Relativistic solitons in magnetized plasmas

Daniela Farina and Maurizio Lontano
Istituto di Fisica del Plasma, Consiglio Nazionale delle Ricerche, 20125 Milan, Italy

Sergei Bulanov
General Physics Institute, Russian Academy of Sciences, Moscow, Russia

(Received 27 April 2000)

The results of analytical and numerical investigations on the properties of one-dimensional (nondrifting) solitons of relativistic amplitude, in the presence of an externally imposed uniform magnetic field $\boldsymbol{B}_0$, are presented and compared with those of the unmagnetized plasma theory (Esirkepov *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **68**, 33 (1998) [JETP Lett. **68**, 36 (1998)]). The presence of a uniform longitudinal magnetic field, the intensity of which corresponds to an electron cyclotron frequency $\Omega_e = eB_0/m_e c$ that is a non-negligible fraction of the laser frequency ω_0, has important consequences on the properties of relativistically intense solitons. The region of the parameter space (ω_0, Ω_e) where magnetized solitons exist is determined analytically, and new conditions of breaking due to the total density depletion are given. It is shown that stable high energy magnetized solitons can be produced.

PACS number(s): 52.40.Nk, 52.35.Mw, 52.35.Sb, 52.40.Db

I. INTRODUCTION

Recent multidimensional particle-in-cell (PIC) simulations [1–4] have shown that, during the interaction of a relativistically strong laser pulse with an underdense plasma, up to 30–40% of the pulse energy becomes trapped in quasistationary density cavities which appear behind the laser pulse itself. These electromagnetic field ''bunches'' drift at a velocity much smaller than the group velocity of the laser, and, in the presence of density gradients, are accelerated toward decreasing density values; when they reach the plasma-vacuum interface, they disappear suddenly, emitting the trapped electromagnetic (e.m.) energy in the form of bursts of radiation at a strongly dowshifted (with respect to ω_0) frequency [2]. Generally speaking, this process represents an important mechanism of laser energy loss, since the radiation involved in these dynamics does not participate to the physical processes which should have been triggered by the laser pulse itself. Therefore, in recent years, the old problem of the formation and stability of relativistic solitons and solitonlike structures in collisionless plasmas has begun to be reconsidered in the light of new interaction conditions, in order to give an analytical basis to the results of the recent numerical experiments. Among the wide literature on solitons, here we wish to quote earlier papers on one-dimensional relativistic solitons where several of their peculiarities have been discussed [5–9]. In 1998, Esirkepov *et al.* [10] determined an analytical expression for a subcycle stationary soliton. The conditions for the stability of such a solitary structure indicate that the allowed frequency of the trapped radiation has a minimum, to which a maximum field amplitude corresponds. One-dimensional PIC simulations support these conclusions.

The solitons found in Ref. [10] are circularly polarized. If they are assumed to be generated by a circularly polarized e.m. wave propagating in the plasma, then it is natural to incorporate a magnetic field into the model, which is generated due to the inverse Faraday effect [11–13]. As is known, in this case a magnetic field oriented in the direction of the laser light propagation appears. The magnetic field generated by the laser can be substantially high, as to affect the soliton structure.

In order to include the effect of Faraday rotation in the soliton model, we have carried out an extension of the analysis made by Esirkepov *et al.*, by assuming that a static and uniform magnetic field $\boldsymbol{B}_0 = B_0 \hat{\boldsymbol{e}}_x$ is present throughout the plasma. The existence of localized e.m. field distributions, in the envelope approximation, have been investigated in connection with the problems of electrostatic ion cyclotron waves propagating perpendicularly to the ambient magnetic field [14], of electrostatic electron waves at arbitrary propagation angle [15], and of Alfven waves [16]. A study on the propagation of large amplitude, e.m. waves in a magnetized plasma was developed in Ref. [17]. To our knowledge, the present study represents the first investigation of the effects of a strong magnetic field on soliton formation, within a fully relativistic treatment and without any specific assumption about the characteristic spatial scales of the problem.

The paper is organized as follows: in Sec. II, relevant equations are derived and an integral of motion is obtained. Section III is devoted to an analysis of the structure of the relevant phase space in order to obtain the conditions under which localized solutions are allowed. Peculiar periodic solutions are briefly discussed. In Sec. IV, an implicit analytical expression of the soliton solution is derived, and the conditions for its stability are given. The analytical solution of the problem in the weakly relativistic case is presented, as well. Finally, Sec. V is devoted to concluding remarks.

