximation is possible [11], γ will depend only on the statistical properties of the noise β. If d is the exponent characterizing the diffusion of noise [$\sqrt{\langle\beta^2(n)\rangle} \approx n^d$], then $\gamma = 1 + d$: this is essentially a consequence of the central limit theorem. We have of course $\gamma = 3/2$ for Gaussian noise; for noise driven by fractional Brownian motion different values of γ are clearly possible [14]. Notice that only for these special processes do we get $d = H$, with H the Hurst exponent, so that in this case $\gamma = 1 + H$.

The first experimental evidence of on-off intermittency was reported by Hammer *et al.* [15] in a nonlinear electronic circuit. The distribution of the laminar phases was found to be in agreement with a $-3/2$ power law. Experiments conducted in spin-wave instabilities [16] led to further evidence of such intermittent behavior in solid state physics. A $-3/2$ power law of the distribution of laminar lengths was obtained, whereas a power law with exponent -1 of the mean laminar length was recorded as a function of the control parameter.

In this paper, we report the measurement of signatures compatible with on-off intermittency at the onset of the ion-acoustic instability. However, in our case, due to nonlinear effects, the power law depends on the value of the control parameter.

IV. EXPERIMENTAL SETUP

The experiments are carried out in a multipolar double-plasma device, 35 cm in diameter and 70 cm in length. A stainless steel grid of 80% transparency with 20 lines per cm separates source and target chamber, where plasmas can be produced independently by thermionic discharges. The working pressure (argon gas) is typically $p = 2 \times 10^{-4}$ mbar. The ionizing electrons are emitted by the heated tungsten filament cathodes and accelerated through a discharge voltage U_d ($U_d = 40$ V) toward the respective anodes inside each chamber. In this classical arrangement, the homogeneity and density of the plasmas are greatly enhanced by the multipolar magnetic cusps on the outer walls repelling most of the ionizing electrons accelerating toward the anodes [17]. The potential difference between the target and source anodes can be adjusted using a separate power supply.

In the experiment depicted in this report, the voltage between the source and the target chamber is the external control parameter. It is important to note that the injected ion beam velocity is proportional to the square root of the potential difference $\Delta\phi$ applied between the chambers. The applied potential difference is below 10 V. The exact potential difference is obtained from analysis of the probe characteristics and the injected beam velocity is checked using solitary test waves.

The parameters of the plasma in each chamber are deduced from measurements performed with axially and azimuthally movable plane Langmuir probes. In particular, the electron density and the plasma potential are deduced from the probe characteristics. The plasmas created in both chambers have densities $n = 10^8 - 10^9$ cm^{-3} and electron temperatures $T_e \approx 3$ eV.

The fluctuation signals are recorded with the probes bi-

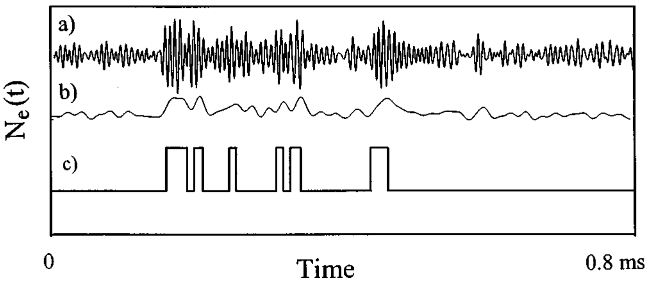

FIG. 1. Typical time series of the density fluctuations recorded on a plane probe in the target plasma 1 cm behind the grid, just at the onset of the instability. Bursts of high frequency waves can be clearly seen (a). The corresponding rectified and low pass filtered signal is displayed on trace (b). Binarization of the time series produces trace (c). The statistics of the laminar phase duration is investigated using this last signal.

ased slightly above the plasma potential. The ion beam is detected by use of an electrostatic energy analyzer consisting in a 12 mm diameter collector and one selecting grid. A resolution of 0.2 eV is obtained in the determination of the beam energy.

