Frequency and width crossing of two interacting resonances in a microwave cavity

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Frequency and width crossing have been observed for two coupled resonances in a microwave cavity. The cavity consisted of two nearly identical rectangular boxes. The boxes were coupled by a slit of variable width. The data are well described by a non-Hermitean S matrix leading to a 2×2 non-Hermitean effective Hamiltonian. The values of the interaction strength $|v|$ for which width crossing and frequency crossing, respectively, were observed, are above and below a critical value $|v_c|$ for which one expects a joint frequency and width crossing.

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I. INTRODUCTION

Two level mixing is a fruitful concept and simple language in physics [1,2]. In the case of bound states the underlying Hamiltonian is Hermitean and owing to this property an off-diagonal coupling between both states causes the level energies to repel each other. For this reason a frequency crossing is only possible, if the off-diagonal coupling is zero [1–3]. The extension of two level mixing from bound to unbound, i.e., decaying states is very natural. The decaying unbound states have complex energies ϵ_k and are described by a non-Hermitean Hamiltonian [3–11], however. The decomposition of the complex energies ϵ_k into the real energies E_k and frequencies ν_k and the widths Γ_k and γ_k is given in Eq. (1), where we have put $h=1$:

$$\epsilon_k = E_k - \frac{i}{2}\Gamma_k = \nu_k - \frac{i}{2}\gamma_k . \tag{1}$$

We will refer to the ϵ_k in the paper alternatively as complex energies or complex eigenfrequencies. The fact that the energies of the decaying states are complex opens a rich scenario of crossings and anticrossings of energies or frequencies and width for the unbound two level system. In particular it has been suggested that whereas energies of bound states can cross only for a vanishing interaction [1] the complex energies of unbound (decaying) states can cross at a nonvanishing interaction strength v [5–11]. A rather surprising theoretical result is given in Ref. [11]. It states that in the two unbound level system a real purely off-diagonal interaction implies that either there is a joint crossing of both the unperturbed frequencies and the perturbed widths or there is a joint crossing of the unperturbed frequencies and of the perturbed frequencies.

The terms frequency crossing or anticrossing refer to the situation that two frequencies or widths are measured as a function of a slowly varying parameter λ. One speaks of frequency crossing if there is a value λ_0 for which the two frequencies become equal, i.e., $\nu_1(\lambda_0)=\nu_2(\lambda_0)$. One speaks of frequency anticrossing if for all values of the parameter λ the two frequencies differ, i.e., $\nu_1(\lambda)\neq\nu_2(\lambda)$ for all λ. This language is also used for the two widths $\gamma_1(\lambda)$ and $\gamma_2(\lambda)$. The language comes from atomic physics, where such crossings and anticrossings are observed as a function of the magnetic field. At this point we want to mention a very early paper on coupled decaying electronic states of molecules by Estrada *et al.* [12], which contains a discussion of the crossing phenomena. In this paper we will give experimental evidence for this theoretical result, by performing experiments on microwave cavities, which have recently been shown very useful in a study of quantum chaos and general resonance phenomena [13–18].

II. EXPERIMENTAL DATA

The experiment was performed with a double box microwave cavity. It consisted of two nearly identical rectangular boxes made of copper. These resonators were coupled by a narrow slit in the partition wall, which allowed the two subsystems to interact, see Fig. 1. The two antennas a and b provided an inductive coupling of the resonator with the transmission cables. Microwaves in the frequency range of 992 to 994 MHz excited the first TM-mode of each box. In order to obtain two nearly equal complex eigenfrequencies the two boxes were made nearly identical. In addition there were two tools, which allowed to vary the parameters of the cavity: A small block of plastic P, of variable position y in the first box allowed to vary the eigenfrequency ν_1^0 of the first box, a slit of variable size x in the wall separating the

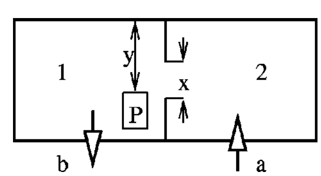

FIG. 1. Sketch of the twin microwave cavity. The antennas a and b are coupled to the generator and to the detector, respectively. The wall between the two cavities has an opening of width x, which couples them. The body P depicts a small block of plastics, which can be moved inside cavity 1 by an amount y from the wall.

