Phase diagram of congested traffic flow: An empirical study

H. Y. Lee,[1] H.-W. Lee,[2] and D. Kim[3]
[1]*Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*
[2]*School of Physics, Korea Institute for Advanced Study, 207-43 Cheongryangri-dong, Dongdaemun-gu, Seoul 130-012, Korea*
[3]*School of Physics, Seoul National University, Seoul 151-742, Korea*
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We analyze traffic data from a highway section containing one effective on-ramp. Based on two criteria, local velocity variation patterns and expansion (or nonexpansion) of congested regions, three distinct congested traffic states are identified. These states appear at different levels of the upstream flux and the on-ramp flux, thereby generating a phase digram of the congested traffic flow. Observed traffic states are compared with recent theoretical analyses and both agreeing and disagreeing features are found.

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In the last decade, there has been growing interest in the traffic flow, which is partly motivated by the fact that the traffic flow is an easily perceivable realization of heavily studied driven nonequilibrium systems [1]. Another important motivation is the hope that complex traffic behaviors may be understood with the help of physical approaches. Such understanding can be used to optimize traffic and even to forecast traffic situations.

A reasonable first step along this line of thinking will be the classification of distinct traffic states and separate investigation of their properties. Various traffic models are proposed [2] and compared with real traffic data [3]. With the help of these models, free flow and so-called wide traffic jams are well understood. On the other hand, the nature of congested traffic flow (or synchronized flow), which appears near road inhomogeneities mostly, yet remains unclear despite various empirical [4–6] and theoretical [7–11] efforts.

A recent theoretical study [9] proposed an intriguing possibility that the congested traffic flow may not be a single dynamic phase but rather a collection of multiple phases, each of which is realized under different conditions. A similar conclusion is also reported from the investigation of another theoretical model [10]. In the empirical investigations, however, although qualitatively distinct congested traffic states are reported [4,12], no empirical evidence is found for the existence of any characteristic parameters that distinguish their appearance conditions [6].

In this paper, we report empirical investigation of traffic congestion in a highway section containing one effective on-ramp. Details of the section are given in our preliminary report [13], so we provide here only a brief description (Fig. 1). All ramps are connected to the outermost lane (lane 4) and a stretch of lane divider (from $x=3.5$ to 8.3 km, dashed line in Fig. 1) blocks lane change from the two outer lanes (lane 3 and 4) to the two inner ones (lane 1 and 2) and vice versa. In a short road segment near the end of the lane divider (from $x=8.3$ km to the location of the detector D9 approximately), many vehicles in the outer lanes switch into the inner lanes, which is also enhanced by vehicle flux through the on-ramp ON3 at $x=8.6$ km. As a result, this segment works as an *effective* on-ramp region for the traffic flow of the inner lanes and traffic congestion often occurs in the road section with the lane divider, where the inner lanes are decoupled from the outer ones.

We investigate the traffic congestion in the inner lanes using the 30 s averaged traffic data from June to September, 1999 (a total of 107 days, a much larger data set compared to 14 days in Ref. [13]). All quantities below are averaged over the two inner lanes. For each realization of a particular congested traffic state, which is stably maintained about 30 min or longer, the effective ramp flux f_{rmp} [defined as the difference between two flux values measured at the detectors D10 and D7, $q(\mathrm{D10})-q(\mathrm{D7})$] and the upstream flux f_{up} [$q(\mathrm{D}n)$ is used when the congestion extends up to D$n+1$] are averaged over the time interval of its duration and the resulting average values $\langle f_{\mathrm{rmp}}\rangle$, $\langle f_{\mathrm{up}}\rangle$ are marked in Fig. 2(a). Note that three congested traffic states, which we call CT2, CT4, and CT5 states, respectively (see below), occupy distinct regions, providing a supporting evidence for the prediction [9,10] that f_{up} and f_{rmp} are characteristic parameters of congested traffic flow. Thus Fig. 2(a) becomes an empirical phase diagram of the congested traffic flow. Figure 2(b) shows an alternative phase diagram obtained from 10 min averaging of the flux values instead. Two phase diagrams are qualitatively the same.