II. FORMULATION OF THE PROBLEM

In a static magnetic field $\boldsymbol{B}_0$, Maxwell's equations for the wave vector potential $\boldsymbol{A}$ and for the scalar potential ϕ, and the hydrodynamic equation for the kinetic momentum $\boldsymbol{p}$, can be written

$$\triangle \boldsymbol{A} - \partial_{tt}\boldsymbol{A} - \boldsymbol{\nabla}\partial_t \phi - \frac{n}{\gamma}\boldsymbol{p} = 0, \tag{1}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{A} = 0, \tag{2}$$

$$n = 1 + \triangle\phi, \tag{3}$$

$$\partial_t(\boldsymbol{p}-\boldsymbol{A}) = \boldsymbol{\nabla}(\phi-\gamma) + \boldsymbol{v}\times\boldsymbol{\nabla}\times(\boldsymbol{p}-\boldsymbol{A}) - \boldsymbol{v}\times\boldsymbol{B}_0, \tag{4}$$

where $\boldsymbol{v} = \boldsymbol{p}/\gamma$ is the fluid velocity, and $\gamma = \sqrt{1+|\boldsymbol{p}|^2}$. Here the following dimensionless quantities have been introduced: $\boldsymbol{r}\omega_{pe}/c \to \boldsymbol{r}$, $t\omega_{pe} \to t$, $\boldsymbol{p}_e/m_e c \to \boldsymbol{p}$, $n_e \to n$, $e\boldsymbol{A}(\phi)/m_e c^2 \to \boldsymbol{A}(\phi)$, and $e\boldsymbol{E}(\boldsymbol{B})/m_e\omega_{pe}c \to \boldsymbol{E}(\boldsymbol{B})$, where $\omega_{pe} = \sqrt{4\pi n_0 e^2/m_e}$ is the electron plasma frequency, n_0 the unperturbed electron density, and m_e the electron rest mass.

Let us assume that the magnetic field is directed along x, $\boldsymbol{B}_0 = \Omega \boldsymbol{e}_x$, and consider the one-dimensional case in which $\partial_y = \partial_z = 0$. Since we want to find a localized solution, from Eq. (2) we obtain $A_x = 0$. We look for a solution of the system of equations (1)–(4) corresponding to a circularly polarized radiation; then we assume that the vector potential and the electron momentum, under the action of the fields, have the forms

$$A_\perp \equiv A_y + iA_z = A(x)\exp(i\omega t), \tag{5}$$

$$p_\perp \equiv p_y + ip_z = p(x)\exp(i\omega t), \tag{6}$$

where A and p are assumed real, with the other quantities n, ϕ, γ, and p_x depending only on the spatial variable x. Then, Eqs. (1)–(4) read

$$n = 1 + \phi'', \tag{7}$$

$$A'' + \omega^2 A = n\frac{p}{\gamma}, \tag{8}$$

$$(\phi-\gamma)' + \frac{p}{\gamma}(p-A)' = 0, \tag{9}$$

$$v_x(p-A)' + i\omega(p-A) = i\Omega\frac{p}{\gamma}, \tag{10}$$

where the prime denotes differentiation with respect to x.

When a static magnetic field is considered, Eq. (10) splits into

$$v_x(p-A)' = 0, \tag{11}$$

$$p - A = \frac{\Omega}{\omega}\frac{p}{\gamma}. \tag{12}$$

From Eq. (11), and due to the localized character of the searched solution, it follows that $v_x = 0$. Note that in the case without magnetic field, the relation $p = A$ holds (see Ref. [10]).

Then, for $\boldsymbol{B}_0 \neq 0$, the system of equations (7)–(10) reduces to the following system:

$$n = 1 + \phi'', \tag{13}$$

$$A'' + \omega^2 A = n\frac{p}{\gamma}, \tag{14}$$

$$\left(\phi - \gamma + \frac{1}{2}\frac{\Omega}{\omega}\left(\frac{p}{\gamma}\right)^2\right)' = 0, \tag{15}$$

$$\left(1 - \frac{\Omega}{\omega\gamma}\right)p = A. \tag{16}$$

By means of Eqs. (15) and (16), the density [Eq. (13)] reads

$$n = 1 + \gamma'' + \frac{1}{2}\frac{\Omega}{\omega}\left(\frac{1}{\gamma^2}\right)''. \tag{17}$$