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two cavities allowed to vary the coupling strength v of the two cavities.

A sinusoidal electrical signal from a generator of variable frequency ν and amplitude $I_a(\nu)$ was fed into the cavity through the input antenna a and a signal of complex amplitude $O_b(\nu)$ was observed from the output antenna b by a detector. Both, amplitude and relative phase of the output signal $O_b(\nu)$ was measured. The ratio of complex amplitudes $S_{ab}(\nu)=O_b(\nu)/I_a(\nu)$ was measured as a function of the generator frequency ν. It was analyzed using an S matrix. The form of the S matrix was taken from nuclear physics and is discussed extensively by Mahaux and Weidenmüller [19,20]. In nuclear physics the antennae correspond to the channels, which are denoted c, c' there [16,21]. The form of the S matrix is

$$S_{cc'}(\nu)=e^{i\phi_c}\left(\delta_{cc'}-i\sum_{nn'}W_{cn}D^{-1}_{nn'}W_{c'n'}\right)e^{i\phi_{c'}}. \quad (2)$$

Here the propagator $D_{nn'}$ defines an effective symmetrical non-Hermitean Hamiltonian $H_{nn'}$ by the relation

$$D_{nn'}=\nu\delta_{nn'}-H_{nn'}. \quad (3)$$

The amplitudes W_{cn}, $W_{c'n'}$ describe the coupling of channel c, respectively, c', to the resonance n. As the walls absorb energy (the quality factor Q of the cavity has been of the order of 10^3) the S matrix is nonunitary. Thus the parameters ϕ_c, $\phi_{c'}$, W_{cn}, $W_{c'n'}$ will be complex in general.

In the experiment we investigated the neighborhood of the two nearly degenerate fundamental modes of the two box cavity ϵ_1, ϵ_2. These two eigenmodes are well separated in frequency from the other eigenmodes of higher order and thus the data can be accurately described by taking only these two modes into account. Therefore the effective Hamiltonian H becomes a complex symmetrical 2×2 matrix. The respective complex eigenvalues ϵ_1 and ϵ_2 of this effective Hamiltonian matrix are identical with the poles of the corresponding S matrix. Strictly speaking resonance states do not have complex energies which are constants. The resonances which we are considering are however extremely narrow. For narrow resonances one can use, however, effective energy independent parameters. According to the size x of the coupling slit in the microwave cavity one can decompose the effective Hamiltonian H into an unperturbed (uncoupled) part $H^0(y)$, which depends on the position y of the plastic block P, and a purely nondiagonal interaction $V(x)$, which is a function of the slit size x only:

$$\begin{aligned} H&=H(x,y)\\ &=H^0(y)+V(x)\\ &=\begin{pmatrix}\epsilon_1^0(y) & 0\\ 0 & \epsilon_2^0\end{pmatrix}+\begin{pmatrix}0 & v(x)\\ v(x) & 0\end{pmatrix}. \end{aligned} \quad (4)$$

Here we have assumed further that only the eigenmode ϵ_1^0 of box 1 in which the plastic P is moved is affected by the movement. Thus we assume $\epsilon_1^0=\epsilon_1^0(y)$ and $\epsilon_2^0=\text{const}$. The decomposition $H=H^0+V$ allows us to define besides the perturbed complex eigenfrequencies ϵ_1, ϵ_2 of H also the corresponding unperturbed complex eigenfrequencies ϵ_1^0, ϵ_2^0 of H^0. As is well known [1–3] the ϵ_n are related to the ϵ_n^0 and the interaction strength v by

$$\left(\epsilon_{1,2}-\frac{1}{2}(\epsilon_1^0+\epsilon_2^0)\right)^2=\frac{1}{4}(\epsilon_1^0-\epsilon_2^0)^2+v^2. \quad (5)$$

