Coherent resonance in a one-way coupled system

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We describe the resonancelike behavior of a cooperative phenomenon involving noise, nonlinear systems with intrinsic limit cycle dynamics, and coupling in the absence of an external signal. We show that coupling can significantly sustain the propagation of coherent resonance with considerable enhancement or suppression along a one-way chain. In addition, coherent resonance can occur without tuning for a proper noise level and coupling constant.

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The phenomenon of stochastic resonance (SR) has attracted considerable attention in different fields of science [1–4]. The typical SR is the result of a cooperative effect of noise and an external signal acting upon a nonlinear system such as a bistable or an excitable system. The response of the system to a weak external signal is enhanced by the presence of noise. Recently, a new kind of SR-like behavior without external signal as a consequence of the intrinsic dynamics of nonlinear system was presented [5–10], which has been called automous SR [5–7], internal SR [8], or coherent resonance (CR) [9,10].

It's known that coupled dynamical systems are an important field in nonlinear science. Very recently, the significant phenomena of array enhanced stochastic resonance [11–13] and noise enhanced propagation [14–16] in coupled systems have been demonstrated. In these studies, a cooperative phenomenon involving external signal, noise, nonlinearity, and coupling is concerned. If there are only intrinsic dynamics in coupled systems driven by noise in the absence of an external signal, what will it happen? More recently, experimental observation of CR in an array of cascade excitable systems using a monovibrator circuit has been shown by Postnov *et al.* [17]. When the first monovibrator is exicted by the external noise, while each successive stage gets the input signal from the output of the preceding circuit, CR can be propagated with enhancement along the chain. In this paper, a one-way coupled brusselators subject to additive noise in the absence of external signal is investigated. When we inject Gaussion white noise to the first cell of the system, not surprisingly, noise-induced oscillations and CR in the cell are observed. More interestingly, CR can be propagated with considerable enhancement or suppression along the coupled chain. In addition, a new phenomenon of CR without tuning at proper coupling constant and noise level is shown.

The well-known brusselator model is given by [18,19]

$$dx/dt = x^2 y - Bx - x + A = f_1(x,y),$$
$$dy/dt = Bx - x^2 y = f_2(x,y), \tag{1}$$

where A, B, x, and y are dimensionless concentrations. For $B < A^2 + 1$, the system is in a steady state, and for $B > A^2 + 1$, limit cycle oscillations appear. A is the control parameter. The evolution equations for the one-way coupled system are

$$dx_n/dt = f_1(x_n, y_n) + D_n(x_{n-1} - x_n),$$
$$dy_n/dt = f_2(x_n, y_n). \tag{2}$$

We only consider that x is coupled. Where D_n is coupling constant joining the $(n-1)$th cell and nth cell, $n = 1,2 \ldots N$ and N is the end number of coupled cells. Let us fix $x_0 = 0.446$. Gaussian white noise is only imposed on the control parameter A of the first cell,

$$A_1 = A[1 + \xi(t)] \tag{3}$$

with $\langle \xi(t) \rangle = 0$, $\langle \xi(t)\xi(s) \rangle = \sigma^2 \delta(t-s)$, and σ is the noise level. In our simulation, $A = 0.46$, $B = 1.2$, stochastic noise is imposed per 100 time steps.

For a single oscillator $(N = 1)$, let us fix $A = 0.46$ such that the system is placed in a steady state. When noise is

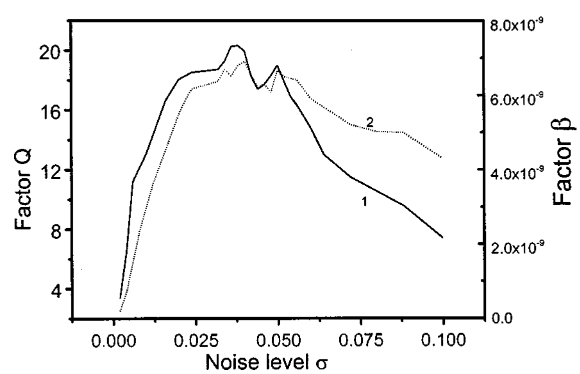

FIG. 1. The quality factor Q and β as a function of noise level σ (curves 1, 2 correspond to Q and β, respectively). $A = 0.46$; $B = 1.2$.