Finally, we obtain an equation for the amplitude of the vector potential A,

$$A'' + A\left[\omega^2 - \frac{1 + \gamma'' + \frac{1}{2}\frac{\Omega}{\omega}(\gamma^{-2})''}{\gamma - \frac{\Omega}{\omega}}\right] = 0, \tag{18}$$

where the following relationship holds:

$$(\gamma^2 - 1)(\gamma - \Omega/\omega)^2 - \gamma^2 A^2 = 0. \tag{19}$$

Equations (18) and (19) completely define the soliton structure. By means of the substitutions

$$p = \sinh u, \quad \gamma = \cosh u, \quad A = \sinh u - \frac{\Omega}{\omega}\tanh u, \tag{20}$$

Eq. (18) can be written

$$\frac{d}{dx}\left[F(u)\frac{du}{dx}\right] + G(u) = 0, \tag{21}$$

where

$$F(u) = 1 - \frac{\Omega/\omega}{\cosh^3 u}, \tag{22}$$

$$G(u) = \omega^2 \sinh u\left(\cosh u - \frac{\Omega}{\omega} - \frac{1}{\omega^2}\right). \tag{23}$$

Equation (21) can be easily reduced to quadratures, since it admits an integral of motion

$$H = \frac{1}{2}\left(F(u)\frac{du}{dx}\right)^2 + U(u) = E, \tag{24}$$

where $dU/du = F(u)G(u)$, and E is an integration constant. A similar procedure was used in the determination of the trasverse field distribution in a problem of e.m. radiation self-focusing [18]. Introducing, for the sake of simplicity, the parameters

