Growth with surface curvature on quenched potentials

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A discrete growth model driven by the Laplacian of the surface curvature in quenched random media is discussed. The interface width W at the saturated regime obeys scaling $W \sim L^{\alpha}$ with $\alpha \approx 2.3$, where L is the system size. Starting from an initial sine wave condition of a selected wavelength, we measure an autocorrelation function, and obtain the dynamic critical exponent $z \approx 3.1$. The model is expected to be described by the quenched Mullins-Herring equations.

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Kinetic roughening of driven interface in quenched random media has attracted very much attention recently [1]. It is related through various mapping to many other physical phenomena such as the immiscible displacement of fluids in porous media [2], the domain walls in magnetic systems [3], flux movement in a superconductor [4], and the invasion of liquid in porous media [5]. When an interface in disordered media is driven by an external force F, the motion of it shows a pinning-depinning transition. The quenched disorder generates random pinning forces effectively. If the driving force F is sufficiently weak compared to the random pinning force, the interface is pinned by the disorder. If F is strong enough, the interface moves indefinitely with an average velocity v. At F_c, which is the critical force, $v(t)$ decreases following a power law of time and approaches zero.

The roughness of the interface is an interesting quantity in this kind of growth phenomena. In a finite system of lateral size L, the surface width W, which is the standard deviation of the height, scales as [6]

$$W^2(L,t) \sim \begin{cases} L^{2\alpha} g(t/L^z) \\ t^{2\beta}, \quad t \ll L^z \\ L^{2\alpha}, \quad t \gg L^z, \end{cases} \tag{1}$$

where the scaling function $g(x)$ is $x^{2\beta}$ for $x \ll 1$ and constant for $x \gg 1$. The exponents α, β, and z are called the roughness, the growth, and the dynamic exponents, respectively. They are connected by the relation $z\beta = \alpha$. Recently a more general scaling form of W is suggested for the self-organized critical models to explain other correlation length that reflects the amount of self-organization that has taken place [7].

There are some trials to classify the surface roughness, with each universality class corresponding to a particular continuum growth equation for the coarse-grained height variables $h(\mathbf{x},t)$, which describes the growing interface as a function of the lateral surface coordinate $\mathbf{x}$ and time t. For example, the model of random deposition with diffusion to the local height minima [6] is known to belong to the Edwards and Wilkinson (EW) universality, described by the EW equation [8]. In this case the noise is a thermal noise, which depends on both space and time. Here, we focus on the effect of the quenched disorder, which is frozen in the medium.

One of the simple equations for the interface roughening in quenched random media is the quenched Edwards-Wilkinson (QEW) equation

$$\frac{\partial h(x,t)}{\partial t} = \nu_2 \nabla^2 h + \eta(x,h) + F, \tag{2}$$

where F is the external driving force and $\eta(x,h)$ is the quenched random potential satisfying the relation $\langle \eta(x,h)\eta(x',h')\rangle = 2D\delta(x-x')\delta(h-h')$. Many studies are devoted to the QEW equation and the related models [9–15]. One of them is the linear-interface model (LIM) [9,10,15]. Paczuski *et al.* studied the LIM with driving force [10]. Here, the model is described briefly. Each site has a random noise $\eta(x,h)$ to represent quenched random pinning forces and there is a linear configurational term $f_{conf} \sim \nabla^2 h$, where h is the local height. Thus the local total force

$$F_{tot}(x,t) = \nu_2 \nabla^2 h(x,t) + \eta(x,h) \tag{3}$$

is calculated and the height of the site that has the maximum total force is advanced by one unit at each time. So the model is driven by the extremal dynamics. It is generally believed that the model is expected to follow the QEW equation.

As pointed out before [16], if the surface current is driven by the differences in the surface chemical potential and the chemical potential is proportional to the surface curvature, one has to replace $\nabla^2 h$ by $-\nabla^4 h$ in the QEW equation. So it would be interesting to consider a quenched Mullins-Herring (QMH) equation [17]

$$\frac{\partial h(x,t)}{\partial t} = -\nu_4 \nabla^4 h + \eta(x,h) + F. \tag{4}$$

The equation could be relevant to the dynamics of the liquid in porous media.

Here we consider a discrete growth model, which is expected to follow the QMH equation. In one substrate dimension, $\alpha \approx 2.3$ is obtained. We also measure the dynamic exponent z independently using the relaxation function method [18], and find $z \approx 3.1$.