Three congested traffic states, CT2, CT4, and CT5, are classified according to the two criteria given below. It is

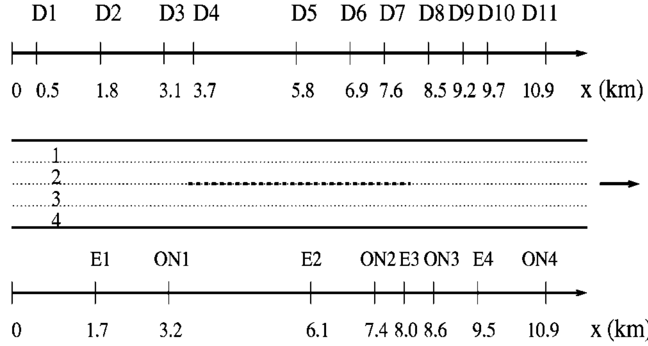

FIG. 1. Schematic diagram of a road section in the Olympic Highway in Seoul. Locations of detectors (Dn), on-ramps (ONn), and off-ramps (En) are marked. The dashed line in the middle denotes the lane divider and the arrow indicates the driving direction.

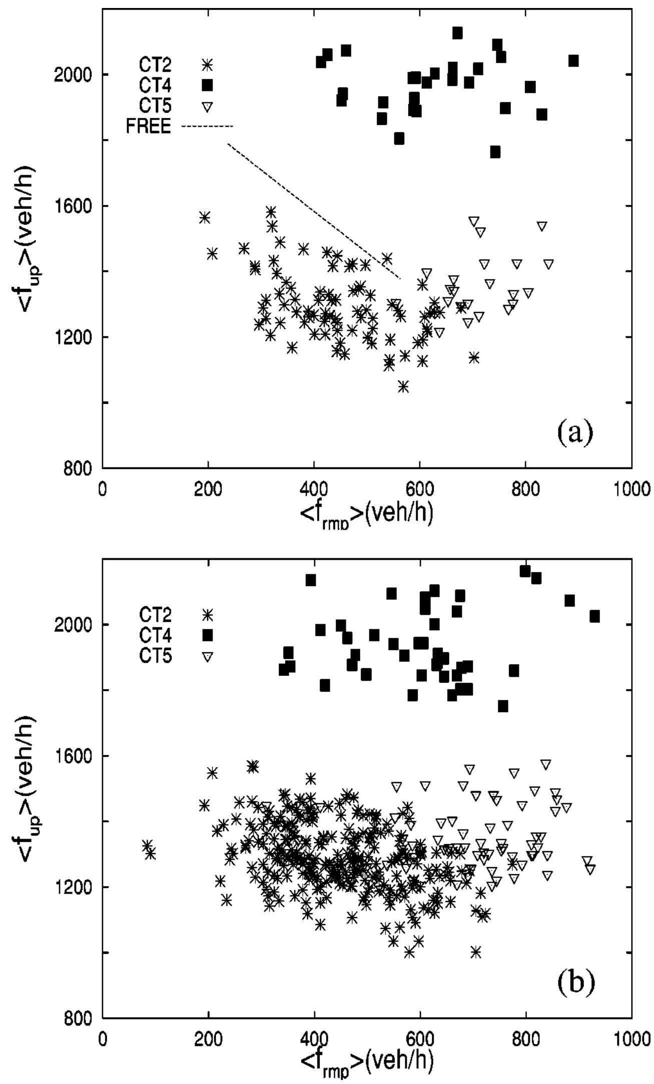

FIG. 2. (a) Empirical phase diagram of the congested traffic flow. $\langle f_{\rm up}\rangle$ and $\langle f_{\rm rmp}\rangle$ represent the average upstream and on-ramp flux values over the time interval during which a particular congested traffic state is maintained (also lane averaged). The dashed line is an empirical estimation of the free flow phase boundary below which the free flow can remain linearly stable. (b) Same diagram using 10 min averaged flux values.

previously reported [6] that the congested region may consist of backward (towards upstream) traveling clusters and the size of the clusters grow spontaneously during their backward propagation. As a result, large amplitude oscillation of velocity develops spontaneously. On the other hand, recent theories [9,10] predict that large velocity oscillation may or may not develop. Thus our criterion (i) is whether such spontaneous growth of velocity oscillation appears (CT5) or not (CT2, CT4). This criterion can be examined by comparing temporal variation of velocity at different detectors.