In the following we study the relation between the ϵ_n and the ϵ_n^0 in the neighborhood of the crossing point of the unperturbed frequencies $\nu_1^0\approx\nu_2^0$. In the experiment we measured the complex number $S_{ab}(\nu)=O_b(\nu)/I_a(\nu)$ as a function of the generator frequency ν. The data far from the crossing point showed two strong resonances corresponding to the two fundamental complex eigenfrequencies ϵ_1 and ϵ_2 of the cavity. The data can in principle be analyzed directly with the S matrix of Eq. (2). Instead of applying this description we used a special form Eq. (6) of the S matrix, which is numerically stable in the vicinity of the complex energy crossing point ($\epsilon_1=\epsilon_2$), however. We give only the S matrix element S_{ab} between the input antenna a and the output antenna b:

$$S_{ab}(\nu)=-ie^{i(\Phi_a+\Phi_b)}\frac{vW_{a2}^0W_{b1}^0}{\epsilon_1-\epsilon_2}\left(\frac{1}{\nu-\epsilon_1}-\frac{1}{\nu-\epsilon_2}\right), \quad (6)$$

where Φ_a and Φ_b are defined as in Eq. (2). The interaction v is defined in Eq. (4) and W_{a2}^0 and W_{b1}^0 are the amplitudes W_{a2} and W_{b1}, respectively, which are appropriate to the unperturbed system ($v=0$). In deriving Eq. (6) from Eq. (2) we assume that in the unperturbed system there is no direct coupling from the antenna a to box 1 and from antenna b to box 2, i.e., we assume $W_{a1}^0=W_{b2}^0=0$.

From the fits to the data far away from the crossing point we obtained unique values of the two complex eigenfrequencies ϵ_1 and ϵ_2. The ϵ_1, ϵ_2 were obtained for several values of the parameters x and y of the two perturbing elements of the cavity, namely, the coupling strength $v=v(x)$ and the position y of the block P. We measured ϵ_1 and ϵ_2 at 9 values y_k of y and for 2 values x_i of x: $x=2.1$ and $x=1.3$ cm. Thus we obtained 18 complex eigenfrequencies $\epsilon_1(x,y)$, $\epsilon_2(x,y)$. In fitting the resonances in the vicinity of the crossing some numerical stability problems were encountered even for the form of Eq. (6) for the S matrix. This is not surprising, because in the vicinity of the crossing point $\nu_1^0\approx\nu_2^0$ the data show only one resonance structure in $|S|$ and a smooth behavior of the phase Φ_s of the S_{ab} defined by $S=e^{i\Phi_s}|S|$. In order to get unique fit parameters ϵ_1 and ϵ_2 also in the vicinity of the crossing point we used two extra constraints. First it was assumed that the sum of the two complex energies $\epsilon_1+\epsilon_2$ is at most a quadratic polynomial in y:

$$\epsilon_1+\epsilon_2=\epsilon_1^0+\epsilon_2^0=a+ib+cy+dy^2, \quad (7)$$

where the real coefficients a, b, c, d may depend on v, respectively, x but not on y. Furthermore it was assumed that the product $W_{a2}^0W_{b1}^0$ is independent of y. These assumptions were tested for the data far away from the crossing point, which gave unique fitted values for ϵ_1 and ϵ_2. Then these assumptions were used to get unique values for ϵ_1 and ϵ_2 also in the vicinity of the crossing.

From the perturbed complex frequencies ϵ_1 and ϵ_2 one can calculate the two unperturbed complex frequencies $\epsilon_n^0(x,y)$ from the perturbed complex frequencies $\epsilon_n(x,y)$ using Eq. (5). In doing this we note first that $\epsilon_2^0(x,y)=\epsilon_2^0(x)$ because the plastic P moves only in box 1. Thus from measurements at the two positions y and y' at a constant x one obtains eight real parameters from the following four complex quantities: $\epsilon_1(x,y)$, $\epsilon_1(x,y')$, $\epsilon_2(x,y)$, $\epsilon_2(x,y')$. From these eight real parameters one can determine the quantities $\epsilon_1^0(x,y)$, $\epsilon_1^0(x,y')$, $\epsilon_2^0(x,y)=\epsilon_2^0(x,y')$ and v, which contain also eight real parameters, if we allow v to be complex. Actually the measurements were performed at 9 different values of the distance y of the block P from the wall. Thus the parameters ϵ_n^0 and v are overdetermined. As $v=v(x)$ is thus fixed we will use v instead of x (Fig. 1) in the further discussion. In a similar way we replace the distance y (Fig. 1) by the more physical parameter $\delta\nu_1^0$ which is the difference between the unperturbed frequencies:

$$\delta\nu_1^0(x,y)=\nu_1^0(x,y)-\nu_2^0(x,y). \tag{8}$$

One finds, that $\delta\nu_1^0$ depends essentially only on y and is nearly independent of x. Thus we will use $\delta\nu_1^0$ instead of the distance y in the following.