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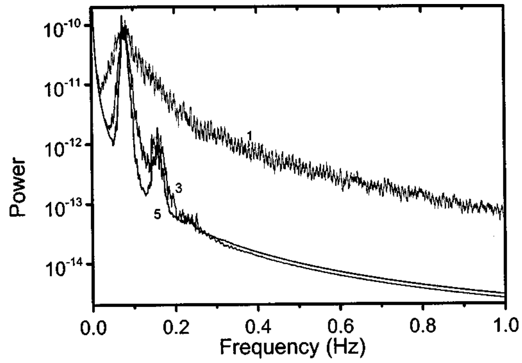

FIG. 2. The power spectra for C_1, C_3, and C_5, noise background is suppressed with the increment of the coupling number (curves 1, 3, and 5 correspond to C_1, C_3, and C_5, respectively). $\sigma=0.12$; $N=5$; $D_n=0.2$.

imposed, noise-induced oscillations appear. We study quantitatively the effect of noise through the power spectra $x(t)$, noticing the power spectra are obtained by averaging over 50 runs. To observe CR the degree of coherent β of the spectra can be calculated as defined in Ref. [5], $\beta=f_0h/\Delta f$, where f_0 is the principal peak frequency of the spectrum, h is the maximum peak height, and Δf the width of the peak at half-maximum height. Because β does not directly take into account the noise floor around the principal peak as proposed in Ref. [6], in this paper, and the effect of noise to the peak height h is much stronger than that to the width Δf with the increasing of coupling number n, β isn't properly used as a quality factor to compare the changes in the degree of coherence of different coupling cells, but it is a useful quantity for a separate cell. Here, we define another amplification factor $Q=S/S_n$, which is a similar definition as coherent signal-to-noise ratio in Ref. [6]. S is the total area of noise-induced principal peak at f_0 and S_n is the area below a line joining the relative minimum through $(0,f_0)$ to that in $(f_0,2f_0)$ as noise floor. The factor Q and β versus various noise levels are plotted in Fig. 1. The fundamental character of the both curves are in good agreement, and the curves indicate the occurrence of CR in the single oscillator.

Now, we study the influence of coupling. We fix $A=0.46$, and take $D_n=0.2$, $N=5$. Let us assume that the only C_1 (we use C_1 and C_n to denote the first cell and nth cell,

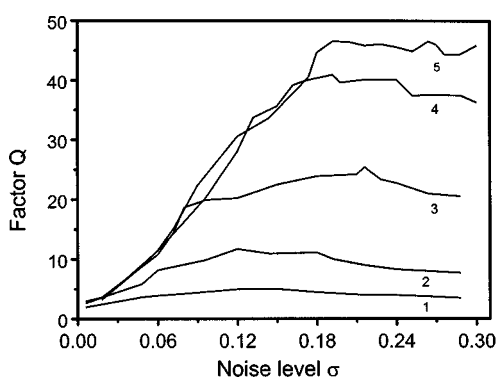

FIG. 3. Propagation of CR with considerable enhancement along the one-way coupled chain (curves 1, 2, 3, 4, and 5 correspond to C_1, C_2, C_3, C_4, and C_5, respectively). $N=5$; $D_n=0.2$.

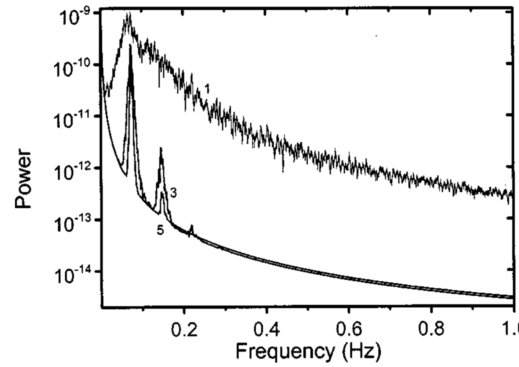

FIG. 4. The power spectra for C_1, C_3, and C_5 (curves 1, 3, and 5 correspond to C_1, C_3, and C_5, respectively). $\sigma=0.24$; $N=5$; $D_n=0.05$.