$$X = \frac{1}{\omega^2}, \quad Y = \frac{\Omega}{\omega}, \tag{25}$$

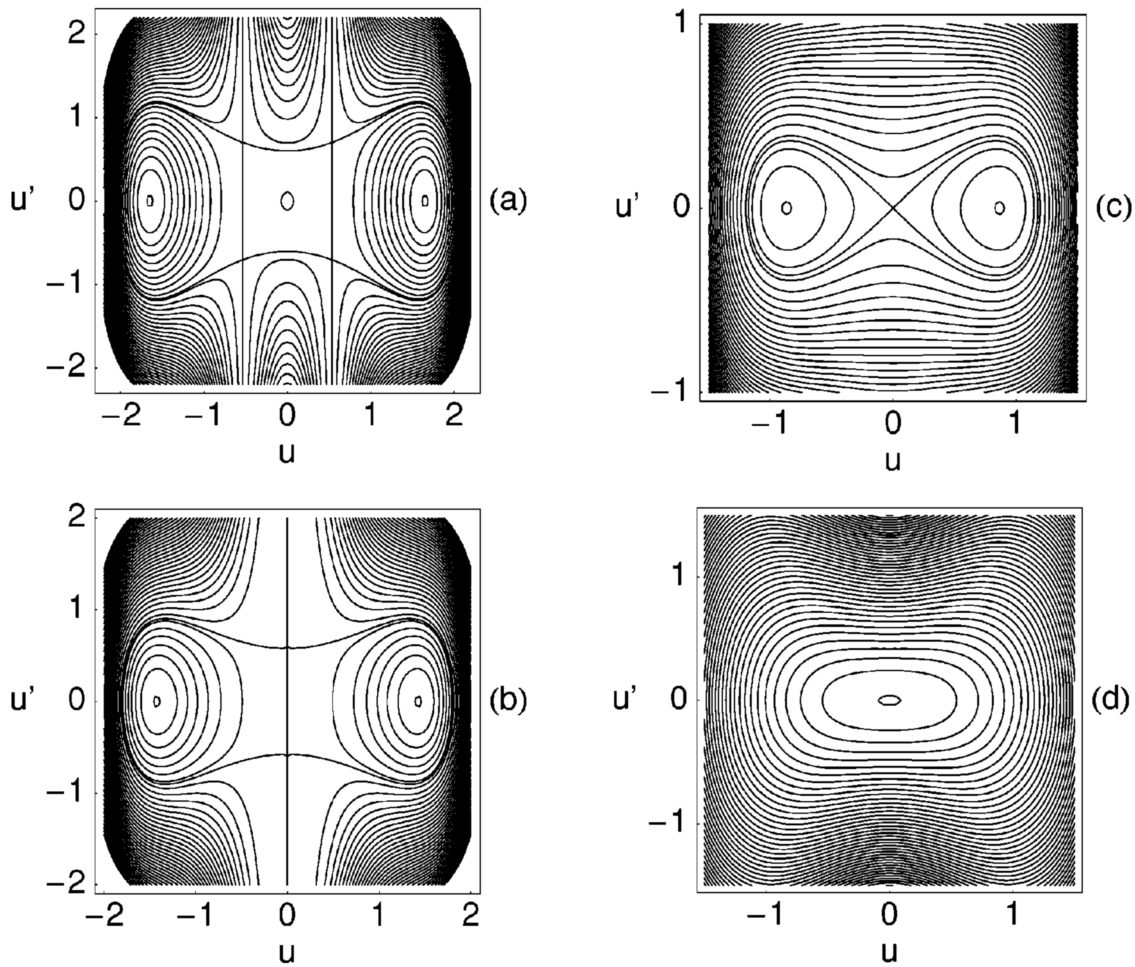

FIG. 1. Phase plot of Eq. (24) in the plane (u,u'). Cases (a), (b), (c), and (d) refer to $X=1.2$, and $Y=1.5$, 1, 0.2, and -0.5, respectively.

U reads, explicitly,

$$U=\frac{\cosh u-1}{2X}[\cosh u-(2X+2Y-1)+Y(X+Y-2)\operatorname{sech} u+Y(X+Y)\operatorname{sech}^2 u]. \tag{26}$$

III. PHASE SPACE ANALYSIS

To analyze the phase space structure of Eq. (24), we look for the solution of the system

$$\frac{\partial H}{\partial u}=0, \quad \frac{\partial H}{\partial u'}=0, \tag{27}$$

in the phase space (u,u'). Three possible types of solution are found, varying the parameters X and Y. The point $u=0$, $u'=0$, exist for any value of X and Y, and corresponds to an X point for $1-X<Y<1$, and to an O point otherwise. The point $u=\operatorname{arccosh}(X+Y)$, $u'=0$, exists only for $X+Y-1\geqslant 0$, and it is always an O point. The point $u=\operatorname{arccosh} Y^{1/3}$, $u'^2=Y^{1/3}(X+Y-Y^{1/3})/(3X)$, exists for $Y\geqslant 1$, and it has no definite character.

From the above considerations, in the parameters plane (X,Y) (with X positive) three regions can be identified, characterized by different structures of the phase space: (i) $Y\geqslant 1$, (ii) $X+Y-1>0$ and $Y<1$, and (iii) $X+Y-1<0$. The relevant contourplots of Eq. (24) are shown in Fig. 1. In the case (i) [Figs. 1(a) and 1(b)], only periodical solutions are found, since the curve which splits the phase plane in different regions is not actually a separatrix in the proper sense. In case (ii) [Fig. 1(c)], a localized solution can be found, corresponding to a true separatrix in the phase plane. Finally, in case (iii) [Fig. 1(d)], again only periodical solutions are found.

To have a better insight of the solutions, we analyze these curves in the plane (A,A'). For $Y>1$, the mapping between (u,u') and (A,A') is not unique, therefore, strictly speaking, (A,A') cannot be considered as a phase space. For $Y\leqslant 1$, the curve is a figure-8 plot. We note that for $Y<1$ the point $A=0$, $A'=0$ is a standard X point, while for $Y=1$ the curve is tangent to the axis $A=0$ at $A'=0$. Note that in the latter case, $A'\propto A^{2/3}$, around the point (0,0), which means that this point is reached in a finite ''time,'' as $A(x)\propto x^3$. As a consequence, the correponding solutions become zero over a finite spatial length, and they present an inflection point of cubic type. After that, the same e.m. pattern manifests itself indefinitely with alternating sign. Therefore, strictly speaking, $Y=1$ does not correspond to a localized solution.

In conclusion, a localized solution can be found only in case (ii). Note that region (ii) corresponds to the evanescent case for the wave under consideration in the linear approximation. From Eqs. (1)–(4), assuming a dependence of the kind $\exp(i\omega t-ikx)$, the following linear dispersion relation can be obtained:

$$k^2=\frac{1}{X}-\frac{1}{1-Y}, \tag{28}$$

which gives $k^2<0$ for $X+Y-1>0$ and $Y<1$.

