Wave-beam coupling in quadratic nonlinear optical waveguides: Effects of nonlinearly induced diffraction

A. D. Boardman,[1] K. Marinov,[2,*] D. I. Pushkarov,[2] and A. Shivarova[3]
[1]*Photonics and Nonlinear Science Group, Joule Laboratory, Department of Physics, University of Salford, Salford M5 4WT, United Kingdom*
[2]*Institute of Solid State Physics, Bulgarian Academy of Sciences, BG-1784 Sofia, Bulgaria*
[3]*Faculty of Physics, Sofia University, BG-1164 Sofia, Bulgaria*

(Received 8 February 2000)

Beam coupling influenced by nonlinearly induced diffraction, an effect stemming from the div $\vec{E}$-term in the wave equations, is stressed in the study. The system considered consists of two beams carried by TE-modes at frequencies of ω and 2ω in quadratic nonlinear planar optical waveguides. The power-conservation law, the Lagrangian and the Hamiltonian of the system, as well as the equations governing its stationary states are derived. It is shown that the nonlinearly induced diffraction modifies the second-order nonlinear terms and acts as an effective third-order nonlinearity. The procedure for dealing with modifications caused by effects like the nonlinearly induced diffraction within the framework of a paraxial approach is discussed. The numerical analysis carried out has the nonlinear wave-number shift and the linear phase mismatch as parameters. The influence of the nonlinearly induced diffraction on the shape (the amplitude and the width) of the solitary waves is demonstrated.

PACS number(s): 42.65.Wi, 42.65.Jx, 42.82.Et, 42.82.−m

I. INTRODUCTION

The second-order nonlinear effects and nonlinear energy exchange in nonlinear wave interactions at fundamental and second-harmonic frequencies have been a subject of intensive study since the beginning of the research in nonlinear optics. The solitonlike waves in second-order nonlinear media predicted in 1974 [1] form nowadays a tremendously enlarged field called cascaded nonlinearities [2,3]. The higher efficiency of the second-order nonlinear processes compared to those related to third-order nonlinearity justifies their consideration as effects of great importance for applications to all-optical switching. Solitonlike wave formation in both self-phase modulation and self-focusing have been covered by many studies (see, e.g., Refs. [4–14]). A new family of soliton solutions have been found in cascaded nonlinearities, stability analysis have been performed in different dimensions and problems concerning soliton interactions and soliton formation in the presence of both second- and third-order nonlinearities have been treated.

This study stresses the modifications of the beam coupling in quadratic optically-nonlinear media which are related to the div $\vec{E}$ term in the wave equations, a term which introduces effects due to spatial inhomogeneity of the nonlinear polarization. Although quite often neglected in nonlinear optics, such effects have been treated before in studies both on temporal [15,16] and spatial [17–19] solitary waves in Kerr-type nonlinear media. With respect to beam propagation in third-order nonlinear optical waveguides, the rate of the spatial variation of the nonlinear polarization shows up as a nonlinearly induced diffraction [20]. Modifying the nonlinear Schrödinger equation (by changing the cubic nonlinear term in it and inducing an effective fifth-order nonlinearity) it leads to a new type of solitary waves. In a way, the nonlinearly induced diffraction appears as a factor which partially controls the balance between the linear diffraction of the beam and the nonlinearity of the media. Here, in quadratically nonlinear optical waveguides, the nonlinearly induced diffraction modifies the second-order nonlinear terms in the coupled beam equations and introduces an effective third-order nonlinearity in them. Therefore, it could strongly influence phenomena which are governed by simultaneous action of second- and third-order nonlinearities. The nonlinearly induced diffraction affects the energy distribution in the system of the two beams carried at the fundamental (ω) and second-harmonic (2ω) frequencies and modifies its stationary states. This is demonstrated by both the analytical results derived here for the power of the two-beam system, its Hamiltonian and Lagrangian, as well as the corresponding Euler-Lagrange equations, and the numerical analysis performed for obtaining the shape and the parameters (amplitudes and widths) of the solitary-wave stationary states. The procedure for the derivation of the coupled equations, when modifications caused by effects like the nonlinearly induced diffraction are considered in the framework of the paraxial approach, is also discussed.

II. COUPLED BEAM EVOLUTION EQUATIONS

Propagation of two beams [with field space dependence $\vec{E}(x,y,z)$] carried by TE modes [$\vec{E}=(E_X,0,0)$] at fundamental (ω) and second-harmonic (2ω) frequencies

$$E_{\omega}=\tfrac{1}{2}(\overline{E}_{\omega}e^{-i\omega t}+\text{c.c.}),\tag{1a}$$

$$E_{2\omega}=\tfrac{1}{2}(\overline{E}_{2\omega}e^{-2i\omega t}+\text{c.c.})\tag{1b}$$

along the z axis of a planar nonlinear optical waveguide is considered in a scalar approach. The extension of the beams is in the x direction and the guiding confinement is in the y direction. Waveguide medium with quadratic nonlinearity

*Also at Faculty of Physics, Sofia University, BG-1164 Sofia, Bulgaria.

