# Relativistic focusing and ponderomotive channeling of intense laser beams

B. Hafizi,[1] A. Ting,[2] P. Sprangle,[2] and R. F. Hubbard[2]
[1]*Icarus Research, Inc., P.O. Box 30780, Bethesda, Maryland 20824-0780*
[2]*Plasma Physics Division, Naval Research Laboratory, Washington, DC 20375-5346*
(Received 10 March 2000)

The ponderomotive force associated with an intense laser beam expels electrons radially and can lead to cavitation in plasma. Relativistic effects as well as ponderomotive expulsion of electrons modify the refractive index. An envelope equation for the laser spot size is derived, using the source-dependent expansion method with Laguerre-Gaussian eigenfunctions, and reduced to quadrature. The envelope equation is valid for arbitrary laser intensity within the long pulse, quasistatic approximation and neglects instabilities. Solutions of the envelope equation are discussed in terms of an effective potential for the laser spot size. An analytical expression for the effective potential is given. For laser powers exceeding the critical power for relativistic self-focusing the analysis indicates that a significant contraction of the spot size and a corresponding increase in intensity is possible.

PACS number(s): 52.40.Db, 52.40.Nk, 42.65.Jx, 42.65.Sf

## I. INTRODUCTION

Propagation of an intense laser beam in plasma has applications in x-ray lasers and laser-driven accelerators, among others [1–3]. On average the quiver motion of electrons in a laser beam leads to their expulsion from regions of high intensity. The expulsion is due to the ponderomotive force and sets up a space-charge (ambipolar) field that retards the electrons and eventually a quasi-steady-state may be established. The effect of the quiver motion is to reduce the local plasma frequency and can lead to so-called relativistic focusing of a laser beam [4–7]. The expulsion can enhance the focusing effect and is referred to as ponderomotive channeling [8–15]. When the laser beam is sufficiently intense complete expulsion—i.e., cavitation—can occur. Experimental observations of relativistic focusing and ponderomotive channeling have been reported in Refs. [16–21].

Analysis of the relativistic effect has shown that focusing occurs when the laser power $P$ exceeds a critical power $P_c$. In Ref. [6] an envelope equation for the laser spot size was derived with the relativistic effect included. An envelope equation describes the variation of the spot size with propagation distance as a function of the ratio $P/P_c$. Numerous other analytical and numerical studies of intense laser beam propagation in plasmas have revealed many fascinating details. In some applications the time scales of interest are such that the ions can be assumed to be stationary. Making use of the relativistic cold electron fluid equations along with the Maxwell equations, key physics issues related to beam dynamics, including the effects of relativistic focusing and ponderomotive channeling, can be studied. For underdense plasma the equations can be greatly simplified, requiring the solution of a reduced, nonlinear wave equation in three space dimensions. An analysis of the wave equation in Ref. [8] established the possibility of electron cavitation for sufficiently large $P/P_c$ and, based on numerical solutions, an estimate for the threshold value was obtained. A highlight of the studies in Refs. [10] and [11] was the elucidation of the detailed radial mode structure of the laser beam, from which an improved value for the threshold power was obtained. Following a transient period the propagation of a laser beam in plasma can settle to a stationary regime wherein the beam profile is invariant. This is referred to as *matched* beam propagation. In Ref. [9] special matched beam solutions in slab geometry were obtained and compared with results from a particle simulation code. The studies in Ref. [12] show that although the central portion of a finite-length laser pulse can be guided, the leading edge of the pulse is subject to diffractive spreading. In Ref. [13] propagation of an intense laser beam in plasma was analyzed in general terms by making use of two global invariants (constants of motion) associated with the reduced nonlinear wave equation. Writing the envelope equation for the laser beam in terms of these two invariants (and other quantities), it is possible to derive a necessary condition for focusing of the laser beam. Relativistic focusing and ponderomotive channeling are not, of course, the only processes taking place in plasma in the presence of an intense laser field. For example, the generation of plasma waves and Raman scattering can modify the propagation dynamics significantly [12,15]. Moreover, on the longer time scale, ion motion will affect cavitation and guiding [22].