In order to have access to the full spatiotemporal dynamics of the system, a linear array of eight cylindrical probes of 0.5 mm diameter and 4 mm length each, equally spaced 5 mm from one another, is installed. The array is inclined by an angle of 15° to the axis of the device to reduce shadowing effects. Digital scopes and a spectrum analyzer allow for real-time analysis of the fluctuation signals. The statistical investigation of the distribution of the recurrence times of the bursts is performed on line using acquisition and processing software. In practice, a time series of 2.5×10^4 samples of the fluctuating probe current is transferred to the computer every 2 s and the display of the distribution of laminar lengths is refreshed at the same rate.

V. RESULTS: STATISTICAL ANALYSIS OF THE INTERMITTENCY

The electron density and plasma potential are adjusted in each plasma chamber in order to get an ion beam-plasma system with a beam density close to 0.1 and a beam velocity close to c_s. The control parameter (bias of the source anode) is slowly increased in order to slightly accelerate the ion beam. At the threshold voltage U_1 bursts of unstable waves are recorded on the probe located 3 cm behind the grid in the target plasma. As mentioned in the previous section, the exact potential difference between source and target plasma may be different from this voltage due to a residual plasma potential difference when no external voltage is applied.

The fluctuation level is of the order of 5% and the frequency is found to be close to half the ion plasma frequency, typically between $f=450$ kHz and 750 kHz. In Fig. 1 is depicted a typical time series of the electron saturation current just above the threshold. The raw data display a high level of broadband low frequency noise. The high frequency bursts are clearly seen on the first trace after high pass filtering of the signal. The bursts are of short duration, typically 10 to 20 periods of the unstable waves, although long bursts can occasionally be seen. The temporal distribution of the wave trains seems to be very irregular. On increasing the control parameter further, the bursts are more frequent and of a longer mean duration. Finally, the instability is completely established and a permanent high frequency signal is recorded on the probe.

In order to perform a statistical investigation that allows a comparison with a dynamical model, it is convenient to define the laminar length as the time lag between two successive bursts. At first glance, it is clear that the mean laminar length decreases rapidly when the control parameter is increased.

It is important to note that the noise level recorded on the probe just below the threshold of the instability or during the laminar phases is not negligible. Indeed, during the bursts, the signal to noise ratio is very poor.

The requirements of the statistical analysis imply careful signal processing. After transfer to the computer of a long time series, the first step consists in a numerical high pass filtering of the signal. The lower cutoff filter frequency is chosen such that the main peak of the time averaged spectrum is beyond the cutoff. It is typically $f_{low}=300$ kHz in the case shown. The absolute value of the signal is then calculated and a low pass filtering produces the envelope of the bursts. The last step consists in digitizing the signal, choosing a threshold level close to half the mean amplitude of the bursts. The resulting signal is of binary form corresponding to a null signal during the laminar phases and a value 1 during the bursts. Figure 1 displays the low pass rectified signal and the binary signal.

The statistical analysis is then performed by calculating the histogram of the laminar lengths. It is important that this signal processing and the related statistical analysis is performed on line during the experiment. The display of the distribution of the lengths is updated every two seconds and a reliable distribution is obtained after a few minutes. The number of analyzed events is then of the order of 10^4.

The result of the investigation is presented in Fig. 2, where the evolution of the distribution of the duration of the laminar phases for increasing values of the control parameter is plotted on a log-log scale. The slope of the power law at the threshold is found to be -1.4. On increasing the control parameter (external voltage), the exponent of the power law increases slightly approaching the value 2.0. During this evolution, the power law distribution is progressively changed to an exponential decay law. The main result obtained here is that the exponent of the power law at the threshold is very close to $-3/2$, but departs from this value as the control parameter increases, driving the system to a more strongly nonlinear regime. This point can be compared with the evolution of the mean laminar length when the control parameter is increased. Figure 3 displays the recorded evolution in our experiment. A power law with exponent -1 as a function of the voltage between the plasmas is obtained with relatively good accuracy. This is also in agreement with earlier numerical and analytical results on on-off intermittency [11].