It follows from Eq. (5), that the labeling of the two complex eigenfrequencies as ϵ_1 and ϵ_2 is arbitrary at each frequency $\delta\nu_1^0$. To get a unique labeling of ϵ_k, we note, that far to the left of the crossing, i.e., for $\delta\nu_1^0 \ll 0$ and for $|\delta\nu_1^0| \ll |v|$. The perturbed and unperturbed complex eigenfrequencies must be approximately equal, i.e.,

$$\epsilon_k \approx \epsilon_k^0, \quad k\in\{1,2\}. \tag{9}$$

By requiring continuity we obtain unique values of both ϵ_1 and ϵ_2 as a function of $\delta\nu_1^0$ for given values ϵ_1^0 and ϵ_2^0. In the Figs. 2 and 3 we have plotted the values of the unperturbed frequencies ν_1^0, ν_2^0 and widths γ_1^0, γ_2^0 and of the perturbed frequencies ν_1, ν_2 and widths γ_1, γ_2 as a function of the difference $\delta\nu_1^0$ of the unperturbed frequencies for two values of the coupling strength v.

In Fig. 2 the coupling strength has a value $|v|=0.063$ MHz and one observes width crossing and frequency anticrossing. In Fig. 3 the interaction strength v has the smaller value $|v|=0.015$ MHz and one observes frequency crossing and width anticrossing.

The curves in the upper part of Figs. 3 and 2 show the unperturbed frequencies and widths as a function of $\delta\nu_1^0$. These curves are obtained from the relations

$$\epsilon_1^0(v)=\epsilon_1^{00}(v)+\delta\nu_1^0, \tag{10}$$

$$\epsilon_2^0(v)=\epsilon_2^{00}(v), \tag{11}$$

where $\nu_1^{00}(v)=\nu_2^{00}(v)$ is assumed to make $\delta\nu_1^{00}$ well defined.

In Table I we give two sets of parameters $\epsilon_1^{00}(x)$, $\epsilon_2^{00}(x)$, and $v(x)$ obtained at two values of $v(x)$ and x. The curves in the lower part of Figs. 2 and 3 were calculated with Eq. (5) from the unperturbed complex frequencies, the interaction strength v obtained from Table I and the relations (10),(11). We note that the perturbed frequencies and widths are well reproduced by the corresponding unperturbed quantities and

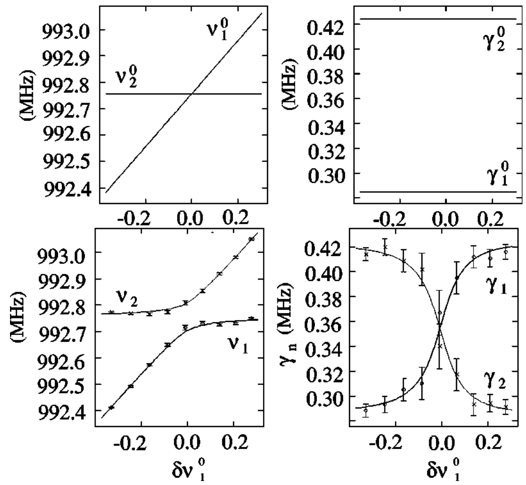

FIG. 2. Frequency anticrossing left-hand side (LHS) and width crossing right-hand side (RHS). The frequencies ν_1^0, ν_2^0 and widths γ_1^0, γ_2^0 of the unperturbed system $(v=0)$ and the corresponding frequencies ν_1, ν_2 and widths γ_1, γ_2 of the perturbed system are shown in dependence of the parameter $\delta\nu_1^0=\nu_1^0-\nu_2^0$. ψ_1^0 and ψ_2^0 denote the states of the isolated cavities 1 and 2. This is the strong coupling case 0.063 Mhz$=|v|>v_c=0.037$ Mhz. The lines through the data points are calculated from Eqs. (5),(9) in the main text.

by an essentially real interaction strength $v=e^{i\phi_v}|v|$. Ideally the unperturbed complex frequencies should be independent of the interaction strength v. This is essentially true although there are some small discrepancies in particular in the widths $\gamma_1^0(v=0.063)$ and $\gamma_1^0(v=0.015)$, which differ by about 10%.