In this paper the propagation of an intense laser beam in plasma is analyzed. In the expression for the refractive index the contributions due to relativistic effects and radial ponderomotive displacement of electrons are identified. Analysis of the wave equation leads to explicit formulas that can be numerically evaluated, allowing the effects of ponderomotive channeling and relativistic focusing to be readily computed. Specifically, an envelope equation for the spot size, including relativistic focusing *and* ponderomotive expulsion of electrons, is derived. Solutions of the envelope equation are discussed in terms of an effective potential for the spot size. An analytical expression for the effective potential is obtained and discussed. Solutions of the envelope equation are compared with previous results that neglected ponderomotive channeling. The utility of the generalized envelope equation is that it permits analytical determination of the spot size. In particular, for a given $P/P_c$, the matched (i.e., equilibrium) spot size may be readily calculated. For laser powers exceeding the critical power for relativistic self-focusing the analysis indicates a significant contraction of the spot size and a corresponding increase in intensity.

## II. THREE-DIMENSIONAL NONLINEAR REFRACTIVE INDEX

Propagation of a laser beam in a medium may be described in terms of the refractive index $\eta$. To derive an expression for the refractive index, including relativistic effects and ponderomotive channeling, the relativistic cold fluid equations and the wave equation may be employed. The equations are simplified by effecting the change of variables $(z,t)\to(\xi,\tau)$, where $\xi=z-ct$, $\tau=t$, and then making the quasistatic approximation that the fluid variables change little during transit through a laser pulse of duration $\tau_L$ [23,24]. The field and fluid variables $Q$ are expanded as $Q_f+Q_s$, where $|\partial Q_f/\partial\xi|\sim|kQ_f|$ for the rapidly varying (suffix $f$) parts and $|\partial Q_s/\partial\xi|\sim\min[k_p|Q_s|,|Q_s|/(c\tau_L)]$ for the slowly varying (suffix $s$) parts. Here, $k=\omega/c$ is the free-space wavenumber along the propagation direction $z$, $\omega$ is the laser frequency, and $k_p=[4\pi|e|^2n_0/(mc^2)]^{1/2}$ is the plasma wave number evaluated with the unperturbed density $n_0$. The resulting equations are then expanded to first order in powers of $(kr_s)^{-1}\ll 1$ and $k_p/k\ll 1$, where $r_s$ is the laser spot size.

The wave equation for the normalized vector potential can be written as [3]

$$\left(\nabla_\perp^2+2ik\frac{\partial}{\partial z}\right)\hat{\mathbf{a}}_f=k^2(1-\eta^2)\hat{\mathbf{a}}_f, \tag{1}$$

where the refractive index for circularly polarized electromagnetic waves in the long pulse limit $(ck_p\tau_L\gg 1)$ is expressible as

$$\eta(r,z)=1-\frac{k_p^2[1+k_p^{-2}\nabla_\perp^2(1+|a|^2)^{1/2}]}{2k^2(1+|a|^2)^{1/2}}. \tag{2}$$

Here, $\mathbf{a}=|e|\mathbf{A}/(mc^2)$, $\mathbf{A}$ is the vector potential, $\mathbf{a}_f=(\hat{\mathbf{a}}_f/2)\exp(ik\xi)+\text{c.c.}$, $\hat{\mathbf{a}}_f=a(r,z)(\mathbf{e}_x+i\mathbf{e}_y)$, $\mathbf{e}_x$ and $\mathbf{e}_x$ are unit vectors along the $x$ and $y$ axes, respectively, $a$ is the slowly varying amplitude of the vector potential, and the Coulomb gauge $(\operatorname{div}\mathbf{A}=0)$ has been employed. Strictly speaking, the operator $\partial/\partial z$ in Eq. (1) should read $c^{-1}\partial/\partial\tau$. However, for the purpose here it is more convenient to consider the evolution of the laser beam with distance and the error incurred in replacing $c\tau$ with the propagation distance is negligible.

The first term in Eq. (2) represents free-space propagation and the two terms in the square brackets correspond to the plasma contribution. The ''1'' in the square brackets, modified by the denominator, leads to self-focusing due to the relativistic variation of mass, while the term involving the transverse Laplacian operator takes account of the decrease in electron density due to the ponderomotive force. This term is responsible for ponderomotive channeling. The relative density perturbation is given by

$$\frac{\delta n(r,z)}{n_0}=k_p^{-2}\nabla_\perp^2(1+|a|^2)^{1/2}, \tag{3}$$

provided the density is everywhere non-negative, i.e., $\delta n+n_0\geq 0$.