VI. A DYNAMICAL MODEL FOR THE ONSET OF ION-ACOUSTIC INSTABILITY

To get further insight into the experimental results, we will consider a simple dynamical system that should capture the essential features of ion-acoustic instability in a noisy

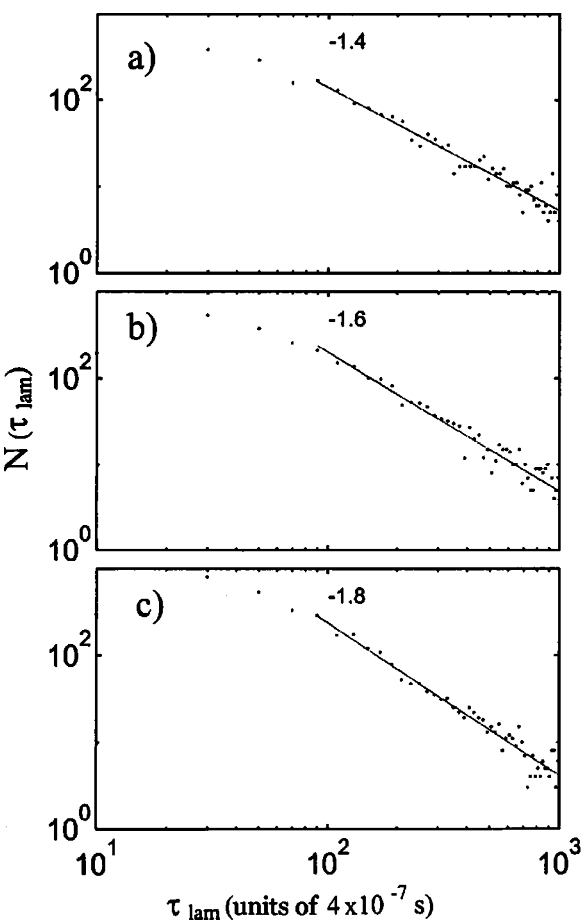

FIG. 2. Distribution of the duration of the laminar phases on a log-log plot for three different values of the control parameter (voltage between the plasma chambers). A power law of exponent -1.4 is recorded at the threshold. This value decreases toward -2 when the control parameter is increased.

plasma. As discussed in Sec. II, this is a supercritical Hopf bifurcation with noisy control parameter.

The normal form of a supercritical Hopf bifurcation in discrete time reads

$$\rho_{n+1}=\beta\rho_n-\rho_n^3, \quad \theta_{n+1}=\theta_n+\omega. \tag{6.1}$$

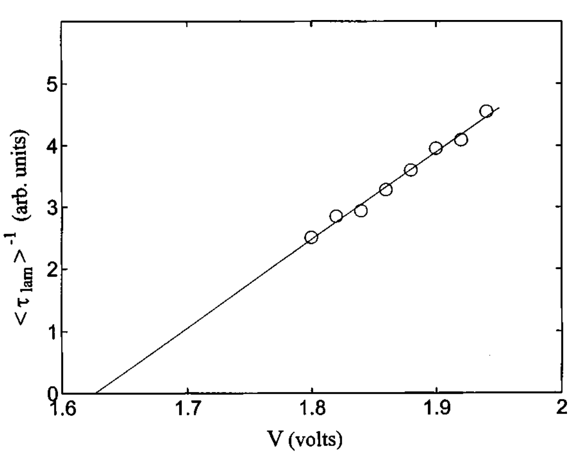

FIG. 3. Evolution of the inverse of the mean laminar duration as a function of the control parameter (bias of the source plasma). The critical value of the control parameter is close to 1.65. A linear law is recorded.