within the 32 crystal class (β quartz) [21] is considered. Therefore, the amplitudes at frequencies ω and 2ω of the second-order nonlinear polarization $[\vec{P}_{NL}^{(2)}=(P_{NL},0,0)]$ are

$$\overline{P_{NL}^{2\omega}}=\frac{\varepsilon_0\chi^{(2)}}{2}\overline{E_\omega^2}, \tag{2a}$$

$$\overline{P_{NL}^{\omega}}=\varepsilon_0\chi^{(2)}\overline{E_\omega^*}\overline{E_{2\omega}}, \tag{2b}$$

where ε_0 is the vacuum permittivity, and $\chi^{(2)}=\chi_{XXX}^{(2)}(2\omega;\omega,\omega)=\chi_{XXX}^{(2)}(\omega;2\omega,-\omega)$ is the second-order susceptibility.

In the wave equation

$$\nabla^2\vec{E}-\vec{\nabla}(\vec{\nabla}\cdot\vec{E})=\frac{\varepsilon_L}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}+\mu_0\frac{\partial^2\vec{P}_{NL}}{\partial t^2} \tag{3}$$

the $(\vec{\nabla}\cdot\vec{E})$ term is kept and expressed through $\vec{P}_{NL}$ by using

$$\vec{\nabla}\cdot\vec{E}=-\frac{1}{\varepsilon_0\varepsilon_L}\vec{\nabla}\cdot\vec{P}_{NL}, \tag{4}$$

a relation which stems directly from $\vec{\nabla}\cdot\vec{D}=0$ with $\vec{D}=\varepsilon_0\varepsilon_L\vec{E}+\vec{P}_{NL}$. In Eqs. (3) and (4), ε_L is the linear dielectric constant, μ_0 is the vacuum permeability, c is the light speed in vacuum, and $\vec{D}$ is the displacement.

The set of equations, obtained from Eqs. (1)–(4) for describing the coupled beam propagation, is

$$\frac{\partial^2\overline{E_\omega}}{\partial x^2}+\frac{\partial^2\overline{E_\omega}}{\partial z^2}+\frac{\omega^2}{c^2}\varepsilon_L(\omega)\overline{E_\omega}+\frac{\omega^2}{c^2}\chi^{(2)}\overline{E_\omega^*}\overline{E_{2\omega}}+\frac{\chi^{(2)}}{\varepsilon_L(\omega)}\frac{\partial^2(\overline{E_\omega^*}\overline{E_{2\omega}})}{\partial x^2}=0, \tag{5a}$$

$$\frac{\partial^2\overline{E_{2\omega}}}{\partial x^2}+\frac{\partial^2\overline{E_{2\omega}}}{\partial z^2}+\frac{4\omega^2}{c^2}\varepsilon_L(2\omega)\overline{E_{2\omega}}+\frac{2\omega^2}{c^2}\chi^{(2)}\overline{E_\omega^2}+\frac{\chi^{(2)}}{2\varepsilon_L(2\omega)}\frac{\partial^2\overline{E_\omega^2}}{\partial x^2}=0. \tag{5b}$$

The last terms in Eqs. (5) come out from the $(\vec{\nabla}\cdot\vec{E})$ term in Eq. (3) and since their form is similar to that of the diffraction terms [the first ones in Eq. (5)], we call them nonlinearly induced diffraction terms.

The transformation

$$\overline{E_\omega}=\tilde{E}_\omega\exp(i\beta_\omega z), \tag{6a}$$

$$\overline{E_{2\omega}}=\tilde{E}_{2\omega}\exp(i\beta_{2\omega} z) \tag{6b}$$

applied involves the total, nonlinear wave numbers $\beta_\omega=k_\omega+\Delta\beta_\omega$, $\beta_{2\omega}=k_{2\omega}+\Delta\beta_{2\omega}$ (where k_ω, $k_{2\omega}$ are the linear wave numbers and $\Delta\beta_\omega$, $\Delta\beta_{2\omega}$ are the nonlinear contributions) and the convenient condition for synchronism

$$\beta_{2\omega}=2\beta_\omega \tag{7}$$

is used for them. For balanced states, the linear mismatch is compensated by the mismatch of the nonlinear contributions in which case the following condition holds exactly:

$$\Delta k_L\equiv k_{2\omega}-2k_\omega=2\Delta\beta_\omega-\Delta\beta_{2\omega}. \tag{8}$$

The transformation (6) should be rather used instead of transformation $\overline{E_\omega}=\tilde{E}_\omega\exp(ik_\omega z)$, $\overline{E_{2\omega}}=\tilde{E}_{2\omega}\exp(ik_{2\omega}z)$ because the latter is not commutative with the paraxial approximation. In addition, the derivation of the power-conservation law when the nonlinearly induced diffraction is taken into account requires before going to the paraxial approach to have the total wave number taken into account.