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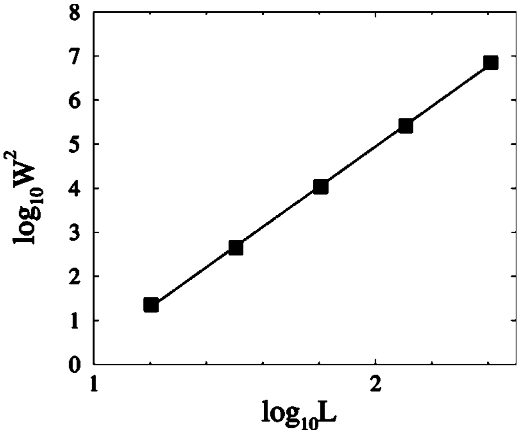

FIG. 1. The plot of $W^2(L)$ at the saturated regime as a function of L in log-log plot with $\nu_4=0.5$ for the system sizes $L=16$, 32, 64, 128, and 256. The guideline is for $\alpha=2.3$.

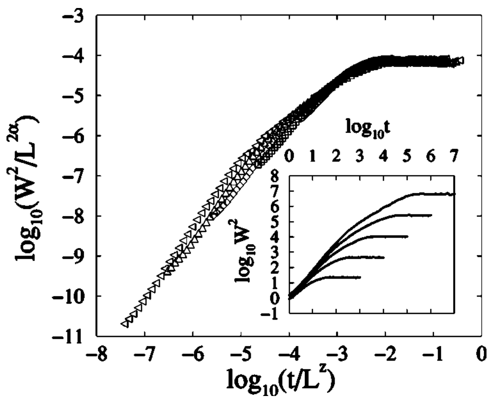

FIG. 2. The scaling collapse of the surface width as a function of time for the system sizes $L=16$, 32, 64, 128, and 256 with $\alpha=2.3$, $z=3.1$. The plot of $W^2(L,t)$ against t in logarithmic scales for the system sizes (inset).

We introduce a discrete model to mimic the QMH equation. At each site, the random quenched noise $\eta(x,h)$ is given with the uniform distribution between -1 and 1. Starting from initial flat surface, the local total force

$$F_{tot}(x,t)=-\nu_4\nabla^4 h+\eta(x,h(x,t)) \tag{5}$$

is calculated and then the height of the maximum total force site is advanced by one unit. We have carried out extensive simulations for this growth model at one substrate dimension. Since one height step is advanced at each trial, the average surface height $\langle h(\mathbf{x},t)\rangle$ is treated as time t. Starting from a flat interface, the entire process is subject to the periodic boundary condition. The simulations are carried out for the system sizes $L=16$, 32, 64, 128, and 256, and the data are averaged over 400 configurations. To obtain α and β, the time-dependent interface width $W(L,t)$ are monitored. As usual, the surface width $W(t)$ increases as t^β for early times and eventually saturates when the parallel correlation $t^{1/z}$ is of the order of the lateral system size L [19].

For the roughness exponent α describing the saturation of the interface fluctuation, we use the relation $W(L)\sim L^\alpha$ in the steady-state regime $t\gg L^z$. Since the value of z is around three, it takes a long time to arrive at the saturated regime. This forced us to restrict our simulation system sizes to $L=256$ in $d=1+1$. As shown in Fig. 1, from the log-log plot of $W(L)$ and size L, we get a nice straight line with

$$\alpha=2.3\pm0.1. \tag{6}$$

To obtain the collapse of the data, we rescale W by L^α with $\alpha=2.3$ and t by L^z for various values of z. In Fig. 2, the rescaled data of different system sizes are collapsed into a curve for a specific value $z=3.1$. In general, β is obtained from the slope of the straight line fit through the data points $W(L,t)$ in log-log plot for $t\ll L^z$. The data collapse is not perfect for the regime, where the line through the data points is not straight. Through the relation $W(t)\sim t^\beta$, we can roughly estimate $\beta=0.8\sim0.9$. Note that the slope varies with time. Even for large system size $L=8192$, we could not get an accurate value of β directly from $\ln W(t)$-$\ln t$ plot.

Again, we try to determine β from the saturated surface width. Start from flat initial condition and wait until the width becomes saturated up to time t_s. The saturated surface height $h_s(x,t)$ is defined as $h(x,t+t_s)-h(x,t_s)$. The surface width $W(t)$ obtained from $h_s(x,t)$ also satisfies the power law behavior $W(t)\sim t^{\beta_s}$. In simulation, we wait a long enough time until the width is saturated. The saturated surface configuration is chosen as the initial configuration. The widths from the saturated surface height are monitored as a function of time. In Fig. 3, the dependence of W on t is shown in a log-log plot. The straight line through the data points indicates that

$$\beta_s=0.75\pm0.01. \tag{7}$$

In comparison with the data from the saturated initial condition in Fig. 3, the plot of surface width as a function of time in Fig. 2 is curved for $t\ll L^z$. The value of β_s is different from that of β. As pointed out by Ref. [7], for the generically critical models [20], the saturated β is the same as β. However, for the self-organized critical models, β_s is usually different from β.