We will now try to understand the observed crossings and anticrossings from the basic relation (5). We follow here the theoretical results and the presentation of Ref. [11]. The variation of the squared differences of the eigenfrequencies is obtained from Eq. (5):

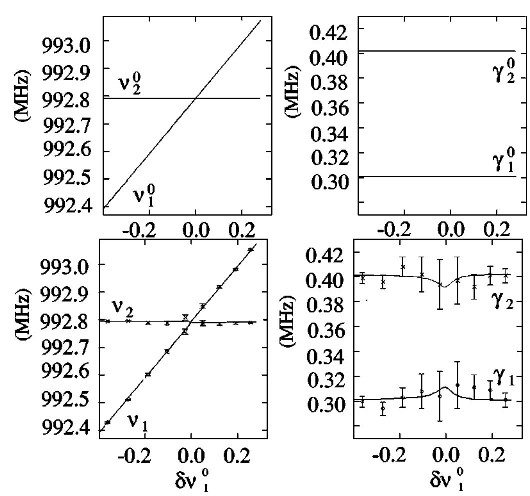

FIG. 3. Frequency crossing (LHS) and width anticrossing (RHS): The frequencies ν_1^0, ν_2^0 and widths γ_1^0, γ_2^0 of the unperturbed system $(v=0)$ and the corresponding frequencies ν_1, ν_2 and widths γ_1, γ_2 of the perturbed system are shown as a function of the parameter $\delta\nu_1^0$ (see caption to Fig. 2). This is the weak coupling case 0.015 Mhz$=|v|<v_c=0.037$ Mhz. The lines through the data points result from Eqs. (5),(9) in the main text.

TABLE I. Calculated unperturbed resonance parameters and interaction strengths v.

x (mm)	$\|v\|$ (MHz)	$\arg v$ (deg)	ν_1^{00} (MHz)	γ_1^{00} (MHz)	ν_2^{00} (MHz)	γ_2^{00} (MHz)
21	0.063(2)	2(2)	992.747(2)	0.285(4)	992.747(1)	0.424(3)
13	0.015(6)	7(23)	992.765(2)	0.321(4)	992.765(1)	0.422(3)

$$(\epsilon_1-\epsilon_2)^2=(\epsilon_1^0-\epsilon_2^0)^2+4v^2. \tag{12}$$

In general the interaction v is complex. But in our special experimental case v is real within the error bars. Therefore we restrict our considerations to a real v.

In order to simplify the calculations one decomposes the differences of the complex eigenfrequencies into real and imaginary parts

$$\epsilon_1^0-\epsilon_2^0=:e^0+ig^0, \quad \epsilon_1-\epsilon_2=:e+ig. \tag{13}$$

From Eqs. (12),(13) one obtains

$$e^0g^0=eg, \tag{14}$$

$$e^2-(e^0)^2=4v^2-(g^0)^2+g^2. \tag{15}$$

One can classify the behavior of the perturbed complex frequencies at a point where the unperturbed real frequencies cross, i.e., for $\delta\nu_1^0=\nu_1^0-\nu_2^0=e^0=0$. One finds three cases, which are distinguished by the value of the interaction strength $|v|$ in comparison to a critical value v_c:

$$v_c:=\tfrac{1}{2}|g^0|=\tfrac{1}{4}|\gamma_1^0-\gamma_2^0|. \tag{16}$$

The three cases are as follows.

Overcritical coupling. If $|v|>v_c$, one finds $\nu_1\neq\nu_2$ and $\gamma_1=\gamma_2$, i.e., frequency anti-crossing and width crossing.

Critical coupling. If $|v|=v_c$, one finds $\nu_1=\nu_2$ and $\gamma_1=\gamma_2$, i.e., a joint frequency and width crossing, which is also referred to as complex frequency crossing.

Subcritical coupling. If $|v|<v_c$, one has $\nu_1=\nu_2$ and $\gamma_1\neq\gamma_2$, i.e., frequency crossing and width anticrossing.