Interaction of an intense laser beam with plasma is usually accompanied by myriad instabilities [2,3,25–35] that can affect the propagation of the pulse. Instabilities are neglected in the present analysis for simplicity; in particular, the generation of plasma waves due to, for example, Raman forward scattering is ignored.

## III. ENVELOPE EQUATION

The operator on the left-hand side of Eq. (1) is in the standard paraxial form. To solve it, the source-dependent expansion (SDE) technique, with Laguerre-Gaussian basis functions, is employed and $a(r,z)$ is expanded as [36,37]

$$a(r,z)=\sum_{m=0}^{\infty}a_m(z)D_m(r,z), \tag{4a}$$

where

$$D_m(r,z)=L_m\left[\frac{2r^2}{r_s^2(z)}\right]\exp\{-[1-i\alpha(z)]r^2/r_s^2(z)\}, \tag{4b}$$

$L_m$ is the Laguerre polynomial of order $m$ and $\alpha$ is proportional to the wave-front curvature. Observe that $r_s$ and $\alpha$ are, in general, functions of the propagation distance $z$. The virtue of the SDE method is that the fundamental amplitude $a_0$ is dominant, i.e., $|a_0|\gg|a_{m>0}|$. Assuming this, the envelope equation for the spot size is readily shown to be given by [37]

$$\frac{\partial^2 r_s}{\partial z^2}-\frac{4}{k^2r_s^3}-\frac{4}{r_s}G=0, \tag{5a}$$

where $G$ is given by

$$G=\tfrac{1}{2}\int_0^\infty d\chi(1-\eta^2)(1-\chi)\exp(-\chi), \tag{5b}$$

and $\chi=2r^2/r_s^2$. Writing $|a_0|=\hat{a}\hat{r}/r_s$, where $\hat{a}$ and $\hat{r}$ are the vacuum amplitude and minimum spot size at focus, respectively, making use of the expression for $\eta$ in Eq. (2) and performing the integral in Eq. (5b) (Appendix), the equation for the laser beam envelope may be written as

$$\frac{d^2X}{dz^2}+(\hat{a}^2Z_R)^{-2}\frac{\partial V}{\partial X}=0, \tag{6a}$$

where $V$ is defined by

$$\frac{\partial V}{\partial X}=-16\frac{P}{P_c}X\{1-(1+X^{-2})^{1/2}-2\ln 2+2\ln[1+(1+X^{-2})^{1/2}]\}-\frac{\ln(1+X^{-2})}{X}, \tag{6b}$$

the scaled spot size is defined by

$$X=\frac{r_s}{\hat{a}\hat{r}}, \tag{6c}$$

$Z_R = k\hat{r}^2/2$ is the Rayleigh range *in vacuo*, and $P/P_c = (k_p \hat{a}\hat{r}/4)^2$ is the ratio of the laser beam power to the critical power for relativistic focusing. The expression for $P/P_c$ given here is based on the fundamental Gaussian representation for the radial profile of the laser beam. More accurate numerical solutions of the wave equation lead to a slightly larger value for $P/P_c$ [8,11].

Equation (6) may be used to analyze the combined effects of relativistic focusing and ponderomotive channeling. Equation (6a) represents oscillations of a ''particle,'' represented by $X$, in an effective potential $V(X, P/P_c)$ and can be integrated once to obtain

$$\frac{1}{2}\left(\frac{dX}{dz}\right)^2 + (\hat{a}^2 Z_R)^{-2} V = \text{const}, \tag{7a}$$

where

$$V = 16\frac{P}{P_c} X^2\{(1+X^{-2})^{1/2} - 1 + \ln 2 - \ln[1+(1+X^{-2})^{1/2}]\} - \tfrac{1}{2}\mathrm{li}_2(-X^{-2}), \tag{7b}$$

and $\mathrm{li}_2(x) = \int_x^0 dt[\ln(1-t)]/t$ is the dialogarithm function [38].

The threshold condition for bound solutions of a collimated (i.e., parallel) incident laser beam with large spot size can be examined by expanding the effective potential for $X \to \infty$. In this limit Eq. (6a) reduces to

$$\frac{d^2X}{dz^2} + (\hat{a}^2 Z_R)^{-2}\left(\frac{P/P_c - 1}{X^3} + \frac{3 - 4P/P_c}{6X^5}\right) \approx 0. \tag{8}$$

It is clear that focusing can take place provided $P$ exceeds $P_c$. Depending on the value of $P/P_c$ the effective potential can have a single minimum, corresponding to a matched (equilibrium) solution for the scaled spot size $X_m$. It follows from Eq. (7) that $P/P_C$ determines the depth and location of the minimum of the effective potential, while the spatial scale length for focusing is set by $\hat{a}^2 Z_R$. The analysis can be generalized to the case of a long but finite laser pulse by the substitution $\hat{a} \to \hat{a}\exp[-(\xi/c\tau_L)^2]$ if the longitudinal pulse profile is a Gaussian, for example.