After introducing notation $\alpha=k_{2\omega}/2k_\omega$, $B=1+(\Delta\beta_\omega/k_\omega)$ and making the transformations $2\beta_\omega z\to z$, $2\beta_\omega x\to x$, $\gamma\tilde{E}_{2\omega}\to E_2$, $\sqrt{2}\gamma\tilde{E}_\omega\to E_1$ [where $\gamma=(\omega/(2\beta_\omega c))^2\chi^{(2)}$], Eqs. (5) take finally the form

$$i\frac{\partial E_1}{\partial z}+\frac{\partial^2 E_1}{\partial x^2}+\frac{1-B^2}{4B^2}E_1+E_1^*E_2+4B^2\frac{\partial^2(E_1^*E_2)}{\partial x^2}=0, \tag{9a}$$

$$2i\frac{\partial E_2}{\partial z}+\frac{\partial^2 E_2}{\partial x^2}+\frac{\alpha^2-B^2}{B^2}E_2+E_1^2+\frac{B^2}{\alpha^2}\frac{\partial^2 E_1^2}{\partial x^2}=0. \tag{9b}$$

In Eqs. (9) the nonparaxial terms are neglected because in the amplitudes E_1, E_2 there is no fast (linear and nonlinear) phase changes. The constants $B^2-1\approx 2\Delta\beta_\omega/k_\omega$ and $\alpha^2-1\approx\Delta k_L/k_\omega$ are, respectively, twice the relative nonlinear wave-number shift at the fundamental frequency and the relative mismatch. Therefore, the coefficients $(B^2-1)/4B^2$ and $(B^2-\alpha^2)/B^2$ in the third terms of Eqs. (9a) and (9b) are related to the relative nonlinear wave-number shift, respectively, of the fundamental and second-harmonic waves.

III. POWER-CONSERVATION LAW

The linear and nonlinear diffraction [i.e., the second and the last terms in Eqs. (9)] are combined and Eqs. (9a) and (9b) are multiplied, respectively, to $E_1^*+4B^2E_1E_2^*$ and $E_2^*+(B^2/\alpha^2)E_1^{*2}$. After some algebra, we obtain the conservation law of the two-beam system:

$$dP/dz=0, \tag{10a}$$

where

$$P=\int_{-\infty}^{\infty}[|E_1|^2+2\alpha^2|E_2|^2+2B^2(E_1^{*2}E_2+E_1^2E_2^*)]dx \tag{10b}$$

is the total power (or mass) carried by the beams. Whereas the first two terms give the power carried separately by each of the beams, the third term, which is the term stemming from the nonlinearly induced diffraction, combines contributions of the two beams. Through the nonlinearly induced diffraction the nonlinear polarization is explicitly involved in the conservation law of the beam system. The derivation of the power-conservation law (10) of the system is possible since a proper consequence in making the transformations

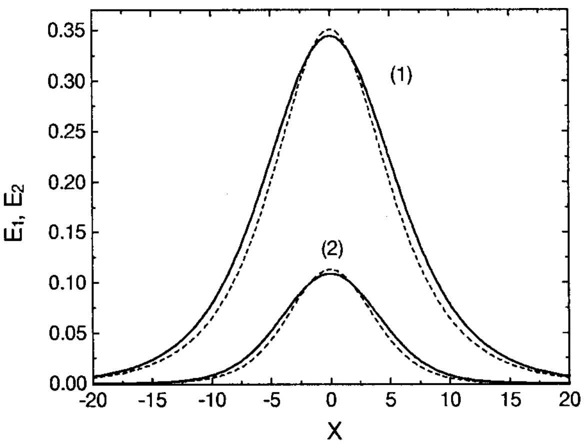

FIG. 1. Comparison of the normalized amplitudes E_1 and E_2 [according to the notation in Eqs. (11)] of the beams at the fundamental (ω) and second-harmonic (2ω) frequencies, marked, respectively, by (1) and (2), for $\alpha=0.9$ ($\Delta k_L/2k_\omega=-0.1$) and $B=1.03$ ($\Delta\beta_\omega/k_\omega=0.03$) and nonlinearly induced diffraction taken (in solid curves) and not taken (dashed curves) into account.

[separation of the phase variation according to (6) and then applying the paraxial approach] has been chosen.

IV. HAMILTONIAN, LAGRANGIAN, AND EULER-LAGRANGE EQUATIONS OF THE SYSTEM

The equations governing the stationary (solitonlike) solutions obtained by dropping the z dependence in Eqs. (9) are

$$\frac{d^2}{dx^2}(E_1+4B^2E_1E_2)-\frac{B^2-1}{4B^2}E_1+E_1E_2=0, \tag{11a}$$

$$\frac{d^2}{dx^2}\left(E_2+\frac{B^2}{\alpha^2}E_1^2\right)-\frac{B^2-\alpha^2}{B^2}E_2+E_1^2=0. \tag{11b}$$

After making the transformation $E_{1,2}/[(B^2-1)/4B^2]\rightarrow E_{1,2}$, $x[(B^2-1)/4B^2]^{1/2}\rightarrow x$, the set of the coupled equations (11) takes the form

$$\frac{d^2}{dx^2}[E_1+(B^2-1)E_1E_2]-E_1+E_1E_2=0, \tag{12a}$$

$$\frac{d^2}{dx^2}\left[E_2+\frac{B^2-1}{4\alpha^2}E_1^2\right]-4\frac{B^2-\alpha^2}{B^2-1}E_2+E_1^2=0, \tag{12b}$$

and can be integrated once to obtain the Hamiltonian

$$H=T+U, \tag{13a}$$

where

$$T=\frac{1}{2}\left[\frac{dE_1}{dx}+(B^2-1)\frac{d(E_1E_2)}{dx}\right]^2+\frac{\alpha^2}{4}\left[\frac{dE_2}{dx}+\frac{B^2-1}{4\alpha^2}\frac{dE_1^2}{dx}\right]^2, \tag{13b}$$

