Theory for matrix elements of one-body transition operators in the quantum chaotic domain of interacting particle systems

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Demonstrating the equivalence between the recent theory of Flambaum and collaborators which is based on smoothed strength functions, with the much earlier formulation due to French and collaborators which is based on embedded random matrix ensembles and smoothed transition strength densities, we derive a theory for matrix elements of one-body transition operators in the quantum chaotic domain of isolated finite interacting particle systems with a mean-field and a chaos generating two-body interaction (V). The role of the bivariate correlation coefficient (ζ) arising out of the noncommutability of V and the transition operator (in the theory of Flambaum *et al.*, $\zeta=0$) is tested in numerical embedded ensemble calculations with a one- plus two-body Hamiltonian generating order-chaos transitions.

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In the last few years the study of quantum chaos in isolated finite interacting particle systems has turned from spectral statistics to properties of eigenfunctions and transition strengths. For the former the classical random matrix ensembles [Gaussian orthogonal ensemble (GOE), Gaussian unitary ensemble (GUE), etc.] provide the predictions. For the latter, it has been recently recognized by a large number of research groups in atomic, molecular, nuclear, and mesoscopic physics [1–6] that embedded random matrix ensembles (EEs) and in particular EGOE(k), the embedded Gaussian orthogonal ensemble of random matrices of k-body interactions, are relevant.

EGOE(k) for many (m) fermion systems is generated by defining the Hamiltonian H, which is, say, k body, to be GOE in k-particle space and then propagating it to m-particle spaces by using the geometry of the m-particle spaces [7]. Here one assumes that the m-particle space is a direct product space, of single-particle states (say, N in number), for example, as in the nuclear shell model. In many situations Hamiltonians for interacting particle systems contain a mean-field producing part (one-body part h) and a two-body residual interaction V mixing the configurations built out of the distribution of particles in the mean-field single-particle states; h is defined by single-particle energies $\epsilon_i, i=1-N$, and V is defined by two-particle matrix elements. Then it is more realistic to use EGOE(1+2), the embedded ensemble of (1+2)-body Hamiltonians defined by $\{H\}=[h(1)]+\lambda\{V(2)\}$ where $\{V\}$ is EGOE(2) and $[h]$ is a fixed Hamiltonian or an ensemble with single-particle energies chosen random but following some distribution; $[h]$ and $\{V\}$ are independent. In the literature EGOE(1+2) [or the more general EE(1+2) where the matrix elements of $\{V\}$ in two-particle space may or may not be Gaussian distributed] is also called TBRIM (two-body random interaction model) [2]. It is to be expected that the generic features of the EGOE(1+2) approach those of EGOE(2) for sufficiently large values of λ and significant results emerge as λ is varied starting from $\lambda=0$; the first study with a λ variation [using EE(1+2) instead of EGOE(1+2)], for observables in rotating nuclei, is due to Åberg [8].

The nature of occupancies (of single-particle states), strength functions, information entropy (S^{info}), inverse participation ratio (IPR), transition strength sums (for example, Gamow-Teller strength sums in nuclei), matrix elements (transition strengths) of one-body transition operators, interaction-driven thermalization, Fock-space localization, etc., in the chaotic domain of interacting particle systems is being studied in several systems in an attempt to characterize quantum chaos in many particle systems [1–6,8–14]. EGOE results for S^{info} and IPR are reported in [3] and there is a newly emerging understanding, obtained via the study of occupancies and strength sums [4,11,14], that in the chaotic domain of isolated finite interacting many-particle systems smoothed densities (they include strength functions) define the statistical description of these systems and these densities follow from EEs. This paper deals with chaos in relation to matrix elements of one-body transition operators. In particular, for systems with a mean-field and a chaos generating two-body interaction V, the seemingly different formulations due to French, Kota, Pandey, Tomsovic, and Majumdar (FKPTM) [9,15–17] and Flambaum, Gribakina, Gribakin, Kozlov, and Ponomarev (FGGKP) [1,13,18] are analyzed, in this paper, in order to establish a theory for matrix elements of one-body transition operators in the quantum chaotic domain of isolated finite interacting particle systems. In addition numerical EGOE(1+2) calculations are presented for testing the theory. Let us begin with a brief discussion of the FKPTM formulation.