The value of α is measured accurately from the saturated surface width and z can be obtained from the data collapse of

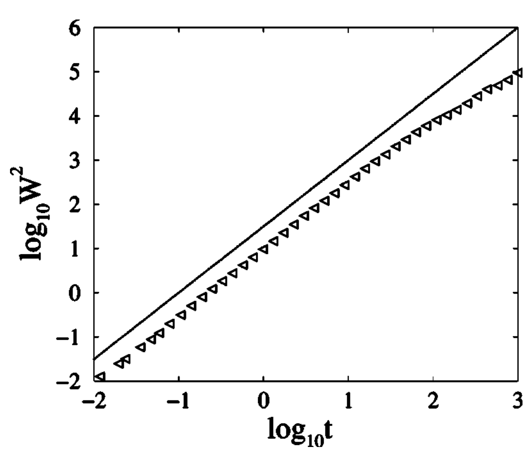

FIG. 3. $W^2(t)$ from the saturated surface height as a function of time t in logarithmic scales for $L=256$. The guideline is for $\beta_s=0.75$.

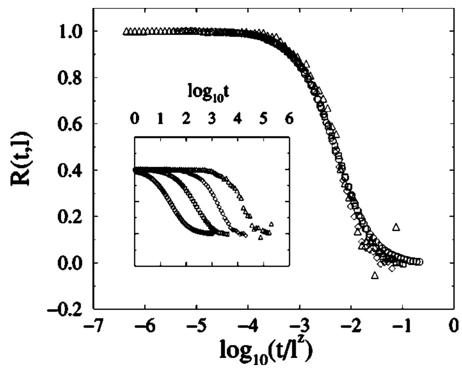

FIG. 4. The data collapse of the relaxation functions for wavelengths $l=32$, 64, 128, and 256 with $z=3.1$. The curves of $R(t)$ against t in logarithmic scales, from the left, represent for $l=32$, 64, 128, and 256 in the inset.

the scaling plot. Here, we use the relaxation function method [18] to measure z independently. The method is described here briefly. A sine wave

$$h(x,0)=C\sin(2\pi x/l) \tag{8}$$

is prepared for the initial condition, where C and l are the amplitude and the wavelength, respectively. The surface is allowed to evolve following the growth rule of the model. We consider an autocorrelation function $C(t,l)$ of the height

$$C(t,l)=\langle h(x,0)h(x,t)\rangle, \tag{9}$$

to characterize the relaxation process of the initial condition. The normalized relaxation functions $R(t,l)$ of C are defined as

$$R(t,l)=C(t,l)/C(0,l). \tag{10}$$

In the long-time limit, the surface height does not have the correlation with the initial condition such that R becomes zero. R shows how the initial fluctuation relaxes with time. In general, the relaxation function decays exponentially

$$R(t,l)\sim e^{-g[t/\tau(l)]}, \tag{11}$$

TABLE I. Exponents of the model.

Exponents		
α	β_s	z
2.3 ± 0.1	0.75 ± 0.01	3.1 ± 0.1

where τ is the relaxation time. The characteristic time of the relaxation function depends on the selected wavelength l of the initial fluctuation. We expect that the relaxation time is proportional to l^z. So, the normalized relaxation function follows the scaling form

$$R(t,l)\sim f(t/l^z). \tag{12}$$

We measure the relaxation functions in our model for various wavelengths l. The normalized relaxation function against $\ln t$ are shown in the inset of Fig. 4 for the wavelengths $l=16$, 32, 64, and 128. We try to rescale the time axes with the characteristic time $\tau\sim l^z$. All curves are excellently collapsed into an universal curve with $z=3.1$ as shown in Fig. 4. The values of the exponents are summarized in Table I.

We study a discrete growth model in quenched random potential that is expected to follow the quenched Mullins-Herring equation. $\alpha\approx2.3$ and $\beta_s\approx0.75$ are obtained from the saturated regime. The relaxation function measurement also allows us to find the dynamic exponent $z=3.1$. It is interesting that the values of the exponents satisfy the relation $\alpha/z=\beta_s$ very well. The value of α is close to 7/3, which can be obtained from the power counting of the equation. There have been many studies on interface in quenched random media. However, theories, simulations, and experiments are not in good agreement yet. We cannot say anything conclusive on the question of universality because of computer limitations and the lack of analytic tools. Experiments, analytic calculation of the exponents for the QMH equation, and the study of the discrete model in higher dimensions are required [21].

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