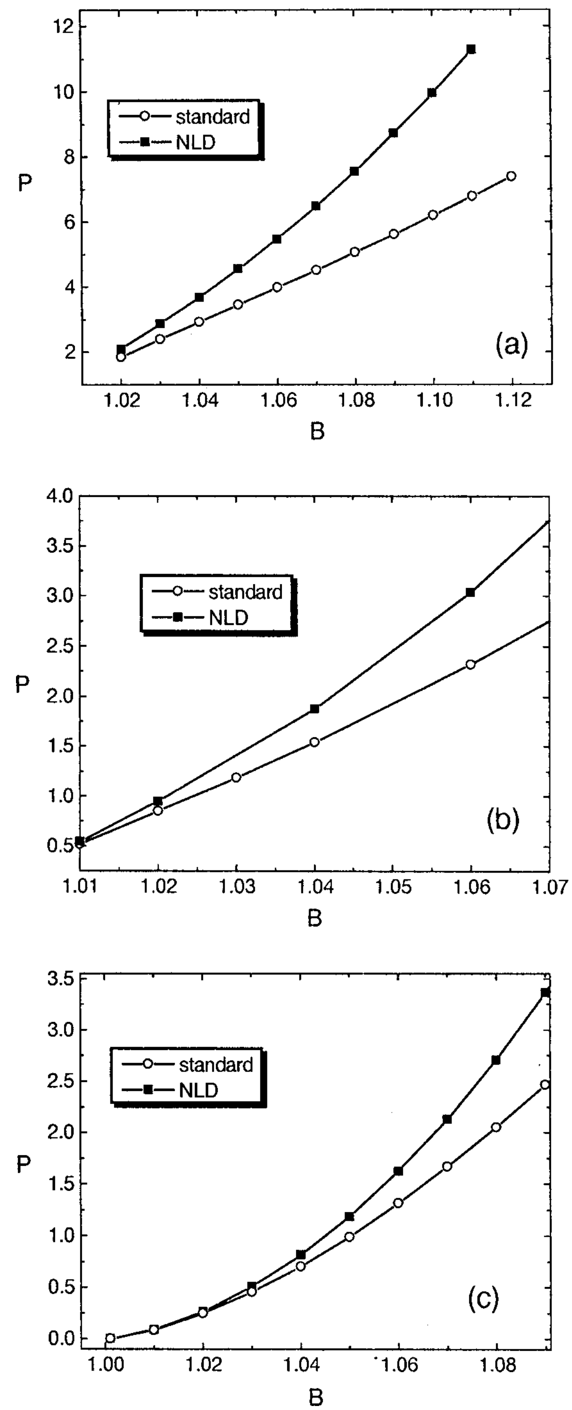

FIG. 2. Comparison of the changes of the normalized total power P of the two-beam system with variation of the dimensionless nonlinear wave-number shift $B=1+(\Delta\beta_\omega/k_\omega)$ with (solid squares, NLD) and without (open circles, standard) nonlinearly induced diffraction taken into account at $\alpha=k_{2\omega}/2k_\omega\equiv 1+(\Delta k_L/2k_\omega)$ equal to (a) 0.7, (b) 0.9, and (c) 1.

$$U=-\frac{1}{2}E_1^2-\alpha^2\frac{B^2-\alpha^2}{B^2-1}E_2^2+E_1^2E_2\left(1+\frac{\alpha^2}{2}-B^2\right)+\frac{B^2-1}{2}(E_1E_2)^2+\frac{B^2-1}{16}E_1^4 \tag{13c}$$