Given a Hamiltonian H and its m-particle eigenstates $|E\rangle$, the transition strengths or matrix elements generated by a transition operator $\mathcal{O}$ are $|\langle E_f|\mathcal{O}|E_i\rangle|^2$. As discussed in detail in [19], in general the state densities $I^{H,m}(E)=\langle\langle\delta(H-E)\rangle\rangle^m=d(m)\rho^{H,m}(E)$ for EGOE(k) take a Gaussian ($\mathcal{G}$) form, i.e., $I^{H,m}(E)\rightarrow I_{\mathcal{G}}^{H,m}(E)$, and similarly $\rho^{H,m}(E)$ which is normalized to unity. Note that $\langle\langle\ \rangle\rangle$ denotes trace, $d(m)$ is the dimension of the m-particle space, and $I_{\mathcal{G}}^{H,m}$ is defined by its centroid (ϵ) and width (σ). In addition, it is also known [9] that the bivariate strength densities (matrix elements of $\mathcal{O}$ weighted by the state densities at the initial and final energies) $I_{biv;\mathcal{O}}^{H;m_i m_f}(E_i,E_f)=\langle\langle\mathcal{O}^\dagger\delta(H-E_f)\mathcal{O}\delta(H-E_i)\rangle\rangle^{m_i}=\langle\langle\mathcal{O}^\dagger\mathcal{O}\rangle\rangle^{m_i}\rho_{biv;\mathcal{O}}^{H;m_i,m_f}(E_i,E_f)$ take bivariate Gaussian form for EGOE(k), i.e., $I_{biv;\mathcal{O}}^{H;m_i m_f}(E_i,E_f)\rightarrow I_{biv-\mathcal{G};\mathcal{O}}^{H;m_i m_f}(E_i,E_f)$, and similarly $\rho^{H;m_i,m_f}_{biv;\mathcal{O}}(E_i,E_f)$ which is normalized to unity. It should be pointed out that, for number nonconserving transition operators $\mathcal{O}$, the number of particles m_i and m_f in the initial and final states, respectively, will not be same. The bivariate Gaussian strength density $I^{H;m_i m_f}_{biv-\mathcal{G};\mathcal{O}}$ is defined by the centroids (ϵ_i, ϵ_f) and widths (σ_i, σ_f) of its two marginals and the bivariate correlation coefficient ζ:

$$\zeta = \left\langle \left\langle \mathcal{O}^\dagger \left(\frac{H-\epsilon_f}{\sigma_f} \right) \mathcal{O} \left(\frac{H-\epsilon_i}{\sigma_i} \right) \right\rangle \right\rangle^{m_i} \Big/ \langle\langle \mathcal{O}^\dagger \mathcal{O} \rangle\rangle^{m_i}.$$

From now on some or all the superscripts over the various densities and traces will be dropped at will when no confusion arises. The FKPTM [9] result for transition matrix elements starts with $H=h(1)+V(2)$ and the bivariate strength density $I^h_{biv;\mathcal{O}}(x_i,x_f)$ due to $h(1)$, the mean-field producing part of H. With $V(2)$ generating chaos and thus represented by EGOE(2), its role is to Gaussian spread (with constant widths) the spikes at the energies x_i and x_f in $I^h_{biv;\mathcal{O}}$. The spreadings, more importantly, are correlated; i.e., the spreading function $\rho^V_{biv;\mathcal{O}}$, in the convolution form $I^H_{biv;\mathcal{O}}(E_i,E_f) = I^h_{biv;\mathcal{O}} \otimes \rho^V_{biv;\mathcal{O}}[E_i,E_f]$ is a bivariate Gaussian $\rho^V_{biv-\mathcal{G};\mathcal{O}}(y_i,y_f;0,0,\sigma_i,\sigma_f,\zeta)$ with ζ arising out of the noncommutability of V and the transition operator $\mathcal{O}$:

$$\zeta \sim \langle \mathcal{O}^\dagger V \mathcal{O} V \rangle / \langle \mathcal{O}^\dagger \mathcal{O} \rangle \langle VV \rangle.$$