## IV. SOLUTIONS OF THE ENVELOPE EQUATION

The envelope equation, Eq. (6), can be used to study the spatial variation of the spot size of a laser beam propagating in plasma. Equation (7) is the first integral of the envelope equation and is useful for visualizing the possible solutions since the evolution of the spot size $X$ is akin to the motion of a particle in an effective potential $V(X, P/P_c)$.

Figure 1 displays the effective potential as a function of scaled spot size $X = r_s/(\hat{a}\hat{r})$ for $P/P_c = 1.2$. The curve labeled (a) in Fig. 1(a) takes into account relativistic focusing only, while for the curve labeled (b) the contributions due to relativistic focusing as well as ponderomotive channeling are included. Figure 1 is shown here for direct comparison with the corresponding plots in Refs. [3] and [6]. In these references, ponderomotive channeling was not considered in the effective potential, as in curve (a). The analysis here and in Refs. [3] and [6] derive an effective potential that has a single minimum. However, it is apparent from Fig. 1 that ponderomotive expulsion of electrons leads to a significant reduction in the matched beam spot size. Additionally, the region around the minimum of the effective potential for curve (b) is observed to be narrower than that for curve (a). Figure 1 shows that in general a laser beam that is propagating in plasma with $P/P_c = 1.2$ will undergo envelope oscillations about a minimum of $V$. Similar envelope oscillations have been previously described, based on numerical solutions of the wave equation [8–13] or by employing an effective potential in slab geometry [9]. A matched (equilibrium) beam solution to Eq. (6a) or (7a) refers to the value of the spot size at the minimum of $V$. The analytical form for the effective potential in Eq. (7b) can be used to obtain the matched beam solution for any value of $P/P_c$.

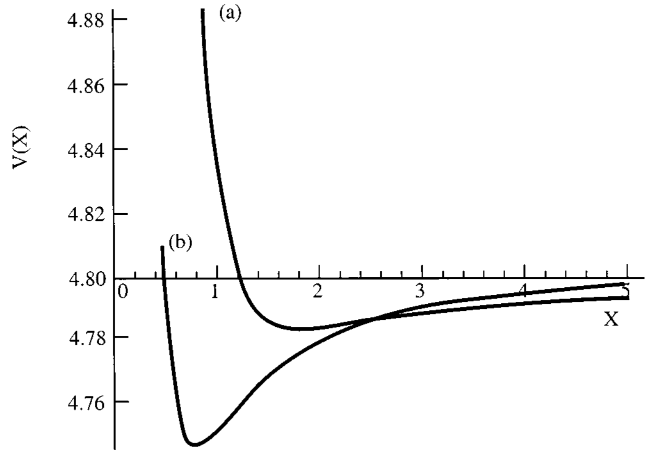


FIG. 1. Plot of effective potential $V(X)$ versus scaled spot size $X = r_s/(\hat{a}\hat{r})$. The ratio of laser power to critical power for relativistic focusing $P/P_c = 1.2$. For curve (a) the effective potential includes only the effects of relativistic focusing while for curve (b) both relativistic focusing and ponderomotive channeling are included.

Multiplying Eq. (1) from the left by $\hat{\mathbf{a}}_f^*$ and adding the resulting expression to that obtained by left multiplying the complex conjugate of Eq. (1) by $\hat{\mathbf{a}}_f$, it follows that

$$\int\int dx\, dy\, \hat{\mathbf{a}}_f^* \cdot \hat{\mathbf{a}}_f = \text{const},$$

independent of $z$ [8,11–13]. This expresses the invariance of power as the laser beam spot size evolves. For the fundamental Gaussian this conservation law reduces to $|a_0| r_s = \hat{a}\hat{r} = \text{const}$. Conservation of power implies that when a collimated, large spot size laser beam is injected into a plasma significant contraction of the beam inevitably results in a correspondingly large rise in the intensity.