IV. LOCALIZED SOLUTIONS

In the following, we shall focus on the parameter regions $X+Y-1>0$ and $Y<1$, for which a separatrix exists in the phase plane (u,u') [and in (A,A')]. Since $E=0$ on the separatrix, from Eq. (24) one obtains

$$dx=du\frac{F(u)}{\sqrt{-2U}}. \tag{29}$$

Transforming from u to $w=\tanh^2(u/2)$, and recalling that $\cosh u=(1+w)/(1-w)$, Eq. (29) can be integrated as

$$x=\int_w^{w_1}\frac{dw}{\sqrt{(w_1-w)(w-w_2)(w-w_3)}}\frac{w+1}{w}F(w), \tag{30}$$

where w_1, w_2, and w_3 are the roots of the third degree polynomial:

$$q(w)=Xw^3+w^2(1+Y)(X+Y+1)-w(2Y^2+2XY+X-2)+(Y-1)(X+Y-1). \tag{31}$$

In the considered parameter range $0<w_1<1$, while w_2, and w_3 are complex for $Y<0$, and real but negative for $0<Y<1$. Equation (30) can be easily integrated in terms of elliptic functions, thus obtaining w (and then u) implicitly as a function of x.

Once the function w is computed from Eq. (30), all the relevant physical quantities can be obtained in terms of w, i.e., $\gamma=(1+w)/(1-w)$, $p=2\sqrt{w}/(1-w)$, $A=p(1-Y/\gamma)$, and

$$\begin{aligned}n&=1+(u'\sinh uF(u))'=1-\sinh uG(u)-2\cosh u\frac{U(u)}{F(u)}\\&=\frac{1}{X(\gamma^3-Y)}[Y^2-Y\gamma-2Y\gamma^3\\&\quad+(2-2X-Y-Y^2)\gamma^4\\&\quad+3(X+Y)\gamma^5-2\gamma^6].\end{aligned} \tag{32}$$

Note that at $x=0$, $w=w_1$, and A, p, and γ are maximum, while n is minimum. Varying the parameters X, and Y, the condition $n\leqslant 0$ can be fulfilled, thus giving rise to a nonphysical result.

For any value of the magnetic field in the range $Y<1$, a *critical frequency* value X_{cr} can be found at which the density is zero at $x=0$. We now look for the occurrence of the condition $n(x=0)\equiv n(w_{1cr})=0$. Since in $x=0$, $u'=0$ and $U=0$, then

$$\begin{aligned}n(x=0)&=1-G(u)\sinh u|_{x=0}\\&=\frac{Xw_{1cr}^3+(4+X+4Y)w_{1cr}^2+(4-X-4Y)w_{1cr}-X}{X(w_{1cr}-1)^3}.\end{aligned} \tag{33}$$

The critical w_{1cr} is solution of the system

$$n(x=0)=0,\quad q(w)=0. \tag{34}$$

Introducing the parameter $S=X+Y-1$ (with $S>0$), and eliminating X from Eqs. (34), one obtains w_{1c} and Y as functions of S, with $(3-\sqrt{5})/2\leqslant S<1$:

$$Y(S-1)^3+1-2S=0, \tag{35}$$

$$w_{1cr}=\frac{S}{2-S}. \tag{36}$$

Equation (35) implicitly defines the *critical S* at which the *density becomes zero*. From Eq. (35), the explicit relations between X and Y can be found:

$$X_{cr}=-2\sqrt{\frac{2}{3Y}}\cos\left[\frac{\pi}{3}+\frac{1}{3}\arccos\left(\frac{3}{4}\sqrt{\frac{3Y}{2}}\right)\right],\quad Y>0, \tag{37}$$

$$\begin{aligned}X_{cr}=&\left(\sqrt{\frac{1}{4Y^2}-\frac{8}{27Y^3}}+\frac{1}{2Y}\right)^{1/3}\\&-\left(\sqrt{\frac{1}{4Y^2}-\frac{8}{27,Y^3}}-\frac{1}{2Y}\right)^{1/3},\quad Y<0.\end{aligned} \tag{38}$$

The minimum X_{cr} occurs at $Y=1$, and is equal to $(3-\sqrt{5})/2\approx 0.382$. In conclusion, physical localized solutions exist in the parameter range $Y<1$, and $\max(1-Y,0)<X<X_{cr}$.

The above results can also be expressed in terms of ω and Ω, although not explicitly: the soliton exists for $\max(\omega_{cr},\Omega)<\omega<\omega_{c-o}$, where $\omega_{c-o}=(\sqrt{\Omega^2+4}+\Omega)/2$ is the cutoff frequency in a magnetized plasma, and ω_{cr} is the upper ω branch of Eq. (35). Note that for $\Omega>(\sqrt{5}+1)/2$, Ω is larger than ω_{cr}. The above region is shown in Fig. 2, and is delimited by the two continuous curves in the plane (Ω,ω).