In the same theoretical works, both expanding and nonexpanding traffic states are predicted, and our criterion (ii) is whether the congested region expands monotonically (CT4, CT5) or not (CT2). Mathematically the expansion rate of the congested region is proportional to the degree of flux mismatch $f_{\rm up}+f_{\rm rmp}-f_{\rm down}$, where $f_{\rm down}$ measures the outflow from the congested region [q(D10) is used]. Thus the comparison of $\langle f_{\rm up}\rangle+\langle f_{\rm rmp}\rangle$ and $\langle f_{\rm down}\rangle$ can be used as a more objective application of the criterion (ii).

In all three states, the density-flow relations show fluctuating behavior [such as Fig. 2(c) in Ref. [4]] and the velocity variations in the lane 1 and 2 are synchronized (such as Fig. 2 in Ref. [5]). Since these properties are already reported in other publications, we do not present similar figures here. Below we discuss other properties of the congested traffic states.

Figure 3(a) depicts the density profile $\rho(x,t)$ of the CT2 state and its evolution with time, where $\rho(x,t)$ is evaluated by $q(x,t)/u(x,t)$ and $u(x,t)$ is the harmonic mean velocity [14] over 30 s intervals. The spontaneous growth of the velocity oscillation mentioned in the criterion (i) does not appear. The flux mismatch $\langle f_{\rm rmp}\rangle+\langle f_{\rm up}\rangle-\langle f_{\rm down}\rangle$ is negligible and the congested region does not expand. Empirical data also show that the length of the congested region increases with $\langle f_{\rm rmp}\rangle$ and its dependence on $\langle f_{\rm up}\rangle$ is rather weak [15].

Figure 3(b) shows the CT4 state. The spontaneous growth of the velocity oscillation does not appear. Due to the large flux mismatch (typical value of $\langle f_{\rm up}\rangle+\langle f_{\rm rmp}\rangle-\langle f_{\rm down}\rangle$ is 500–600 vehicles/h), the congested region expands with time. The expansion rate ranges from 3 to 9 km/h and increases with increasing flux mismatch. An interesting property is that the outflow $\langle f_{\rm down}\rangle$ is practically independent of $\langle f_{\rm up}\rangle$ and $\langle f_{\rm up}\rangle$, and remains almost universal. The average of $\langle f_{\rm down}\rangle$ over the 28 realizations of the CT4 state in Fig. 2(a) is 2010 vehicles/h and its standard deviation is about 65 vehicles/h, which is much smaller than the spread of $\langle f_{\rm up}\rangle$ and $\langle f_{\rm rmp}\rangle$. To our knowledge, this is the first empirical indication of the universal outflow from the congested flow near an on-ramp [16].

Figure 3(c) portrays the CT5 state. An important feature of the CT5 state is the spontaneous growth of the velocity oscillation inside the congested region. Figure 4 shows the temporal variation of the velocity at D5 and D6. The graph for D6 is shifted to the right by 5 min for comparison. Note that the velocity evolutions at the two detectors are correlated after the time shift for D6, and that the velocity oscillates with larger amplitude at D5. These features imply that velocity wave propagates backward (towards upstream) and its amplitude grows during its propagation.

Regarding the criterion (ii), the congested region expands as shown in Fig. 3(c). The upstream front of the congested region initially locates between D5 and D6 and later between D4 and D5. We mention that the flux at D4 remains quasistationary during the depicted time interval. Thus the expansion is not due to the increase of $f_{\rm up}$ but due to the flux mismatch. Compared to the CT4 state, however, it turns out that the flux mismatch (typically 200–250 vehicles/h) is considerably smaller, which implies a slower expansion of the congested region. We estimate the expansion rate by the flux mismatch divided by the density difference at the upstream front of the congested region, and find it ranges 2–4 km/h [17]. The outflow of this state is not universal.