Decomposing the m-particle space into the subspaces $\mathbf{\Gamma}$ defined by $h(1)$ $[m \rightarrow \Sigma \mathbf{\Gamma}$ with $\mathbf{\Gamma}$ labeling the eigenstates of $h(1)]$, $I^H_{biv;\mathcal{O}} = I^h_{biv;\mathcal{O}} \otimes \rho^V_{biv-\mathcal{G};\mathcal{O}}$ can be rewritten as

$$|\langle E_f|\mathcal{O}|E_i\rangle|^2 = \sum_{\mathbf{\Gamma}_i,\mathbf{\Gamma}_f} \frac{I^{\mathbf{\Gamma}_i}_{\mathcal{G}}(E_i) I^{\mathbf{\Gamma}_f}_{\mathcal{G}}(E_f)}{I^{m_i}_{\mathcal{G}}(E_i) I^{m_f}_{\mathcal{G}}(E_f)} |\langle \mathbf{\Gamma}_f|\mathcal{O}|\mathbf{\Gamma}_i\rangle|^2 \times \frac{\rho^{\mathbf{\Gamma}_i,\mathbf{\Gamma}_f}_{biv-\mathcal{G};\mathcal{O}}(E_i,E_f;\epsilon_i,\epsilon_f,\sigma_i,\sigma_f,\zeta)}{\rho^{\mathbf{\Gamma}_i}_{\mathcal{G}}(E_i)\rho^{\mathbf{\Gamma}_f}_{\mathcal{G}}(E_f)}, \quad (1)$$

$$|\langle \mathbf{\Gamma}_f|\mathcal{O}|\mathbf{\Gamma}_i\rangle|^2 = [d(\mathbf{\Gamma}_i)d(\mathbf{\Gamma}_f)]^{-1} \sum_{\alpha \in \mathbf{\Gamma}_i, \beta \in \mathbf{\Gamma}_f} |\langle \mathbf{\Gamma}_f \beta|\mathcal{O}|\mathbf{\Gamma}_i \alpha\rangle|^2,$$

$$I^{\mathbf{\Gamma}_r}(E_r) = \langle\langle \delta(H-E_r) \rangle\rangle^{\mathbf{\Gamma}_r} = d(\mathbf{\Gamma}_r)\rho^{\mathbf{\Gamma}_r}(E_r),$$

$$\epsilon_r = \langle H \rangle^{\mathbf{\Gamma}_r}, \quad \sigma_r^2 = \langle (H-\epsilon_r)^2 \rangle^{\mathbf{\Gamma}_r}, \quad r=i,j.$$

In Eq. (1), $d(\mathbf{\Gamma}_r)$ are dimensions of the subspaces $\mathbf{\Gamma}_r$. Given a one-body transition operator $\mathcal{O} = \Sigma_{\alpha,\beta} \epsilon_{\alpha\beta} a^\dagger_\alpha a_\beta$ where $a^\dagger_\alpha$ creates a particle in the single-particle state α and a_β destroys a particle in state β, it is easy to write down the expression for $|\langle \mathbf{\Gamma}_f|\mathcal{O}|\mathbf{\Gamma}_i\rangle|^2$ in terms of $\epsilon_{\alpha\beta}$ and matrix elements of number operators. References [16,17] give the details of Eq. (1) for one-body operators with $\mathbf{\Gamma}$'s chosen not only to be eigenstates of $h(1)$ but also for their various groupings. Equation (1) is applied in nuclear structure problems with two-body transition operators in [9,15], one-body transition operators, but using the so-called unitary orbits (defined by grouping single-particle states), in [17,20], and one-particle transfer operators in [21]. Although $I^H_{biv;\mathcal{O}} = I^h_{biv;\mathcal{O}} \otimes \rho^V_{biv-\mathcal{G};\mathcal{O}}$ is more general, it is Eq. (1) when applied to one-body transition operators that bring out the relationship between FKPTM and FGGKP.

The FGGKP formulation starts with the transition matrix elements written in terms of matrix elements in the mean-field basis states $|k_i\rangle$ by using the expansion $|E_i\rangle = C^{E_i}_{k_i}|k_i\rangle$:

$$\begin{aligned} |\langle E_f|\mathcal{O}|E_i\rangle|^2 &= \left\{ \sum_{k_i,k_f} C^{E_i}_{k_i} C^{E_f}_{k_f} \langle k_f|\mathcal{O}|k_i\rangle \right\}^2 \\ &= \left[\sum_{k_i,k_f} |C^{E_i}_{k_i}|^2 |C^{E_f}_{k_f}|^2 |\langle k_f|\mathcal{O}|k_i\rangle|^2 \right] \\ &\quad + \left[\sum_{k_i \neq k_i', k_f \neq k_f'} C^{E_i}_{k_i} C^{E_i}_{k_i'} C^{E_f}_{k_f} C^{E_f}_{k_f'} \langle k_f|\mathcal{O}|k_i\rangle \right. \\ &\quad \left. \times \langle k_f'|\mathcal{O}|k_i'\rangle \right] = [\text{diag}] + [\text{offdiag}]. \end{aligned} \quad (2)$$