Typical results of localized solutions are shown in Figs. 3 and 4, at a fixed frequency and different magnetic fields. In Fig. 5, the behavior of the wave vector amplitude A at the critical frequency at which the density becomes zero is shown for different values of magnetic fields. Note that the amplitude and width of the soliton increase with decreasing Ω. At the same time the radiation frequency decreases. From Eqs. (36)–(38) the peak values of the different physical quantities, e.g., ϕ, γ, and A, can be computed as a function of Y at the critical density. They are monotonically decreasing functions of $Y\leqslant 1$, and at $Y=1$ reach their minimum values, $\phi=(\sqrt{5}-1)/4\approx 0.309$, $\gamma=(\sqrt{5}+1)/2\approx 1.618$, and $A=\sqrt{\sqrt{5}-2}\approx 0.486$.

An analytical explicit expression for the localized solution can be found in the limit of small amplitude. Equation (14)

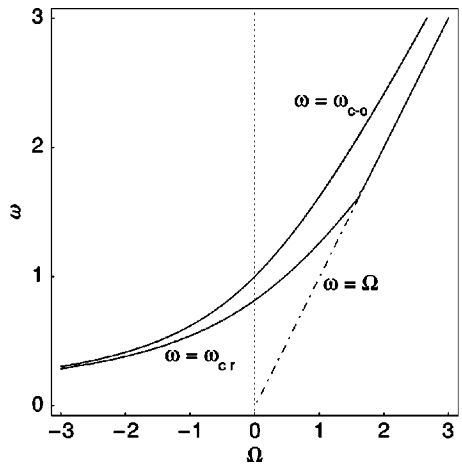

FIG. 2. Frequency region in which a localized solution of Eq. (24) is found. The region is defined by the relation $\max(\omega_{cr},\Omega)<\omega<\omega_{c-o}$, where $\omega_{c\text{-}o}$ is the upper plotted curve, and $\max(\omega_{cr},\Omega)$ the lower curve. The dash-dotted line $\omega=\Omega$ is also plotted for reference.

for A can be simplified assuming quasineutrality, and expanding the expressions for A and A' in the limit $u \ll 1$:

$$n \approx 1, \quad A \approx (1-Y)u + \frac{u^3}{6}(1+2Y),$$
$$A'' \approx (1-Y)u'', \quad \frac{np}{\gamma} \approx u - u^3/3; \tag{39}$$

thus obtaining the following nonlinear equation for u:

$$u'' - u\frac{S}{X(1-Y)} + \frac{u^3}{2}\frac{1-S/3}{X(1-Y)} = 0. \tag{40}$$

The solution of the above equation is

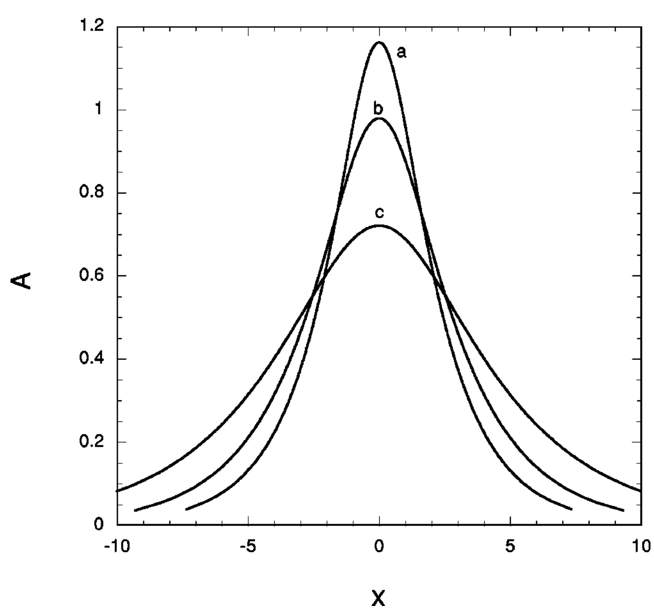

FIG. 3. Behavior of the wave vector amplitude of the soliton, as a function of x. Cases (a), (b), and (c) refer to $X=1.2$, and $Y=0.1$, 0, and -0.1, respectively.