Making use of Eq. (3) the on-axis density perturbation can be written in terms of $P/P_c$ and $X$ as follows:

$$\frac{\delta n(r=0)}{n_0} = -\frac{P_c}{P}\frac{1}{(2X^2)^2(1+X^{-2})^{1/2}}.$$

While the right-hand side of this expression is proportional to $P_c/P$, it should be borne in mind that $X$ is a function of $P/P_c$ and hence this expression does not display the complete scaling of the density perturbation with laser power. It

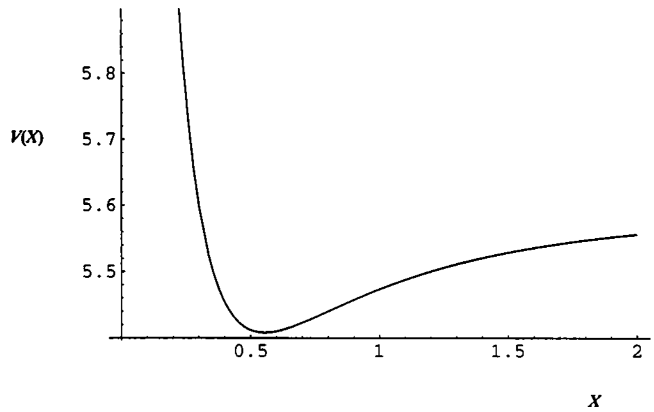


FIG. 2. Plot of effective potential $V(X)$ versus scaled spot size $X=r_s/(\hat{a}\hat{r})$. The ratio of laser power to critical power for relativistic focusing $P/P_c=1.4$.

should be recalled that the only permissible solutions are those for which $\delta n/n_0 \geqslant -1$, and that the analysis here breaks down when (complete) cavitation sets in. The model Eqs. (1) and (2) do not guarantee $\delta n(r,z)/n_0 \geqslant -1$ and thus it is necessary to verify this requirement when solving Eqs. (6) or (7) [8–11,13,22]. All the results presented here satisfy this requirement.

Figure 2 shows the effective potential as a function of $X$ for a slightly larger value of $P/P_c(=1.4)$. This example is of interest since for $P/P_c \approx 1.42$ (complete) cavitation is observed. For $P/P_c=1.4$ the plasma is nearly cavitated at the matched value for the scaled spot size $X_m \approx 0.5543$.

Figure 3 shows a surface plot of the effective potential as a function of both $X$ and $P/P_c$. This plot shows that as the ratio $P/P_c$ is increased the matched beam solution $X_m$ decreases monotonically.

Figure 4 shows plots of the variation of $X$ as a function of the normalized axial coordinate $\overline{z}=z/(\hat{a}^2 Z_R)$. These plots can be obtained from either the numerical solution of Eq. (6a) or a single numerical integration in Eq. (7a). In Fig. 4(a), $P/P_c=1.2$ and the initial conditions are $X=1.8$, $dX/d\overline{z}=0$ (i.e., no initial velocity). In this example the initial conditions are such that the ''particle'' is bound inside the effective potential well. The scaled spot size pinches down by nearly a factor of 3 and oscillates indefinitely about an equilibrium beam radius. Observe that the oscillations are not simple harmonic since the effective potential is not parabolic. For comparison, Fig. 4(b) shows the scaled spot size as a function of scaled axial distance for the same initial conditions as in Fig. 4(a) but with the ponderomotive channeling contribution to the effective potential arbitrarily deleted, i.e., with Eq. (6b) replaced by

$$\frac{\partial V}{\partial X}=-16\frac{P}{P_c}X\{1-(1+X^{-2})^{1/2}-2\ln 2 +2\ln[1+(1+X^{-2})^{1/2}]\}-\frac{1}{X^3}, \tag{9}$$

cf. Refs. [3] and [6]. With only relativistic focusing active, Fig. 4(b) shows that the beam envelope first expands and then performs oscillations with a longer period of oscillation than observed in Fig. 4(a). The oscillation amplitude is small and corresponds to a nearly matched initial condition.

Finally, Fig. 5 shows the variation of $X$ as a function of $\overline{z}$ for $P/P_c=1.01$ and the initial conditions $X=1$, $dX/d\overline{z}=-0.1$ (i.e., an inward initial velocity). In this example the particle is not bound to the relatively weak potential well. Thus, after pinching in, the spot size turns around and expands indefinitely.