are, respectively, the kinetic and potential energies of the

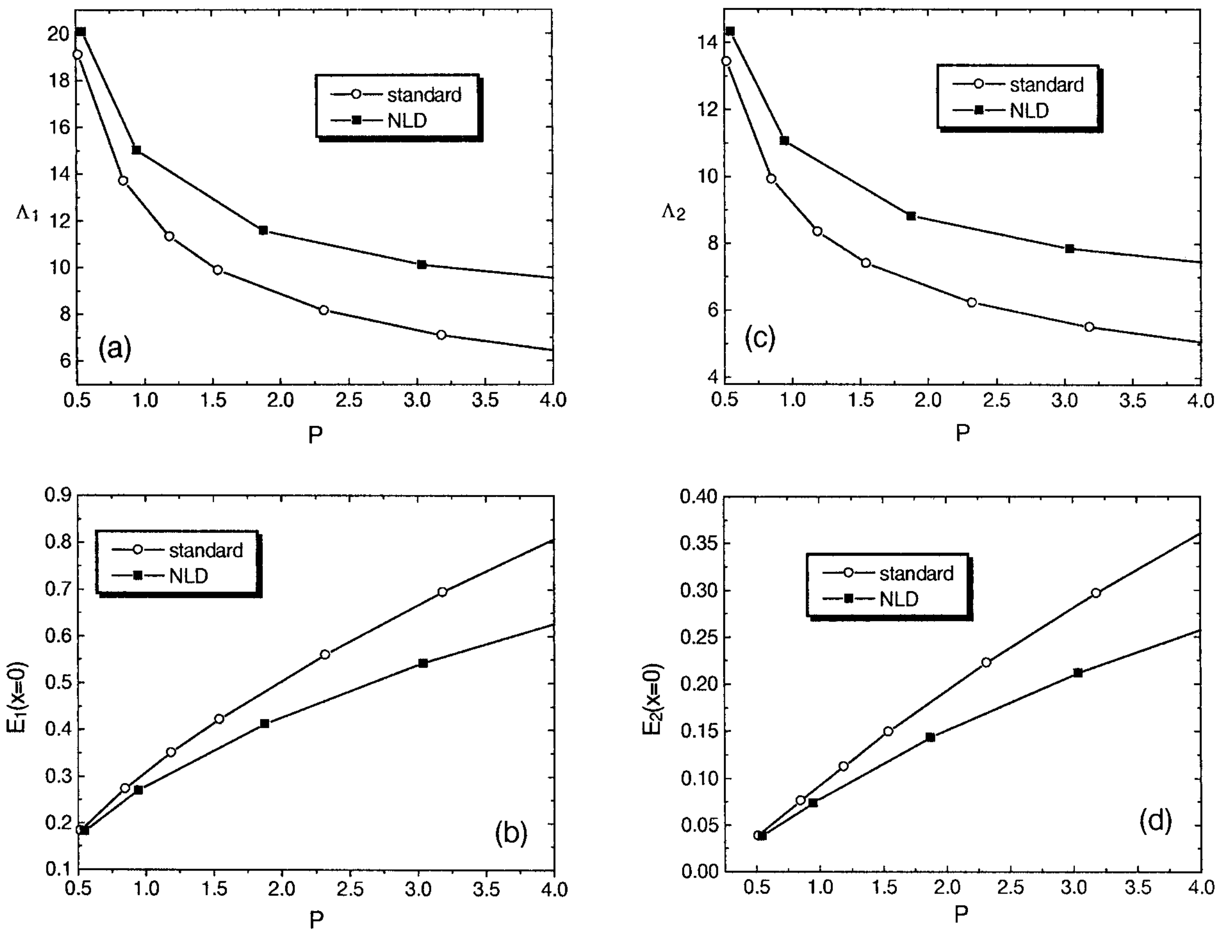

FIG. 3. Comparison of the solitonlike beam properties and their variation with the normalized total power P of the system (for $\alpha=0.9$) without (open circles, standard) and with (solid squares, NLD) nonlinearly induced diffraction taken into account: dimensionless full widths Λ_1 and Λ_2 (at the half-maximum) in (a) and (c) and normalized beam amplitudes $E_1(x=0)$ and $E_2(x=0)$ in (b) and (d), respectively, at the fundamental (index ''1'') and second-harmonic (index ''2'') frequencies.

two-beam system. Therefore, Eqs. (12) are equivalent of particle motion in a two-dimensional potential $U=U(E_1,E_2)$. Comparison with the Hamiltonian $H'=T'+U'$ [where $T'=[(dE_1/dx)^2/2]+[(dE_2/dx)^2/4]$ and $U'=-(E_1^2/2)-[(B^2-\alpha^2)/(B^2-1)]E_2^2+(E_1^2E_2/2)$ are the kinetic and potential energies of the two beam system when the nonlinearly induced diffraction is not taken into account [10]], shows the influence of the latter on the energy of the system. Introducing the paraxial approximation at a wrong stage does not ensure possibility of obtaining the Hamiltonian of the system.

The Lagrangian is

$$L=T-U, \tag{14}$$

where T and U are, as given by expressions (13b) and (13c).

According to Eqs. (11), existence of bound states (at $x\to\infty$) requires $B^2>1$ and $B^2>\alpha^2$. The first inequality is equivalent to $\Delta\beta_\omega>0$ and the second one being equivalent to $2\Delta\beta_\omega>\Delta k_L\equiv k_{2\omega}-2k_\omega$ put a condition for a threshold power which should be ensured in order to have the linear mismatch compensated by the nonlinear wave-number shift in the case of $k_{2\omega}>2k_\omega$.

Assuming considerations close to exact synchronism simplifies the Lagrangian of the system and the drawing of conclusions about the effects associated with the inhomogeneity of the nonlinear polarization and its rate of spatial variation (i.e., about the effects related to the nonlinearly induced diffraction).