Assuming that the transitions between different pairs of mean-field basis states are uncorrelated, the ''offdiag'' term in Eq. (2) is neglected. In the ''diag'' term, for a given k_f and k_i only one $\epsilon_{\alpha\beta}$ in $\mathcal{O}$ will contribute and [16] $|\langle k_f|\mathcal{O}|k_i\rangle|^2 = |\epsilon_{\alpha\beta}|^2 \langle n_\beta(1-n_\alpha)\rangle^{\mathcal{E}_i} \delta(\mathcal{E}_f - [\mathcal{E}_i - \epsilon_\beta + \epsilon_\alpha])$; ϵ_α are energies of the single-particle states α and $\mathcal{E}_i$ are the energies $\langle k_i|H|k_i\rangle$ of the mean-field basis states $|k_i\rangle$. It is well verified by EGOE(1+2) calulations [2] that $\langle n_\beta(1-n_\alpha)\rangle^{\mathcal{E}_i}$ do not vary much over the basis states that contribute to the given initial (E_i) or final (E_f) state in the chaotic domain. With this, Eq. (2) simplifies to

$$\begin{aligned} |\langle E_f|\mathcal{O}|E_i\rangle|^2 &\rightarrow |\langle E_f|\mathcal{O}|E_i\rangle|^2_{\text{diag}} \\ &= \sum_{\alpha,\beta} |\epsilon_{\alpha\beta}|^2 \langle n_\beta(1-n_\alpha)\rangle^{E_i} \\ &\quad \times \left\{ \sum_{\mathcal{E}_i} |C^{E_i}_{\mathcal{E}_i}|^2 |C^{E_f}_{\mathcal{E}_f=\mathcal{E}_i-\epsilon_\beta+\epsilon_\alpha}|^2 \right\}. \end{aligned} \quad (3)$$

The $|C|^2$'s in Eq. (3) are nothing but the strength functions $F_k(E)$, $F_k(E) = |C^E_k|^2/\overline{D(E)}$, where $\overline{D(E)}$ is mean spacing.

With EGOE, the strength functions $F_k(E)$ take a Gaussian form characterized by the spectral width σ_k while the standard form normally employed in many applications is the Breit-Wigner (BW) form $\Gamma_k/[2\pi\{(E-\mathcal{E}_k)^2+\Gamma_k^2/4\}]$ characterized by a spreading width Γ_k. Usually, given (N,m,Δ_{spe}) where Δ_{spe} is the average single-particle level spacing, for EGOE(1+2) the quantum chaotic regime is defined by a critical interaction strength λ_c that is necessary for the emergence of Wigner-Dyson level spacing statistics [5]. However, as is well demonstrated in a recent EGOE(1+2) calculation [22], when the interacting particle system is chaotic, there is yet another border defined by the interaction strength λ_{F_k} (calculations show that $\lambda_{F_k} > \lambda_c$) beyond which the strength functions make a transition from BW to Gaussian form; see also [10,12,14]. Thus the order-chaos transition implies BW form to the Gaussian transition for the $F_k(E)$, with the BW form extending beyond λ_c up to λ_{F_k} (earlier, Georgeot and Shepelyansky [3] also showed that the BW form extends to the chaotic domain).