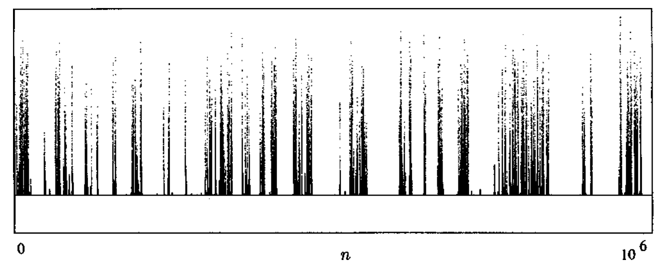

FIG. 4. Time series relative to the application (6.2): ρ_n is plotted as a function of the number of iterations n for 10^6 iterations.

When the control parameter β is smaller than 1, the system has an attracting fixed point at $\rho=0$. When β is larger than 1, the system has an attracting limit cycle at $\rho=\overline{\rho}=\sqrt{\beta-1}$.

Now, allow the control parameter to vary in time, due to noise in the physical system. The evolution of the amplitude ρ, which corresponds to the physical quantity observed, will be

$$\rho_{n+1}=\beta_n\rho_n-\rho_n^3, \quad \rho\in[0,1], \tag{6.2}$$

so that, as β_n varies across the bifurcation value 1, the system will be attracted toward the low signal state $\rho=0$ (when $\beta_n<1$) or toward the high signal state $\rho=\overline{\rho}$ (when $\beta_n>1$). We thus have a bistable system, subject to parametric noise.

We take $\tilde{\beta}=\{\beta_n\}$ to be a random process, of probability ν: classical results on random dynamical system and ergodic theory make this type of situation a well established one [12,13]. An invariant measure on $[0,1]$ exists for ν almost any $\tilde{\beta}$. For the numerical study of Eq. (6.2), we write β_n as

$$\beta_n=1+A+K\left(\epsilon_n-\frac{1}{2}\right) \tag{6.3}$$

ϵ_n is a random variable taking values between 0 and 1. K represents the intensity of noise, which in our case is intrinsic to the system and cannot be modified from the outside. A is an offset, which corresponds to the distance from the critical point in the case where the bifurcation parameter is constant in time; in our case, A must be proportional to the source bias V.

Keeping K constant ($K=0.2$ in our numerical study) and varying A in such a way as to stay close to the instability threshold $s=\langle\ln(\tilde{\beta})\rangle_\nu=0$ (for $K=0.2$, the critical value of A is $A_c=0.0018$), we indeed observe an intermittent behavior between high and low (laminar) signal states, as appears in Fig. 4. We have studied numerically the distribution of durations of laminar phases $P(n)$. A laminar phase of duration m is defined to be an event where the signal ρ is smaller than a threshold $\delta=10^{-10}$ for m consecutive iterations of the application (6.2). $P(n)$ is the fraction of laminar phases whose duration is equal to n.

Figure 5(a) and Fig. 5(b) suggest that $P(n)$ decays, for intermediate times, as an inverse power law, whose exponent γ varies as a function of the distance to threshold. This corresponds to what is observed experimentally. For still larger times we observe a faster decay, which is compatible with theoretical results on on-off intermittency [11].