The transformations

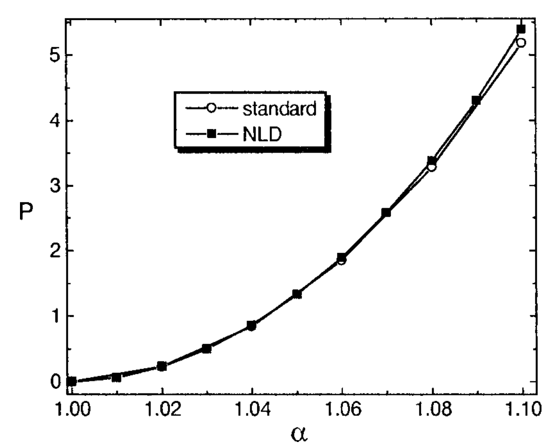

FIG. 4. Comparison of the dependence of the normalized total power of the system P on $\alpha=1+(\Delta k_L/2k_\omega)$ at the soliton threshold ($B=\alpha+0.0001$) with (solid squares, NLD) and without (open circles, standard) nonlinearly induced diffraction taken into account.

$$w = E_1 + (B^2-1)E_1E_2, \tag{15a}$$

$$v = E_2 + \frac{B^2-1}{4\alpha^2}E_1^2 \tag{15b}$$

introduce the displacements w and v at the fundamental and second-harmonic frequencies, respectively. The second terms in the right-hand side of expressions (15) are contributions coming out from the nonlinear polarization which are involved owing to the nonlinearly induced diffraction. By taking them as correction terms, a perturbation procedure in expressing $E_{1,2}$ through w and v can be developed:

$$E_1 \approx w - (B^2-1)wv, \tag{16a}$$

$$E_2 \approx v - \frac{B^2-1}{4\alpha^2}w^2. \tag{16b}$$

Therefore, the Lagrangian (14) of the system expressed in terms of w and v is

$$L = \frac{1}{2}\left\{\left(\frac{dw}{dx}\right)^2 + \frac{\alpha^2}{2}\left(\frac{dv}{dx}\right)^2 + w^2 + 2\alpha^2\frac{B^2-\alpha^2}{B^2-1}v^2 - B^2w^2v\right\} + \frac{1}{2}\left\{\frac{B^2-1}{8}w^4 + (2\alpha^2-1)(B^2-1)w^2v^2\right\}. \tag{17}$$

Comparison with the corresponding expression

$$L' = (1/2)\{(dw'/dx)^2 + [(dv'/dx)^2/2] + w'^2 + 2[(B^2-\alpha^2)/(B^2-1)]v'^2 - w'^2v'\}$$

(with $w' = E_1$, $v' = E_2$) obtained without taking into account the nonlinearly induced diffraction shows that the introduction of the latter gives a new meaning of w and v and changes the coefficients of some of the terms in addition to introducing correction terms [the last two terms in expression (15)]. The Euler–Lagrange equations corresponding to the Lagrangian (17) is

$$\frac{d^2w}{dx^2} - w + B^2wv - \frac{B^2-1}{4}w^3 - (2\alpha^2-1)(B^2-1)wv^2 = 0, \tag{18a}$$

$$\frac{d^2v}{dx^2} - 4\frac{B^2-\alpha^2}{B^2-1}v + \frac{B^2}{\alpha^2}w^2 - \frac{2}{\alpha^2}(2\alpha^2-1)(B^2-1)w^2v = 0. \tag{18b}$$

The nonlinearly induced diffraction involves B^2 and $(B/\alpha)^2$ as coefficients in the third terms in Eqs. (18a) and (18b) and introduces new terms [the last two in (18a) and the last one in (18b)] in the Euler–Lagrange equations of the system. These new terms show that the nonlinearly induced diffraction has a meaning of an effective third-order (defocusing) nonlinearity. Besides, in the equation for the fundamental wave this effective third-order nonlinearity acts both through self- and cross-phase modulation whereas for the second-harmonic wave its action is only through cross-phase modulation. Therefore, the nonlinearly induced diffraction could strongly influence phenomena which are associated with simultaneous action of second- and third-order nonlinearities. When self-phase and cross-phase modulation due to third-order nonlinearity is taken into account together with the second-order nonlinearity, terms of the form $\mu(B^2$