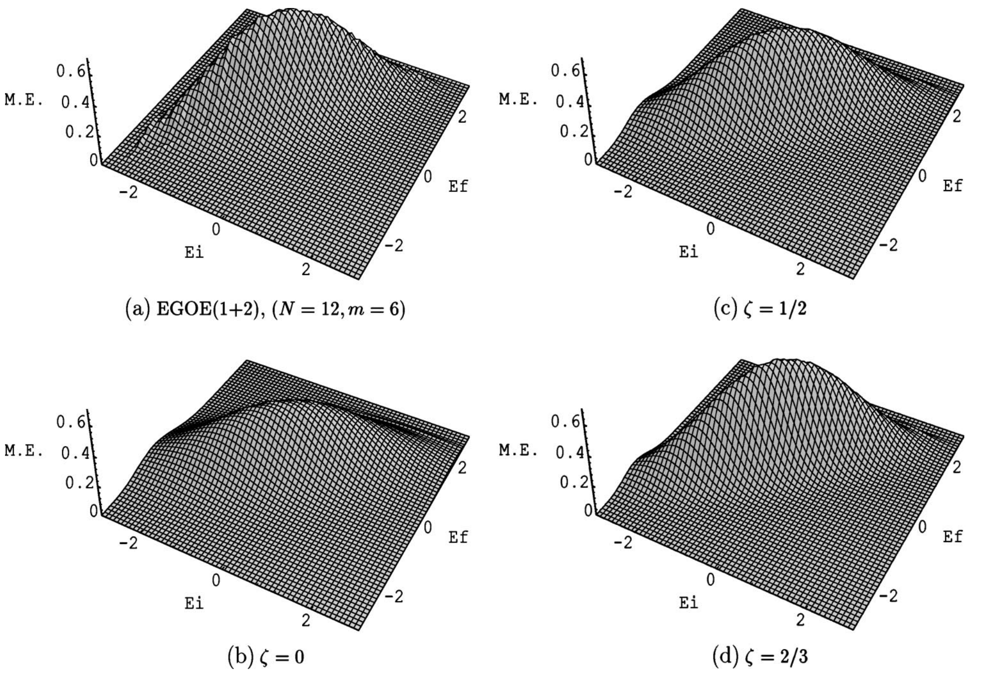

FIG. 1. Transition strengths $|\langle E_f|\mathcal{O}|E_i\rangle|^2$ vs (E_i,E_f). (a) Exact EGOE(1+2) strengths, (b) Eq. (6) with $\zeta=0$, (c) Eq. (6) with $\zeta=1/2$, and (d) Eq. (6) with $\zeta=2/3$. Here Ei$=\hat{E}_i=(E_i-\epsilon)/\sigma$ and Ef$=\hat{E}_f=(E_f-\epsilon)/\sigma$. Similarly M.E. stands for the strengths $|\langle E_f|\mathcal{O}|E_i\rangle|^2$. The EGOE(1+2) system and the one-body transition operator $\mathcal{O}$ are defined in the text. In all the calculations the strengths in the window, $\hat{E}_i\pm\Delta'/2$ and $\hat{E}_f\pm\Delta'/2$, are summed and plotted at $(\hat{E}_i,\hat{E}_f)$; Δ' is chosen to be 0.1. It should be noted that the total strength is 252. An enhancement in strengths due to the bivariate correlation coefficient ζ is clearly seen in (c) and (d) when compared to (b).

With the additional assumptions that (i) strength functions are only a function of $(E-\mathcal{E}_k)/s_k$ where s_k is a scale parameter (Γ_k for BW and σ_k for Gaussian) and (ii) the scale parameters s_k, in the chaotic domain, are constant (for example, $s_k\rightarrow\overline{s_i}$ for the initial many-particle basis states), the sum over $\mathcal{E}_i$ in Eq. (3) can be converted into integrals involving $F_k(E)$ (note that $\Sigma_{\mathcal{E}_i}[\,]\rightarrow\int[\,]d\mathcal{E}_i/\overline{D(E)}$). The final result, in terms of occupancies and the mean spacings, is

$$|\langle E_f|\mathcal{O}|E_i\rangle|^2=\sum_{\alpha,\beta}|\epsilon_{\alpha\beta}|^2\langle n_\beta(1-n_\alpha)\rangle^{E_i}\overline{D(E_f)}\times\int F_{\mathcal{E}_i}(E_i)F_{\mathcal{E}_f=\mathcal{E}_i-\epsilon_\beta+\epsilon_\alpha}(E_f)d\mathcal{E}_i$$
$$=\sum_{\alpha,\beta}|\epsilon_{\alpha\beta}|^2\langle n_\beta(1-n_\alpha)\rangle^{E_i}\overline{D(E_f)}\mathcal{F}(\Delta,\overline{s_i},\overline{s_f}),$$
$$\Delta=E_f-E_i+\epsilon_\beta-\epsilon_\alpha. \tag{4}$$