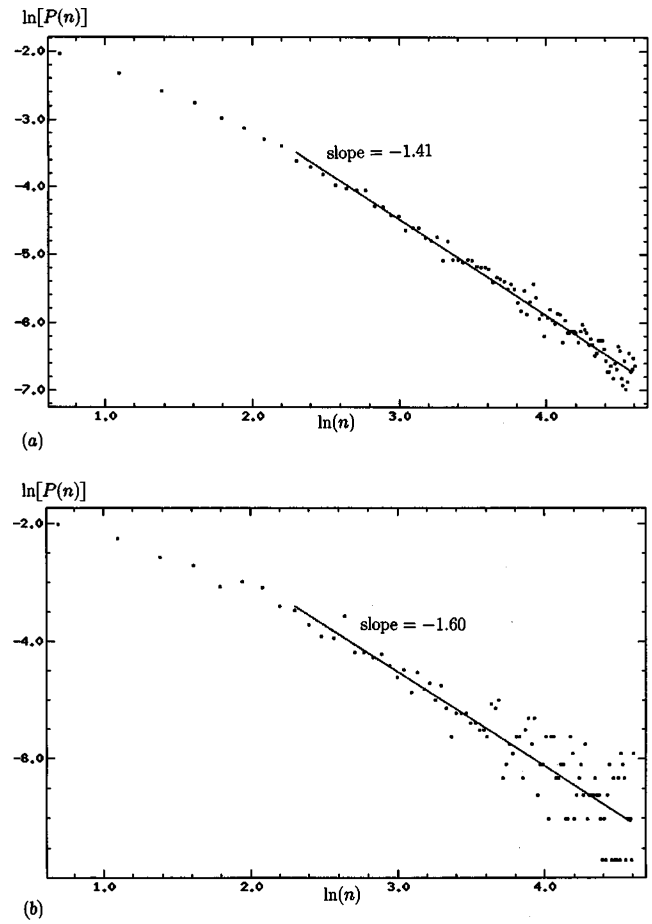

FIG. 5. Log-log plot of the distribution of laminar phases $P(n)$, for two values of the control parameter (model). Distance to threshold increases from (a) to (b). The slope $-\gamma$ is obtained using the least squares method (excluding the first eight points).

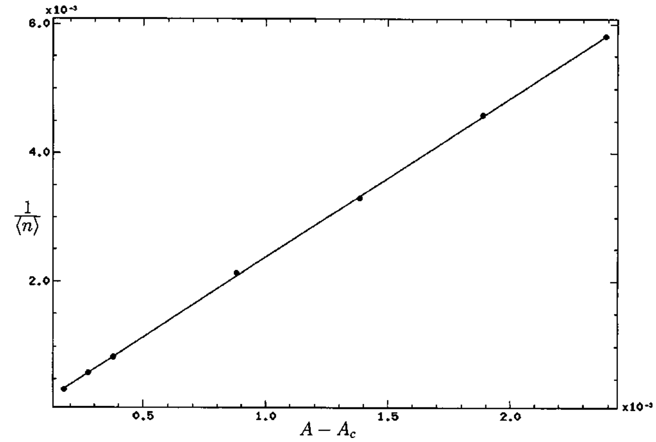

FIG. 6. Inverse of the mean duration of laminar phases as a function of the distance of the control parameter A to its critical value (model).

We argue that the power-law behavior observed for intermediate times must depend on the nonlinearity specific to the Hopf bifurcation. This agrees with recent investigations [14] proving that there exists a dependence of $P(n)$ on the particular form of the mapping considered.

These results do not vary significantly for different choices of the distribution of ϵ_n (discrete or continuous values), as long as the ϵ_n's are independent and the distance to threshold remains the same (this is essentially a consequence of the central limit theorem).

We also studied the mean duration $\langle n \rangle$ of laminar phases in our model. We observe (see Fig. 6) that $\langle n \rangle$ varies as the inverse of the distance of the control parameter to threshold. (Keeping K fixed, as we do, we have $1/\langle n \rangle \propto A$.) This again agrees with the experimental observations.

VII. DISCUSSION

In this paper, we have presented an experimental observation of ion-ion beam instability in a laboratory double-plasma device. We have focused on the presence of intermittent bursts of unstable waves in the plasma, which are recorded just above the instability threshold. The system exhibits an apparently random switching between this bursting state and a laminar state.