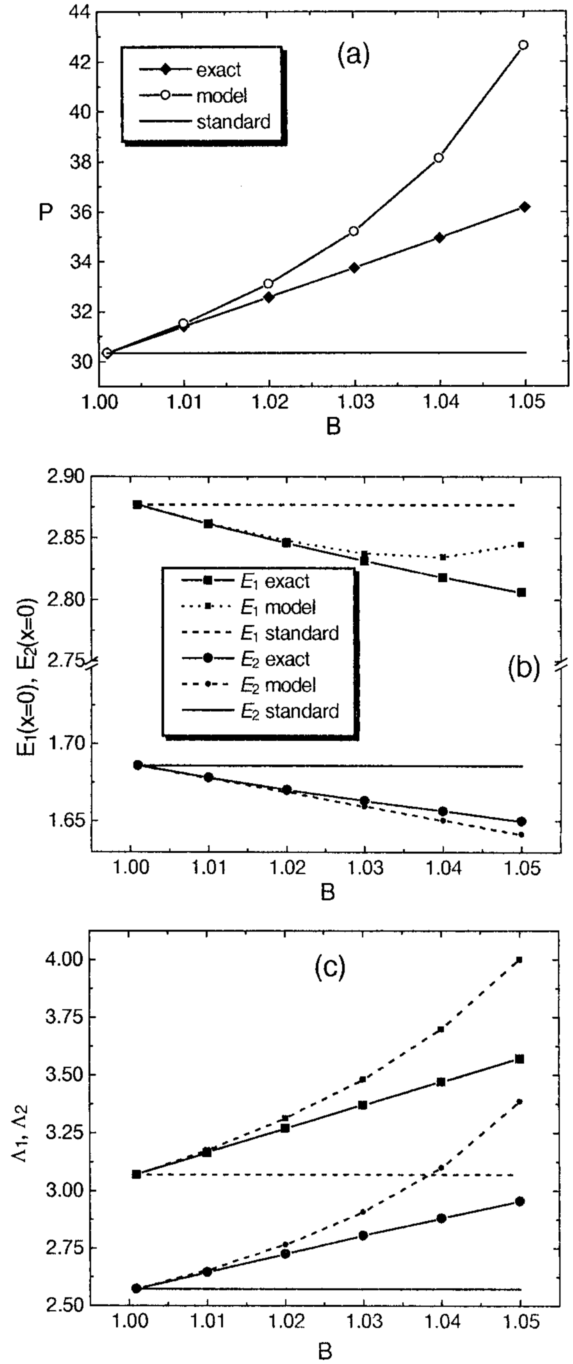

FIG. 5. Comparison (at phase match, $\alpha = 1$) of the solutions of the set of Eqs. (11) (exact solutions) with the solutions of the approximate equations (18) (model). The solutions (standard) in which the nonlinearly induced diffraction is ignored are also shown. In (a), total power P of the system vs $B = 1 + (\Delta\beta_\omega/k_\omega)$ obtained from Eqs. (11) (solid diamonds, exact), and (18) (open circles, model); the solid line represents the solution when the nonlinearly induced diffraction is ignored. Normalized amplitudes $E_1(x=0)$, $E_2(x=0)$ [according to the notation in Eqs. (12)] and dimensionless full widths Λ_1 and Λ_2 (at half-maximum) of the two beams, respectively, in (b) and (c) with notation as specified in (b).

$-1)[(w^2/4)+v^2]w$ and $2\mu(B^2-1)(w^2+v^2)v$, should be added to the left-hand sides of Eqs. (18a) and (18b), respectively. Here $\mu=(3/2)\chi^{(3)}\varepsilon_L(\omega)/(\chi^{(2)})^2$ and $\chi^{(3)}$ is the third-order susceptibility. Therefore, the effects of the nonlinearly induced diffraction predominate over those of Kerr-type nonlinearly for $\mu<1$.

It is worth noting that Eqs. (18) obtained here in the case of nonlinearly induced diffraction taken into account are of the same type as the equations which describe the soliton like solutions in quasi-phase-matched quadratic media [22,23].

V. NUMERICAL ANALYSIS

The set (11) is solved by using an algorithm involving the shooting technique [24]. Since equation $U(E_1,E_2)_{x=0}=0$ relates the amplitudes of the interacting beams at $x=0$, one of the unknown boundary conditions at $x=0$ drops off and, therefore, there is one ''shooting'' parameter left. A solution of the set of Eqs. (11) describing the shape of a coupled two-beam solitonlike state at given B- and α-values is presented in Fig. 1. The role of the nonlinearly induced diffraction is to decrease the amplitudes of the beams and to increase their width. This agrees with the interpretation of the nonlinearly induced diffraction as a defocusing effect.

The dependence of the total power of the system [P, relation (10b)] on the nonlinear wave-number shift at different α values is presented in Fig. 2. The solution without nonlinearly induced diffraction is denoted as ''standard.'' The nonlinearly induced diffraction requires higher power for ensuring a given value of the nonlinear wave-number shift. Increasing mismatch (i.e., larger deviation of α from unity) leads to increased power for a given value of the nonlinear wave-number shift. The lowest value of the power is at phase match [Fig. 2(c)]. At different α values, the relative changes of the power caused by the nonlinearly induced diffraction are almost the same.

The effects associated with the nonlinearly induced diffraction are shown in more details in Fig. 3. The full widths Λ_1, Λ_2 (at half-maximum amplitudes) and the amplitudes E_1, E_2 of the coupled solitonlike beams at the fundamental and second-harmonic frequencies (denoted, respectively, by indices ''1'' and ''2'') and their variation with the total power P of the system (respectively, with the nonlinear wave-number shift) are depicted. Since the effect is nonlinear, its influence (in increasing the beam widths and decreasing their amplitudes) increases with increasing power.

Therefore, the nonlinearly induced diffraction influences the total behavior of the coupled beam system by introducing changes of the shapes and the power of the solitonlike states of the system (Figs. 1–3). Its effect depends on the deviation of $2\Delta\beta_\omega$ from Δk_L and it is the weakest at the threshold ($2\Delta\beta_\omega=\Delta k_L$, respectively, $B=\alpha$). The latter is shown in Fig. 4 where conditions close to threshold for a formation of spatial solitons are simulated.