It is interesting to compare empirically identified states with theoretically predicted states [7–10]. Application of the criteria (i,ii) to both empirical and theoretical states leads to the following pairing between empirical and theoretical states: the CT2 state with the theoretically predicted pinned localized cluster (PLC) state, the CT4 state with the homo-

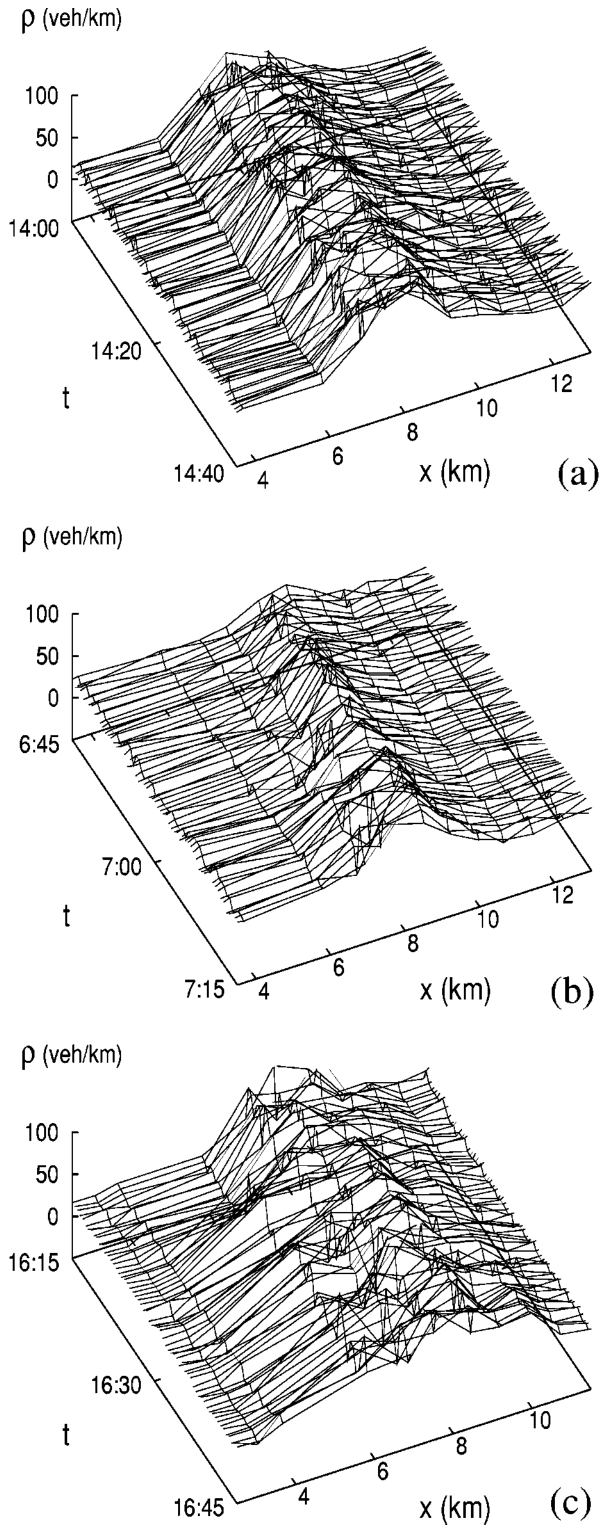

FIG. 3. The $3d$ density profile of the CT2 state (a), CT4 state (b), and CT5 state (c).

geneous congested traffic (HCT) state, and the CT5 state with the oscillating congested traffic (OCT) state.

The pairing motivates further comparison between the paired states. For the CT4-HCT pair, we note that the universal outflow is predicted for the HCT state [9] and the same property is observed for the CT4 state, which strongly motivates the identification of the CT4 state with the HCT states. For the CT5-OCT pair, on other hand, while the out-

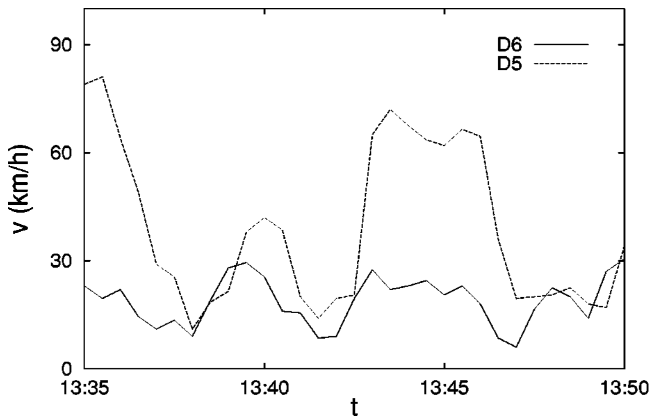

FIG. 4. Spontaneous growth of the velocity oscillation inside the congested region of the CT5 state. The velocity evolution at D6 is shifted to the right by 5 min for clear comparison.