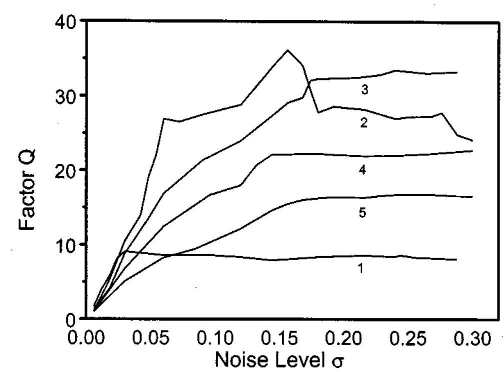

FIG. 5. Propagation of CR with first enhancement and then suppression along the one-way coupled chain (curves 1, 2, 3, 4, and 5 correspond to C_1, C_2, C_3, C_4, and C_5, respectively). $N=5$; $D_n=0.05$.

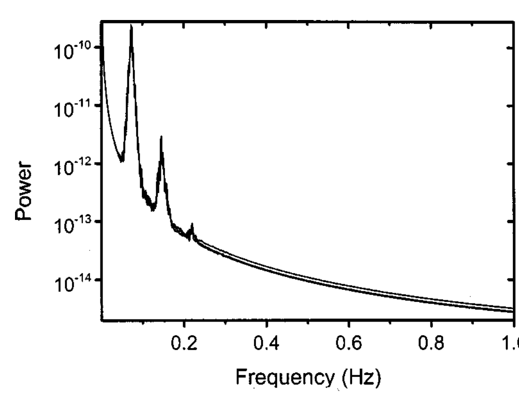

FIG. 6. The spectra in C_3 for different noise levels $\sigma=0.18$, 0.24, and 0.30. The curves change very slightly. $N=5$; $D_n=0.05$.

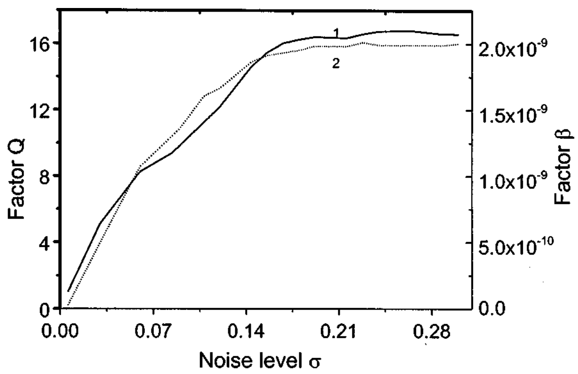

FIG. 7. CR without tuning in C_5. (Curves 1 and 2 correspond to the factor Q and β, respectively.) $N=5$; $D_n=0.05$.

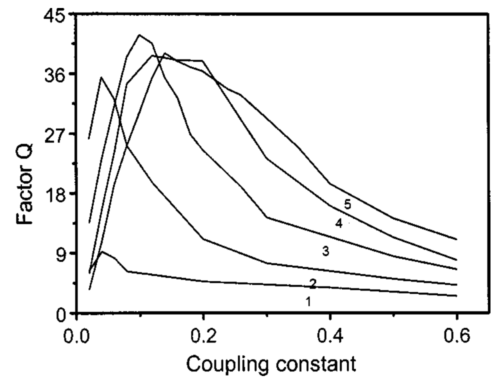

FIG. 8. Plots of the factor Q versus the coupling constant (curves 1, 2, 3, 4, and 5 correspond to C_1, C_2, C_3, C_4, and C_5, respectively). $N=5$; $\sigma=0.16$.