## V. CONCLUSIONS

A powerful laser beam focused on plasma can be stably guided by a combination of relativistic focusing and ponderomotive channeling over extended distances. An envelope equation for the laser spot size has been obtained that can be used to describe the axial evolution of the spot size as a function of the ratio of laser power $P$ to the critical power for relativistic focusing $P_c$. Depending on the initial beam spot size and divergence, the envelope (i.e., radius) of a laser beam that is incident on a plasma will oscillate with propa-

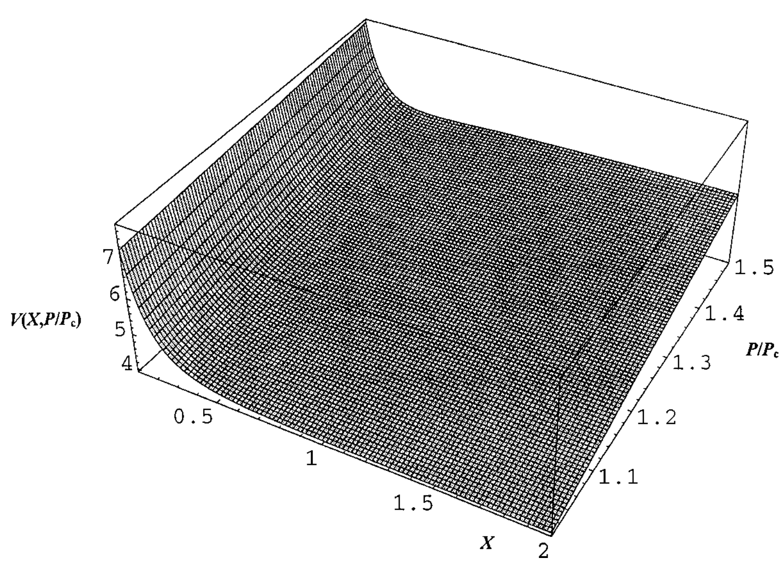


FIG. 3. Surface plot of effective potential $V(X,P/P_c)$ versus scaled spot size $X=r_s/(\hat{a}\hat{r})$ and the ratio of laser power to critical power for relativistic focusing $P/P_c$.

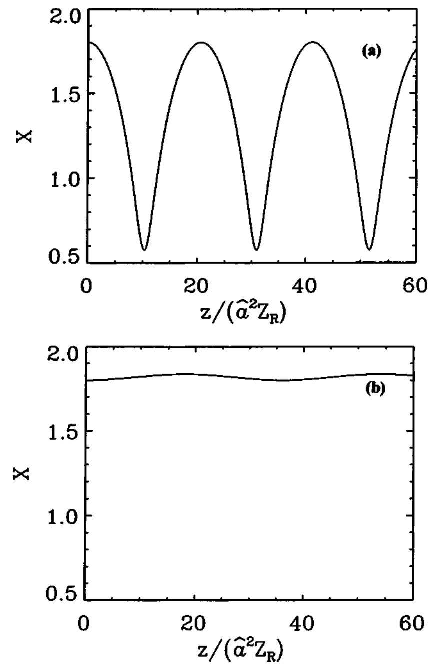


FIG. 4. Variation of scaled spot size $X = r_s/(\hat{a}\hat{r})$ with scaled axial coordinate $\overline{z} = z/(\hat{a}^2 Z_R)$ for $P/P_c = 1.2$. The initial conditions are $X = 1.8$ and $dX/d\overline{z} = 0$, i.e., no initial velocity. In (a) both relativistic focusing and ponderomotive channeling are included, based on Eqs. (6a) and (6b). In (b) only relativistic focusing is included, based on Eqs. (6a) and (9).

gation distance provided $P/P_c > 1$. Oscillations of the spot size or beam spreading can be described in terms of an effective potential that is given by an analytical function and includes the effects of both relativistic focusing and ponderomotive channeling. It is shown that ponderomotive channeling can lead to significant enhancement of the focusing effect.

## ACKNOWLEDGMENTS

The authors are grateful to Dr. A.K. Ganguly for comments and to Dr. J.R. Penano for assistance. This work was supported by the Department of Energy and the Office of Naval Research.