flow is universal for the OCT state [9], it is not for the CT5 state. Thus properties of these two states are only in partial agreement. And for the CT2-PLC pair, we find one quantitative difference: the congested region of the CT2 state is considerably wider than that of the PLC state in Refs. [9] and [10]. The resolution of this discrepancy may require improved traffic theories.

We also compare the theoretical and empirical phase diagrams. It is predicted in Ref. [9] that when the upstream flux and the ramp flux are maintained at *strictly* constant values, f_{up} and f_{rmp}, respectively, the phase boundary between the flux matching and flux mismatching states is practically identical to the stability boundary of the free flow, below which the free flow can remain *linearly* stable, and given by the line $f_{\mathrm{up}}+\alpha f_{\mathrm{rmp}}=\tilde{Q}_{\mathrm{out}}$, where $\alpha=1$ and $\tilde{Q}_{\mathrm{out}}$ is a constant whose value is almost identical to the universal outflow of the HCT state (α is predicted to be a little larger than 1 in Ref. [10]).

Empirical determination of the free flow stability boundary is not an easy task since the free flow near the boundary is quite vulnerable to fluctuations. In the empirical phase diagram [Fig. 2(a)], the dashed line ($\langle f_{\mathrm{up}}\rangle+\alpha\langle f_{\mathrm{rmp}}\rangle=\tilde{Q}_{\mathrm{out}}$, where $\alpha\approx1.3$ and $\tilde{Q}_{\mathrm{out}}\approx2100$ vehicles/h) is an empirical *estimation* of the stability boundary. Here the values of α and $\tilde{Q}_{\mathrm{out}}$ are reliable up to their first significant digits and their second significant digits are rather uncertain. Within this accuracy, α is approximately one and the value of $\tilde{Q}_{\mathrm{out}}$ is close to the universal outflow of the CT4 state. It is also pleasing to note that this line divides (except for a small number of data points) the flux matching and mismatching states, in agreement with the theoretical prediction [9]. This feature is robust and independent of details of the boundary estimation method although the values of α and $\tilde{Q}_{\mathrm{out}}$ depend on the details. We also note that in a later, more refined theory [12], it is predicted that the PLC and OCT states overlap weakly in the phase diagram. Recalling the pairing CT2-PLC and CT5-OCT, the weak overlap of the CT2 and CT5 states in Fig. 2(a) is in agreement with this prediction. Also the locations of the empirical and theoretical overlap regions in the phase diagram are similar.

Regarding the phase boundary between the two expanding traffic states (CT4, CT5), while the recent theory [9] predicts the line $f_{\mathrm{rmp}}=\mathrm{const}$ as the phase boundary between the HCT and OCT states, Fig. 2(a) shows that the boundary

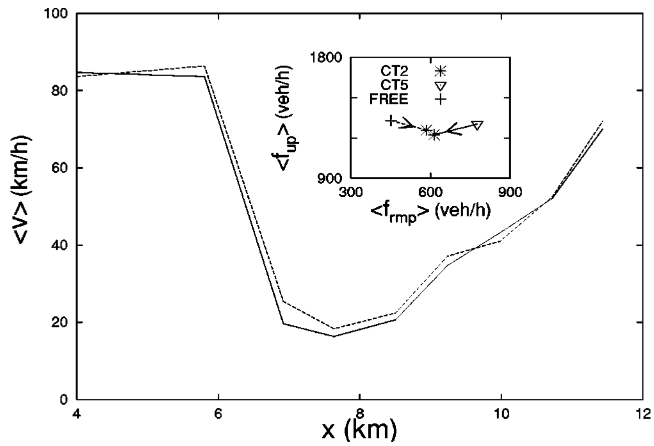

FIG. 5. Time-averaged velocity profiles of two realizations of the CT2 state with almost identical $\langle f_{\mathrm{up}}\rangle$ and $\langle f_{\mathrm{rmp}}\rangle$ but with different evolution histories (solid line for the realization 1 and dashed line for the realization 2). Inset: While the realization 1 has developed from the CT5 state via a phase transition, the realization 2 has evolved from the free flow.

between the CT4 and CT5 states is better described by $\langle f_{\mathrm{up}}\rangle = \mathrm{const}$. Thus as for the phase boundary between the expanding states, empirical results and the theoretical prediction do not agree.