Flambaum *et al.* [13] advocated use of Eq. (4) with BW form for F's which in turn gives a BW form for $\mathcal{F}$,

$$\mathcal{F}(\Delta,\overline{\Gamma}_i,\overline{\Gamma}_f)_{\text{BW}}=\frac{1}{2\pi}\frac{\overline{\Gamma}_i+\overline{\Gamma}_f}{\Delta^2+(\overline{\Gamma}_i+\overline{\Gamma}_f)^2/4}, \tag{5}$$

where $\overline{\Gamma}_i$ and $\overline{\Gamma}_f$ are the average BW spreading widths for the basis states over the initial and final many-particle states, respectively. The FGGKP theory is given by Eqs. (4) and (5) and it was subjected to EGOE(1+2) tests in [2] and also using dipole ($E1$) transitions in a Ce atom [13,18]. It is seen in the EGOE(1+2) calculations that there is always an enhancement (some times it is even by a factor of 2) around $\Delta=0$ in the matrix elements compared to the results given by Eqs. (4) and (5). In other words, the ''offdiag'' term in Eq. (2) in fact gives a coherent contribution. It is also seen that the enhancements grow with number of particles *m*; an estimate for the enhancements is given in [2]. Now we will show that Eq. (1) cures these problems.

Consider Eq. (1) by taking the subspace labels $\mathbf{\Gamma}_i$ and $\mathbf{\Gamma}_f$ to be the configurations defined by distributing the particles in single-particle states. For $\zeta=0$, the $\rho_{biv-\mathcal{G}}/\rho_{\mathcal{G}}\rho_{\mathcal{G}}$ will be unity and Eq. (1) will be identical to Eq. (3). Following the steps that led to Eq. (4), i.e., evaluating $|\langle\mathbf{\Gamma}_f|\mathcal{O}|\mathbf{\Gamma}_i\rangle|^2$ which gives the $\langle n_\beta(1-n_\alpha)\rangle^{\mathcal{E}_i}$ term, replacing it by $\langle n_\beta(1-n_\alpha)\rangle^{E_i}$, assuming constant spectral widths [i.e., σ_r^2 in Eq. (1) do not depend on $\mathbf{\Gamma}_r$, $\sigma_i^2\rightarrow\overline{\sigma_i^2}$ and $\sigma_f^2\rightarrow\overline{\sigma_f^2}$], noting that $I^{\mathbf{\Gamma}}(E)/I^m(E)$ are nothing but $|C|^2$, and converting the sum over $\mathbf{\Gamma}_i(\mathcal{E}_i)$ into an integral give, directly,

$$|\langle E_f|\mathcal{O}|E_i\rangle|^2=\sum_{\alpha,\beta}|\epsilon_{\alpha\beta}|^2\langle n_\beta(1-n_\alpha)\rangle^{E_i}\overline{D(E_f)}\times\int\rho_{biv-\mathcal{G};\mathcal{O}}(E_i,E_f;\mathcal{E}_i,\mathcal{E}_f=\mathcal{E}_i-\epsilon_\beta+\epsilon_\alpha,\overline{\sigma}_i,\overline{\sigma}_f,\zeta)d\mathcal{E}_i$$
$$=\sum_{\alpha,\beta}|\epsilon_{\alpha\beta}|^2\langle n_\beta(1-n_\alpha)\rangle^{E_i}\overline{D(E_f)}\times\mathcal{F}(\Delta=E_f-E_i+\epsilon_\beta-\epsilon_\alpha,\overline{\sigma}_i,\overline{\sigma}_f,\zeta)_{biv-\mathcal{G}}, \tag{6}$$

$$\mathcal{F}(\Delta,\overline{\sigma}_i,\overline{\sigma}_f,\zeta)_{biv-\mathcal{G}}=\frac{1}{\sqrt{2\pi(\overline{\sigma_i^2}+\overline{\sigma_f^2}-2\zeta\overline{\sigma_i}\overline{\sigma_f})}}\times\exp\left(-\frac{\Delta^2}{2(\overline{\sigma_i^2}+\overline{\sigma_f^2}-2\zeta\overline{\sigma_i}\overline{\sigma_f})}\right).$$