Figure 5, where numerical solutions of the coupled equations taken in their exact [Eqs. (11)] and approximate [Eqs. (18)] forms are compared, show that at comparatively small values of the nonlinear wave-number shift the results of the approximate description [Eqs. (18)] coincides with that given by the exact one. This figure also stresses on the effect of the nonlinearly induced diffraction on the beam properties (total power of the system and amplitudes and widths of the beams). The deviations of the solutions of Eqs. (11) and (18), which account for the nonlinearly induced diffraction, from the solution of the corresponding set of equations in which it is ignored, strongly increases with the increase of the nonlinear wave-number shift.

VI. CONCLUSIONS

In conclusion, the nonlinearly induced diffraction stemming from the div $\vec{E}$ term in the wave equation acts on the solitonlike states of a two-beam system at fundamental and second-harmonic frequencies as an effective third-order nonlinearity. With respect to the beam properties at the fundamental frequency this is an effect of both self- and cross-phase modulation whereas at the second-harmonic frequency the nonlinearly induced diffraction acts only through cross-phase modulation. Without influencing the threshold power value, the nonlinearly induced diffraction leads to changes of the properties of the solitons: Widening of their shape accompanied with a decrease of the beam amplitudes. Although the involvement of the nonlinearly induced diffraction complicates the investigation of the two-beam coupled system, derivation of the power-conservation law and getting results for the Lagrangian and the Hamiltonian of the system is still possible if a proper analytical procedure is applied with respect to the stage at which the paraxial approach is made. The nonlinearly induced diffraction could be very important for processes which go simultaneously through second- and third-order nonlinearities.

ACKNOWLEDGMENTS

This work is supported by the National Fund for Scientific Research (Project No. F-810) and by the Foundation for Scientific Research at Sofia University (Project No. 299/99).

[1] Yu.N. Karamzin and A.P. Sukhorukov, JETP Lett. **20**, 339 (1974).
[2] G.I. Stegeman, D.J. Hagan, and L. Torner, Opt. Quantum Electron. **28**, 1691 (1996).
[3] *Advanced Photonics with Second-order Optically Nonlinear Processes*, edited by A.D. Boardman, L. Pavlov, and S.Tanev (Kluwer, Dordrecht, 1998).
[4] R. De Salvo, D.J. Hagan, M. Shiek-Bahae, G. Stegeman, and E.W. Van Stryland, Opt. Lett. **17**, 28 (1992).
[5] R. Schiek, J. Opt. Soc. Am. B **10**, 1848 (1993).
[6] A.G. Kalocsai and J.W. Haus, Opt. Commun. **97**, 239 (1993).
[7] P. Pliszka and P.P. Banerjee, J. Mod. Opt. **40**, 1909 (1993).
[8] S. Trillo and P. Ferro, Opt. Lett. **20**, 438 (1995).
[9] A.V. Buryak and Yu.S. Kivshar, Phys. Lett. A **197**, 407 (1995).
[10] A.D. Boardman, K. Xie, and A. Sangarpaul, Phys. Rev. A **52**, 4099 (1995).
[11] L. Torner, D. Mihalache, D. Mazilu, E. Wright, W. Torruelas, and G.I. Stegeman, Opt. Commun. **121**, 149 (1995).
[12] S. Trillo, A.V. Buryak, and Yu.S. Kivshar, Opt. Commun. **122**,

200 (1996).
[13] L. Torner, D. Mazilu, and D. Mihalache, Phys. Rev. Lett. **77**, 2455 (1996).
[14] C. Conti and G. Assanto, Opt. Express **3**, 389 (1998).
[15] A.D. Boardman, T. Popov, A. Shivarova, S. Tanev, and D. Zyapkov, J. Mod. Opt. **39**, 1083 (1992).
[16] A.D. Boardman, A. Shivarova, S. Tanev, and D. Zyapkov, J. Mod. Opt. **42**, 2361 (1995).
[17] A.W. Snyder, D.J. Mitchel, and Y. Chen, Opt. Lett. **19**, 524 (1994).
[18] E. Granot, S. Sternklar, Y. Isbi, B. Malomed, and A. Lewis, Opt. Lett. **22**, 1290 (1997).
[19] E. Granot, S. Sternklar, Y. Isbi, B. Malomed, and A. Lewis, Opt. Commun. **166**, 121 (1999).
[20] A.D. Boardman, K. Marinov, D.I. Pushkarov, and A. Shivarova, Opt. Quantum Electron. **32**, 49 (2000).
[21] P.N. Butcher and D. Cotter, *The Elements of Nonlinear Optics* (Cambridge University Press, Cambridge, England, 1990).
[22] M.M. Fejer, G.A. Magel, D.H. Jundt, and L. Byer, IEEE J. Quantum Electron. **28**, 2631 (1992).
[23] C.B. Clausen, O. Bang, and Yu. Kivshar, Phys. Rev. Lett. **78**, 4749 (1997).
[24] W.H. Press, S.A. Teukolski, W.T. Vetterling, and B.P. Flannery, *Numerical Recipes in C* (Cambridge University Press, Cambridge, England, 1992).