## APPENDIX

In this appendix an outline of the evaluation of the integral in Eq. (5b) is given. First the expression for $\eta$ in Eq. (2) is squared and inserted into Eq. (5b). The integral of the term proportional to $k_p^2$ leads to the relativistic contributions, proportional to $P/P_c$ in the effective potential, and has been considered in Refs. [3] and [6]. To integrate the term involving the Laplacian, which is due to the ponderomotive force, the change of variables $\chi = 2r^2/r_s^2$ is effected, leading to

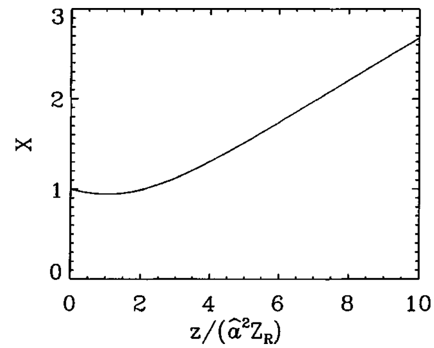


FIG. 5. Variation of scaled spot size $X = r_s/(\hat{a}\hat{r})$ with scaled axial coordinate $\overline{z} = z/(\hat{a}^2 Z_R)$. In this example, $P/P_c = 1.01$, $X = 1$ and the initial velocity is inward, i.e., $dX/d\overline{z} = -0.1$.

$$G_{\text{pond}} = \frac{-|a_0|^2}{k^2 r_s^2} \int_0^\infty d\chi \frac{\exp(-2\chi)}{1 + |a_0|^2 \exp(-\chi)}, \tag{A1}$$

where the suffix ''pond'' indicates that only the contribution due to the ponderomotive term is included. By differentiation it can be shown that

$$\int^{\chi} d\chi' \frac{\exp(-2\chi')}{1 + |a_0|^2 \exp(-\chi')} = -\frac{\exp(-\chi)}{|a_0|^2} + \frac{\ln[1 + |a_0|^2 \exp(-\chi)]}{|a_0|^4}, \tag{A2}$$

whence Eq. (A1) reduces to

$$G_{\text{pond}} = \frac{1}{k^2 r_s^2 |a_0|^2} [\ln(1 + |a_0|^2) - |a_0|^2]. \tag{A3}$$

Rewriting Eq. (A3) in terms of the scaled spot size $X$ leads to the result in Eq. (6b). Interestingly, the second term in Eq. (A3) exactly cancels the $1/X^3$ term in the expression for $\partial V/\partial X$ in Refs. [3] and [6].


[1] E. Esarey, P. Sprangle, J. Krall, and A. Ting, IEEE Trans. Plasma Sci. **PS-24**, 252 (1996).

[2] W. B. Mori, IEEE J. Quantum Electron. **QE-33**, 1879 (1997).

[3] E. Esarey, P. Sprangle, J. Krall, and A. Ting, IEEE J. Quantum Electron. **33**, 1879 (1997).

[4] F. S. Felber, Phys. Fluids **23**, 1410 (1980).

[5] G. Schmidt and W. Horton, Comments Plasma Phys. Control. Fusion **9**, 85 (1985).

[6] P. Sprangle, C. M. Tang, and E. Esarey, IEEE Trans. Plasma Sci. **PS-15**, 145 (1987).

[7] P. Chessa, P. Mora, and T. Antonsen, Phys. Plasmas **5**, 3451 (1998).

[8] G. Z. Sun, E. Ott, Y. C. Lee, and P. Guzdar, Phys. Fluids **30**, 526 (1987).

[9] T. Kurki-Suonio, P. J. Morrison, and T. Tajima, Phys. Rev. A **40**, 3230 (1989).

[10] A. B. Borisov, A. V. Borovskiy, V. V. Korobkin, A. M. Prokhorov, C. K. Rhodes, and O. B. Shiryaev, Phys. Rev. Lett. **65**, 1753 (1990).