We next discuss an implicit but important conceptual implication of the recent theories [7–10], where traffic phases are identified with resulting final states that traffic flow dynamics lead to after sufficient transient time. In the language of nonlinear dynamics, all these states correspond to stable attractors [18] of traffic flow dynamics. An implication of this concept is that for a given external condition (such as f_{rmp}, f_{up}, and ramp geometry), the resulting traffic state is independent of details of the initial traffic state or its ''evolution history'' [19].

This idea can be tested empirically, for example, by comparing realizations with the same external conditions but with different evolution histories. Figure 5 compares the (time-averaged) velocity profile of two realizations of the CT2 state, both with almost the same $\langle f_{\mathrm{rmp}}\rangle$ and $\langle f_{\mathrm{up}}\rangle$. But their evolution histories are different: one has evolved from the free flow and the other from the CT5 state (inset in Fig. 5). Note that two profiles almost overlap with each other despite qualitatively different evolution histories. This insensitivity to the evolution history confirms the conceptual implication of the recent theoretical works.

Obviously there is a fundamental difference between theoretical and empirical situations: while theories [7–10] assume strictly constant upstream and ramp fluxes, in real traffic the fluxes fluctuate all the time. Thus in order to make a rigorous comparison with theories, a good understanding of fluctuation effects is necessary. However such an understanding is not available at present, so in this paper, the fluctuation effects are essentially ignored and the time-averaged values $\langle f_{\mathrm{rmp}}\rangle$ and $\langle f_{\mathrm{up}}\rangle$ are instead used in the analysis. Hence in a strict sense, the present investigation is only a correlation analysis between the time-averaged flux values and the traffic states that are maintained for a sufficiently long time. In retrospect, however, it turns out that many empirical results can be explained by the theories if one accepts that $\langle f_{\mathrm{rmp}}\rangle$ and $\langle f_{\mathrm{up}}\rangle$ play the roles of the constant ramp flux and upstream flux, respectively, assumed in the theories. Hence it appears that the neglect of the fluctuation effects can be justified at least *a posteriori*.

Its justification can be partly understood from two arguments: first, while the fluctuation effects are usually crucial near the phase boundary, traffic states far from the phase boundary are relatively insensitive to fluctuations. Thus when a traffic state occupies a sufficiently wide region in the phase diagram, the use of the time-averaged flux values for the phase diagram can be justified. Second, dominant contributions to flux fluctuations come from short-time scale of order 1 min, and in the time scale of about 10 min or longer, flux variation is almost quasistationary. Thus, provided that the relevant time scale of a traffic state (not very close to the phase boundary) is longer than 1 min, the short-time scale fluctuations will be effectively averaged out by the traffic dynamics itself and can be neglected indeed.

In support of these arguments, we find that our results are not sensitive to the length of the averaging time interval as long as it is sufficiently longer than 1 min. When the same analysis is repeated with 10 min averaging, Fig. 2(b) is obtained. Note that this new phase diagram is qualitatively the same as the former one [Fig. 2(a)] and all discussions above remain unchanged. Changes occur only in a quantitative level. With the 10 min averaging, the standard deviation of the outflow $\langle f_{\mathrm{down}}\rangle$ from the CT4 state increases to about 85 vehicles/h.

We remark on a few details of the analysis. First, there exists another on-ramp (ON4) at 2.3 km downstream from ON3. Sometimes vehicle flux through this on-ramp causes traffic congestion and the resulting congestion extends to the region studied in this paper. All such events are excluded from the present analysis to focus on one particular inhomogeneity [20]. Second, in the analyzed road section, there are two spots (one between D5 and D6 and the other near D7), where the lane divider is imperfect. All time intervals with non-negligible vehicle fluxes through those spots are not included in the analysis.