With Gaussian form for the strength functions, FGGKP theory given by Eq. (4) will coincide with Eq. (6) for $\zeta=0$. Thus, unlike FGGKP, FKPTM equation (6) includes correlated Gaussian spreadings and, as already pointed out, in the chaotic domain (i.e., for $\lambda>\lambda_{F_k}$) Gaussian spreadings are more appropriate than BW spreadings [22]. Second, Eq. (6) gives, for $\Delta=0$ and $\zeta\rightarrow 1$, an enhancement in the matrix elements compared to the diag($\zeta=0$) approximation; for $\overline{\sigma_i}\sim\overline{\sigma_f}$, the enhancement is $1/\sqrt{(1-\zeta)}$. The so-called binary correlation approximation [9] for EGOE gives $\zeta\sim 1-2/m$. Therefore ζ grows with m and hence the enhancements grow with m. Thus Eq. (6) reproduces all the peculiar results observed in the EGOE(1+2) calculations in [2].

For further confirmation that Eq. (6) is a proper theory, in the chaotic domain defined by $\lambda>\lambda_{F_k}$, for matrix elements of one-body transition operators, numerical calculations are carried out for various λ values using a 25-member EGOE(1+2) ensemble $\{H\}=h(1)+\lambda\{V(2)\}$ in 924-dimensional $N=12$, $m=6$ space; $h(1)$ is defined by the single-particle energies $\epsilon_i=(i)+(1/i)$, $i=1,2,\ldots,12$ just as in [2]. The one-body transition operator employed in the calculations is $\mathcal{O}=a_2^\dagger a_9$. Results for $\lambda=0.3$ are shown in Fig. 1. For the present EGOE(1+2) system, $\lambda_{F_k}=0.2$ [22]. Thus, for $\lambda=0.1$ the strength functions are close to BW form and comparing the EGOE(1+2) numerical results, for matrix elements of the one-body operator, with Eqs. (4) and (5) the disagreement between the two is found to be quite similar to the comparision between Figs. 1(a) and 1(b); i.e., it is essential to construct a theory with correlated BW forms. For $\lambda=0.3$, the strength functions are close to Gaussian and the bivariate Gaussian form is a good approximation for strength densities. Therefore for this case Eq. (6) applies as shown in Fig. 1. The comparisons in Fig. 1 clearly emphasize the role of the bivariate correlation coefficient ζ and without ζ it is not possible to get a meaningful description of the transition matrix elements. Numerical calculations give $\zeta=1/2$ and shown in the figure is also the plot for $\zeta=2/3$ as given by the binary correlation approximation. Here the agreement between theory and calculations is even better. This is encouraging as in practice it is often difficult [9,15,17] to calculate the exact values of ζ.

In summary, analyzing FKPTM and FGGKP formulations, a theory (6) for the matrix elements of one-body transition operators in the chaotic domain (with $\lambda>\lambda_{F_k}$) of isolated finite interacting particle systems is derived and tested. Further investigations of Eq. (6) in real systems are highly desirable.

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[22] For a 25-member EGOE(1+2) with $N=12$, $m=6$, and single-particle energies as used in generating Fig. 1, the strength functions $F_k(E)$ are constructed for $\lambda=0.01$, 0.05, 0.1, 0.2, 0.3, and 0.5. It is seen that, for $\lambda=0.01$, 0.05, and 0.1 only the BW form, for 0.2 both the BW and ED (ED is Edgeworth corrected Gaussian [23]) forms, and for 0.3 and 0.5 only the ED form are found to be good. Thus, clearly there is BW form to the Gaussian transition in $F_k(E)$ and the value of $\lambda=\lambda_{F_k}$ for the onset of this transition is $\lambda_{F_k}\approx 0.2$. This should be compared with $\lambda=\lambda_c\approx 0.08$, for the present EGOE(1+2), for the onset of chaos in level fluctuations, occupancies, and strength sums of one-body operators [4,5,14] (the numerically determined λ_c is in agreement with the estimate obtained from the theory given in [5]). In the nuclear shell model example considered in [10], $\lambda_c\sim 0.3$ and $\lambda_{F_k}\sim 0.6$. Let us point out that unlike in the case of λ_c, no theory is available yet for determining λ_{F_k} for EGOE(1+2) given (N,m,Δ_{spe}) and this problem is being addressed.

[23] M. G. Kendall and A. Stuart, *Advanced Theory of Statistics*, 3rd ed. (Hafner, New York, 1969), Vol. 1.