[11] A. B. Borisov, A. V. Borovskiy, O. B. Shiryaev, V. V. Ko-

robkin, A. M. Prokhorov, J. C. Solem, T. S. Luk, K. Boyer, and C. K. Rhodes, Phys. Rev. A **45**, 5830 (1992).
[12] L. A. Abramyan, A. G. Litvak, V. A. Mironov, and A. M. Sergeev, Zh. Eksp. Toer. Fiz. **102**, 1816 (1992) [Sov. Phys. JETP **75**, 978 (1992)].
[13] X. L. Chen and R. N. Sudan, Phys. Rev. Lett. **70**, 2082 (1993).
[14] R. Annou, V. K. Tripathi, and M. P. Srivastava, Phys. Plasmas **3**, 1356 (1996).
[15] K. C. Tzeng and W. B. Mori, Phys. Rev. Lett. **81**, 104 (1998).
[16] A. B. Borisov, A. V. Borovskiy, V. V. Korobkin, A. M. Prokhorov, O. B. Shiryaev, X. M. Shi, T. S. Luk, A. McPherson, J. C. Solem, K. Boyer, and C. K. Rhodes, Phys. Rev. Lett. **68**, 2309 (1992).
[17] P. Monot, T. Auguste, P. Gibbon, F. Jakober, G. Mainfray, A. Dulieu, M. Louis-Jacquet, G. Malka, and J. L. Miquel, Phys. Rev. Lett. **74**, 2953 (1995).
[18] K. Krushelnick, A. Ting, C. I. Moore, H. R. Burris, E. Esarey, P. Sprangle, and M. Baine, Phys. Rev. Lett. **78**, 4047 (1997).
[19] R. Wagner, S. Y. Chen, A. Maksimuchuk, and D. Umstadter, Phys. Rev. Lett. **78**, 3125 (1997).
[20] S. Y. Chen, G. S. Sarkisov, A. Maksimchuk, R. Wagner, and D. Umstadter, Phys. Rev. Lett. **80**, 2610 (1998).
[21] A. B. Borisov, J. W. Longworth, K. Boyer, and C. K. Rhodes, Proc. Natl. Acad. Sci. USA **95**, 7854 (1998).
[22] M. D. Feit, J. C. Garrison, and A. M. Rubenchik, Phys. Rev. E **56**, R2394 (1997).
[23] P. Sprangle, E. Esarey, and A. Ting, Phys. Rev. Lett. **64**, 2011 (1990); Phys. Rev. A **41**, 4463 (1990).
[24] M. D. Feit, J. C. Garrison, and A. M. Rubenchik, Phys. Rev. E **53**, 1068 (1996).
[25] T. M. Antonsen and P. Mora, Phys. Rev. Lett. **69**, 2204 (1992).
[26] C. J. McKinstrie and R. Bingham, Phys. Fluids B **4**, 2626 (1992).
[27] N. A. Andreev, L. M. Gorbunov, V. I. Kirsanov, A. A. Pogosova, and R. R. Ramazashvili, Pis'ma Zh. Eksp. Toer. Fiz. **55**, 551 (1992) [JETP Lett. **55**, 571 (1992)].
[28] J. Krall, A. Ting, E. Esarey, and P. Sprangle, Phys. Rev. E **48**, 2157 (1993).
[29] X. L. Chen and R. N. Sudan, Phys. Fluids B **5**, 1336 (1993).
[30] W. B. Mori, C. D. Decker, D. E. Hinkel, and T. Katsouleas, Phys. Rev. Lett. **72**, 1482 (1994).
[31] C. D. Decker, W. B. Mori, and T. Katsouleas, Phys. Rev. E **50**, R3338 (1994).
[32] K. C. Tzeng, W. B. Mori, and C. D. Decker, Phys. Rev. Lett. **76**, 3332 (1996).
[33] P. Mora and T. M. Antonsen, Phys. Plasmas **4**, 217 (1997).
[34] P. Sprangle, E. Esarey, and B. Hafizi, Phys. Rev. Lett. **64**, 2011 (1997); Phys. Rev. E **41**, 4463 (1997).
[35] P. Sprangle and B. Hafizi, Phys. Plasmas **64**, 2011 (1999).
[36] P. Sprangle, A. Ting, and C. M. Tang, Phys. Rev. Lett. **59**, 202 (1987); Phys. Rev. A **36**, 2773 (1987).
[37] P. Sprangle, B. Hafizi, and J. R. Penano, Phys. Rev. E **61**, 4381 (2000).
[38] The function $li_2(x)$ can be expressed as $li_2(x)=f(1-x)$, the latter function being given by I. A. Stegun in *Handbook of Mathematical Functions*, edited by M. Abramowiitz and I. A. Stegun, National Bureau of Standards Applied Mathematics Series No. 55 (U.S. Government Printing Office, Washington, D.C., 1964), p. 1004. For $|x|<1$ the dialogarithm function has the uniformly and absolutely convergent series expansion $li_2(x)=\Sigma_{k=1}^{\infty}x^k/k^2$.