The proof of these statements taken from Ref. [11] is straightforward from the above. From Eq. (14) and $e^0=0$ it follows $eg=0$, i.e. $e=0$ or $g=0$. From $e^0=0$ and Eq. (15) one obtains for $|v|>\frac{1}{2}|g^0|$ the relation $e^2>0$ and thus $g=0$ follows from $eg=0$. For $e^0=0$ and $|v|<\frac{1}{2}|g^0|$ it follows from Eq. (15) that $g^2>0$ and thus $e=0$. Finally, for $e^0=0$ and $|v|=\frac{1}{2}g^0$ we have $|e|=|g|=0$.

The full complex energy crossing $|e|=|g|=0$ has been discussed in detail in Ref. [10]. We note that width crossing for $|v|>v_c$ can be understood rather easily. Namely for an appropriately large interaction strength v the two eigenmodes are equally mixed and as a consequence the widths are equal. It is also plausible that a larger interaction strength v is needed if $(g^0)^2=\frac{1}{4}(\gamma_1^0-\gamma_2^0)^2$ is large, i.e., if the difference in the unperturbed width of the two eigenmodes is large. An interesting aspect is that there is no subcritical case, $|v|<v_c$, if the unperturbed widths are equal: $\gamma_1^0=\gamma_2^0$. In this case there is no frequency crossing for $|v|\neq 0$. This is a direct generalization of the von Neumann–Wigner case, which forbids crossing at $v\neq 0$ for bound states, i.e., for $\gamma_1^0=\gamma_2^0=0$.

We now compare the data in Figs. 2 and 3 with the theoretical expectations. One finds, 0.063 MHz$=|v|>|v_c|=0.037$ MHz for the data of Fig. 2. This is the overcritical coupling case. And indeed frequency anti-crossing and width crossing is observed. For the data in Fig. 3 one has 0.015 Mhz$=|v|<|v_c|=0.025$ MHz this is the undercritical coupling case and indeed one finds frequency crossing and width anticrossing. We note the small difference of $|v_c|$ in the experiments done at different couplings v. Complex frequency crossing is expected around $|v|=0.030$ MHz and lies thus in between the two measured interaction strengths $|v|=0.015$ MHz and $|v|=0.063$ MHz. It would be very interesting to perform experiments very close to $|v|=|v_c|$. To do this requires however a much more stable setup, which we are developing at present. As all data are well described by the model we can claim to have established from these data also complex frequency crossing. Clearly it would be more convincing if data points much nearer to the complex frequency crossing would be measured. But to perform this experiment requires a much more stable apparatus, which we are presently designing.

We want to add a plausibility argument for width crossing at large interactions. For this we consider the states ψ_n^0, which are the eigenstates of the isolated cavities $n=1,2$ with unperturbed complex eigenfrequencies ϵ_n^0. In analogy to the bound state case we find for a large $|v|$ frequency anticrossing, see Fig. 2. This and the requirement of continuity imply as explained below Eq. (8), that the state ψ_1^0 turns into the state ψ_2^0 with increasing $\delta\nu_1^0$. As a consequence the width $\gamma_1(\delta\nu_1^0\ll 0)\approx\gamma_1^0$ changes into $\gamma_1(\delta\nu_1^0\gg 0)\approx\gamma_2^0$ and vice versa. From the continuity of γ_1 and γ_2 one finds then that γ_1 and γ_2 must cross if $\gamma_1^0\neq\gamma_2^0$.

III. CONCLUSION

Summing up, we have investigated a doublet of two interacting resonances in a two box microwave cavity. We have observed experimentally width and frequency crossing at a nonvanishing interaction strength $|v|\neq 0$. These experimental results have corroborated the theoretical frequency and width crossing and anticrossing relations proposed in Ref. [11]. The frequency and width crossing and anticrossing are obtained at an interaction strength above and below a critical interaction strength $v_c=\frac{1}{4}|\gamma_1^0-\gamma_2^0|$. By measuring above and below of the critical interaction strength v_c we have found signatures which allow to identify full complex frequency crossing, which should occur at $|v|=v_c$. We note that if the two widths differ from each other, one has a full complex frequency crossing at a nonvanishing interaction $v\neq 0$. This clearly differs from the bound state case, where a frequency crossing is only allowed for $v=0$. We think that this paper demonstrates that the study of the unbound two level system in two coupled microwave cavities is a very rewarding enterprise. In this respect a new prediction of the change of the relative phases as a function of parameters of two interacting resonances by Heiss [22] is mentioned.

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