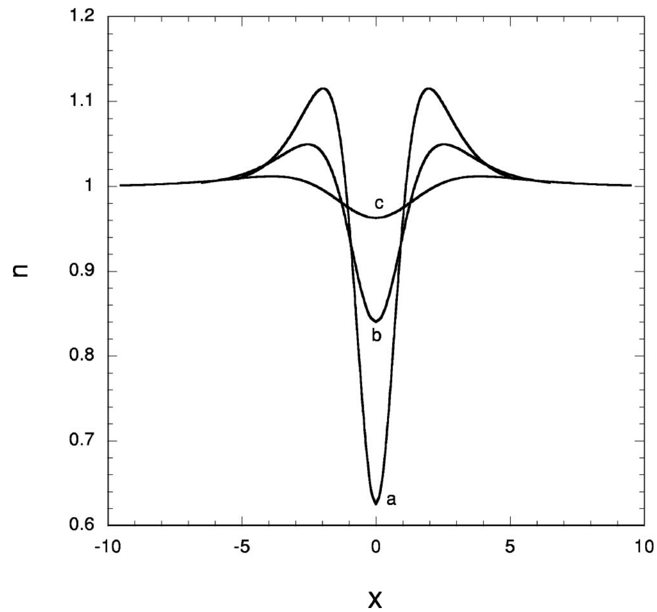

FIG. 4. Behavior of the density as a function of x, for the same parameters as in Fig. 3.

$$u = 2\sqrt{\frac{S}{1-S/3}}\,\mathrm{sech}\sqrt{\frac{S}{X(1-Y)}}\,x, \tag{41}$$

and is valid for $S \ll 1$. For $Y=0$ and $S \ll 1$, it reduces to the limit of the corresponding solution in Ref. [10]:

$$u = 2\sqrt{X-1}\,\mathrm{sech}\sqrt{\frac{X-1}{X}}x. \tag{42}$$

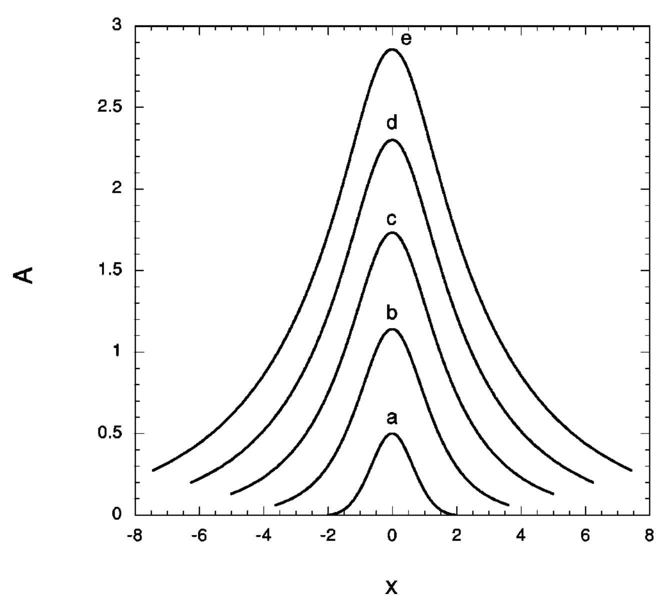

FIG. 5. Behavior of the wave vector amplitude of the soliton as a function of x, for different values of the magnetic field at the critical frequency at which the density in the center is zero. The parameters are as follows: (a) $Y=0.99$, $X_{cr}=0.395$; (b) $Y=0.5$, $X_{cr}=0.96$; (c) $Y=0$, $X_{cr}=1.5$; (d) $Y=-0.5$, $X_{cr}=2.02$; and (e) $Y=-1$, $X_{cr}=2.55$.

V. DISCUSSION AND CONCLUDING REMARKS

The problem of the existence of one-dimensional, non-drifting, localized solutions (solitonlike distributions of e.m. energy density) of the full Maxwell equations, coupled with relativistic hydrodynamic equations for the electron component, in the presence of a constant and uniform magnetic field, was investigated in the case of circularly polarized e.m. radiation. The considered model was intended to represent the occurrence of the inverse Farady effect which is known to accompany the propagation of a circularly polarized radiation wave packet, as is the case for an intense laser pulse propagating in a plasma, or for a relativistic soliton produced in its wake.

In a one-dimensional model microscopic currents associated with the circular motion of the electrons under the action of the rotating e.m. fields cancel each other; then the effects that we observe are due only to strongly nonlinear modifications of the e.m. wave dispersion properties of a magnetized plasma acted upon by relativistically intense radiation. In this paper, the net magnetization which would occur if the soliton had a finite transverse dimension was modeled through an externally given magnetic field.