To characterize this intermittent behavior, we have estimated two quantities: the distribution of duration of laminar phases $P(n)$, and the mean duration of laminar phases $\langle n \rangle$. $P(n)$ is found to decay as an inverse power law with an exponent that depends on the value of the control parameter; $\langle n \rangle$ is proportional to the inverse of the distance of the control parameter to threshold.

These observations agree with the evolution deduced from a theoretical model that combines the normal form of a supercritical Hopf bifurcation and the parametric noisy deviations from threshold. This interpretation of the intermittency observed in ion-acoustic instability has also been proposed by Franck *et al.* [9]. However, we carry further the statistical analysis of the properties of $P(n)$ for both the experiment and the model. We also argue that the scenario we present, characterized by the interplay of noise and nonlinearity (and usually designated in the literature as on-off intermittency) should be placed in the framework of random dynamical systems: in this framework, ergodic measures and Lyapunov exponents do have a meaning and a relevance, contrary to what is stated in [9].

Let us also remark that one of the main points of the discussion of intermittency in [9] is to show that the observed behavior is not compatible with that characteristic of type-I intermittency, as claimed in [8]. This is indeed unambiguously proved by the fact that the exponent $-\gamma$ of the power-law decay of $P(n)$ is always different from $-1/2$.

Two directions for future investigation appear naturally. The first is to study the dependence of $P(n)$ on the properties of noise, for example, injecting noise of a defined spectrum into the experimental system using an arbitrary wave-form generator. The second is to investigate analytically how the observed inverse power-law behavior for $P(n)$ can be generated by the dynamical model (6.2) [18].

ACKNOWLEDGMENT

One of the authors (H.K.) benefited from the financial support of the Alexander von Humboldt Foundation (Germany), which she gratefully acknowledges.

[1] Y. Pomeau and P. Manneville, Commun. Math. Phys. **74**, 189 (1980).
[2] C. Grebogi, E. Ott, and J. A. Yorke, Phys. Rev. Lett. **48**, 1507 (1982).
[3] B. D. Fried and A. Y. Wong, Phys. Fluids **9**, 1084 (1966).
[4] F. Doveil and D. Grésillon, Phys. Fluids **18**, 1756 (1975).
[5] R. J. Taylor and F. V. Coroniti, Phys. Rev. Lett. **29**, 34 (1972).
[6] Y. Kiwamoto, J. Phys. Soc. Jpn. **37**, 466 (1974).
[7] T. Ueki, Y. Nishida, and N. Yugami, Phys. Rev. Lett. **76**, 4171 (1996).
[8] K. Taniguchi, H. Kuwae, N. Hayashi, and Y. Kawai, Phys. Plasmas **5**, 401 (1998).
[9] C. Franck, T. Klinger, and A. Piel, Phys. Lett. A **259**, 152 (1999).
[10] N. Platt, E. A. Spiegel, and C. Tresser, Phys. Rev. Lett. **70**, 279 (1993).
[11] J. F. Heagy, N. Platt, and S. Hammel, Phys. Rev. E **49**, 1140 (1994).
[12] Y. Kifer, *Ergodic Theory of Random Transformations* (Birkhäuser, Boston, 1986).
[13] Pei-Dong Liu and Min Qian, *Smooth Ergodic Theory of Random Dynamical Systems*, Vol. 1606 of Lecture Notes in Mathematics (Springer, Berlin, 1995).
[14] H. L. Yang, Z. Q. Huang, and E. J. Ding, Phys. Rev. E **54**, 3531 (1996).
[15] P. Hammer, N. Platt, S. Hammel, J. Heagy, and B. Lee, Phys. Rev. Lett. **73**, 1095 (1994).
[16] F. Rodelsperger, A. Cenys, and H. Benner, Phys. Rev. Lett. **75**, 2594 (1995).
[17] R. Limpaecher and K. R. Mackenzie, Rev. Sci. Instrum. **44**, 721 (1973).
[18] E. Floriani, A. Lambert, R. Lima, and T. Pierre (unpublished).