In summary, three congested traffic states are identified based on local velocity variation patterns and expansion (or nonexpansion) of the congested region. It is found that the appearance of these congested traffic states is strongly correlated with the time-averaged flux values, $\langle f_{\mathrm{up}}\rangle$ and $\langle f_{\mathrm{rmp}}\rangle$, providing a strong supporting evidence to the prediction [9,10] that these flux values are characteristic parameters of the congested traffic flow. An empirical phase diagram is constructed and compared with theoretical predictions. The prediction on the phase boundary between the flux matching and mismatching states is consistent with the empirical phase diagram and the prediction of the universal outflow are confirmed. However some deviations from theoretical predictions are also found. Finally we mention that there exist regions in the $\langle f_{\mathrm{rmp}}\rangle - \langle f_{\mathrm{up}}\rangle$ plane that are not probed. Thus it is possible that additional congested traffic states exist in those regions.

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[14] The harmonic mean velocity u $[1/u \equiv (1/N)\Sigma_{j=1}^{N} 1/v_j]$ is used instead of v $[v \equiv (1/N)\Sigma_{j=1}^{N} v_j]$ since q/v systematically underestimates the density when the velocity fluctuations are significant; see, for example, L. Neubert, L. Santen, A. Schadschneider, and M. Schreckenberg, Phys. Rev. E **60**, 6480 (1999) whereas q/u takes a better account of the fluctuation effects. Also it can be shown that when both density and flux are temporally averaged [for example, $\rho(x,t) = (1/T)\int_{-T/2}^{T/2} dt' \Sigma_j \delta(x_j(t+t') - x)$, $q(x,t) = (1/T)\int_{-T/2}^{T/2} dt' \Sigma_j v_j(t+t') \delta(x_j(t+t') - x)$], the ratio q/ρ is exactly equal to u.

[15] Usually the boundary between the congested region and the upstream free region locates *between* detectors. But due to the dependence of the congested region size on $\langle f_{\rm rmp} \rangle$, the boundary will locate *at* a particular detector when $\langle f_{\rm rmp} \rangle$ is close to a certain special value. In such a case, small fluctuations of the boundary can cause large amplitude fluctuations of velocity at that special detector since the velocity varies greatly across the boundary. Such large velocity fluctuations are indeed observed in narrow ranges of $\langle f_{\rm rmp} \rangle$ (for example, at D6 when $\langle f_{\rm rmp} \rangle = 330 \pm 30$ vehicles/h). We remark that these fluctuations should be distinguished from the velocity oscillations [6] in the criterion (i). While the former originates from the boundary fluctuations and is localized near the boundary, the latter grows continuously inside the congested region and is extended over relatively wide congested region.

[16] At 30 s scale, $f_{\rm down}$ [q(D10)] fluctuates within the range 2010 ± 140 vehicles/h, which is equivalent to 16.8 ± 1.2 vehicles/30 s. Here the inaccuracy of ± 1.2 vehicles/30 s is quite a small number considering that the vehicle number counting by detectors always results in integer numbers and thus the inaccuracy of $\sim \pm 1$ vehicles/30 s is unavoidable in 30 s data.

[17] It may be hard to detect slow expansion from the $3d$ density plot due to the finite (~ 1 km) spatial resolution given by the average detector spacing. For this reason, this state is incorrectly interpreted as a nonexpanding state in our preliminary report [13] (using only 14 day traffic data), where a different name, CT1, is used for the same state.

[18] E.A. Jackson, *Perspectives of Nonlinear Dynamics* (Cambridge University Press, Cambridge, 1989), Vol. 1.

[19] When there exist multiple (locally) stable attractors for a given external condition, initial traffic state is not completely irrelevant. But even in this case, the initial state affects only the choice of a particular attractor as a final state. Or in the language of nonlinear dynamics, details of the initial state are irrelevant as long as the initial state remains within the basin of attraction [18].

[20] It is found that the on-ramp flux through ON4 may generate still another type of congested traffic state (called CT3 state in Ref. [13]), which is not included in the present analysis since this state appears under different road geometry.