As is known, a right (left) circularly polarized wave propagating along the positive x axis produces a net magnetic field directed along the negative (positive) x axis. In our investigation the right (left) polarization is recovered for $\Omega<0$ $(\Omega>0)$, with $\omega>0$. Generally speaking, the present investigation demonstrates that a stationary magnetic field heavily affects the characteristics of solitons, as it can be argued by inspection of Fig. 2. The frequency interval of stability strongly depends both on the amplitude and sign of the magnetic field. Ranges of frequency values lower and larger than in the unmagnetized case are allowed, for $B_0<0$ and $B_0>0$, respectively. The maximum field amplitudes which characterize the corresponding soliton depends on B_0, as well. Figure 5 shows an important result of our study: if the axial magnetic field generated by a right-polarized wave is such that the corresponding electron cyclotron frequency is of the same order as the radiation frequency, a much larger fraction of e.m. energy than assumed by the unmagnetized model [10] can be trapped in the density cavity which is formed. Moreover, the stability of such high energy solitons is guaranteed by a strong lowering of the frequency, to values much below the local unperturbed electron plasma frequency, as can be seen from the behavior of the allowed region in the left half-plane in Fig. 2. Since the width $\Delta\omega$ of such a region scales as $1/\Omega$ for $\Omega\to-\infty$, then, if the soliton releases its energy in vacuum, from the measurement of the radiated frequency spectrum it is possible, in principle, to infer a characteristic value of the magnetic field present in the plasma.

[1] S. V. Bulanov, T. Zh. Esirkepov, N. M. Naumova, F. Pegoraro, and V. A. Vshivkov, Phys. Rev. Lett. **82**, 3440 (1999).

[2] Y. Sentoku, T. Zh. Esirkepov, K. Mima, K. Nishihara, F. Califano, F. Pegoraro, H. Sakagami, Y. Kitagawa, N. M. Naumova, and S. V. Bulanov, Phys. Rev. Lett. **83**, 3434 (1999).

[3] S. V. Bulanov, F. Califano, T. Zh. Esirkepov, K. Mima, N. M. Naumova, K. Nishihara, F. Pegoraro, Y. Sentoku, and V. A. Vshivkov, J. Plasma Fusion Res. **75**, 506 (1999).

[4] S. V. Bulanov, F. Califano, T. Zh. Esirkepov, K. Mima, N. M. Naumova, K. Nishihara, F. Pegoraro, Y. Sentoku, and V. A. Vshivkov, Physica D (to be published).

[5] N. L. Tsintsadze and D. D. Tskhakaya, Zh. Éksp. Teor. Fiz. **72**, 480 (1977) [Sov. Phys. JETP **45**, 252 (1977)].

[6] V. E. Kozlov, A. G. Litvak, and E. V. Suvorov, Zh. Éksp. Teor. Fiz. **76**, 148 (1979) [Sov. Phys. JETP **49**, 75 (1979)].

[7] S. V. Bulanov, I. N. Inovenkov, V. I. Kirsanov, N. M. Naumova, and A. S. Sakharov, Phys. Fluids B **4**, 1935 (1992).

[8] P. Kaw, A. Sen, and T. Katsouleas, Phys. Rev. Lett. **68**, 3172 (1992).

[9] S. V. Bulanov, T. Zh. Esirkepov, F. F. Kamenets, and N. M. Naumova, Plasma Phys. Rep. **21**, 550 (1995).

[10] T. Zh. Esirkepov, F. F. Kamenets, S. V. Bulanov, and N. M. Naumova, JETP Lett. **68**, 36 (1998).

[11] A. Sh. Abdullaev, Fiz. Plasma **14**, 365 (1988) [Sov. J. Plasma Phys. **14**, 214 (1988)].

[12] I. V. Sokolov, Usp. Fiz. Nauk. **161-163**, 175 (1991) [Sov. Phys. Usp. **34**, 925 (1991)].

[13] L. M. Gorbunov and R. R. Ramazashvili, JETP **87**, 461 (1998).

[14] K. Nishinari, K. Abe, and J. Satsuma, Phys. Plasmas **1**, 3728 (1994).

[15] C. Yinhua, L. Wei, and M. Y. Yu, Phys. Rev. E **60**, 3249 (1999).

[16] C. E. Seyler and R. L. Lysak, Phys. Plasmas **6**, 4778 (1999).

[17] A. I. Akhiezer and R. V. Polovin, Zh. Éksp. Teor. Fiz. **30**, 696 (1956) [Sov. Phys. JETP **3**, 696 (1956)]; also see A. I. Akhiezer, I. A. Akhiezer, R. V. Polovin, A. G. Sitenko, and K. N. Stepanov, *Plasma Electrodynamics* (Pergamon, Oxford, 1975).

[18] T. Kurki-Suonio, P. J. Morrison, and T. Tajima, Phys. Rev. A **40**, 3230 (1989).