respectively) is imposed with noise. The power spectra for the C_1, C_3, and C_5 cell are given as an example in Fig. 2. (Curves 1, 3, and 5 correspond to C_1, C_3, and C_5, respectively.) Observe from the figure, the spectrum of C_1 is very coarse, and the spectra become smoother with the increment of the coupling number. Accordingly, the noise background becomes lower with the increment of the coupling number. We can intuitively see the coherence of the system becomes stronger with the increment of the coupling number from Fig. 2. The curves of factor Q of each cell versus noise level σ is shown in Fig. 3 (curves 1, 2, 3, 4, and 5 correspond to C_1, C_2, C_3, C_4 and C_5), which clearly indicates CR has been transmitted and enhanced by coupling. The phenomena are in accord with the experimental observation in Ref. [17]. We understand the enhancement from the dependence of S and S_n on the coupling number at fixed coupling constant and noise level. S and S_n both become smaller with the increment of the coupling number, but the effect of coupling number to S_n is stronger than the effect to S, so the competition of S and S_n makes their ratio Q become larger and larger.

We change the coupling constant $D_n=0.05$. The power spectra C_1, C_3, and C_5 oscillator are given in Fig. 4. (Curves 1, 3, and 5 correspond to C_1, C_3, and C_5, respectively.) As the spectra at $D_n=0.2$, the spectrum of C_1 is very coarse, and the spectrum become smoother with the increment of the coupling number. But, differing from Fig. 3, it's obvious the coherence in C_3 is stronger than that in C_5. The factors Q versus noise level σ are plotted in Fig. 5. The figure has some different features as in Fig. 3. At lower noise level, curve 2 is the highest and curve 3 is the highest at higher noise level. This suggests CR can also be transmitted at lower-coupling constant, but the CR are first enhanced and then suppressed with the increment of coupling number. The phenomenon of suppression can also be understood from the dependence of S and S_n on the coupling number at fixed noise level as coupling constant $D_n=0.2$. S and S_n both become smaller with the increment of the coupling number, but the effect of the coupling number to S is stronger than the effect to S_n, so the competition of S and S_n makes their ratio Q become smaller and smaller. Another interesting feature is that there exists a plateau in curves 3, 4, and 5 at higher-noise level. Three typical power spectra for noise level $\sigma=0.18,0.24,0.30$ in C_3 are shown in Fig. 6, and the curves change very slightly. Curves of the factor Q and β versus noise level σ in C_5 are given as an example of an existing obvious plateau in Fig. 7. (This phenomenon in the present of external periodic signal is called ''stochastic resonance without tuning'' [20].) Here, we call the phenomenon in the absence of external signal CR without tuning, and we understand it as a result of the coupling acting as a type of nonlinear noise filter.

We fix $\sigma=0.16$ and $N=5$. The factors Q versus different coupling constants are plotted in Fig. 8. The curves increase reaching a maximum and then drop with the increment of the coupling constant showing the existence of CR. It also shows that CR can be transmitted at various coupling constants, but CR will be strongly suppressed at a too low- or high-coupling constant. Notice with fixed σ and n, S increases, reaching a maximum then decreasing and S_n continuously increases with the increment of coupling constant, which is different from S and S_n both increasing with the increment of noise level at fixed D_n and n.

How far can CR propagate with considerable enhancement or suppression? First we study the coupling number $N=10$ and $D_n=0.2$. The highest height of the CR curves appears in C_7, and the highest height of the CR curve in the end cell (C_{10}) reaching about 48.5 is larger than the highest height, about 46.8 of the CR curve in the C_5. It indicates there exists a maximum propagation distance with enhancement in a one-way couple chain. Second, we choose the coupling number $N=5$, let $D_1=D_2=D_3=0.05$, and $D_4=D_5=0.5$, we find CR in C_4 and C_5 is enhanced than for CR with $D_4=D_5=0.05$, the maximum in the C_4 reaching 34 and the maximum in the C_5 reaching about 37. This may suggest that proper coupling constant and noise level can be used to control the propagation of CR with enhancement or suppression along the coupling chain.

In conclusion, we have investigated that coupling can significantly sustain the propagation of CR with considerable enhancement or suppression along the one-way coupling chain. Here, the constructive effect of noise to nonlinear system has been successfully transferred. Proper coupling constant and noise level can be used to control the propagation of CR with enhancement or suppression along a one-way coupling chain. Considering the fact that CR without tuning appears, might one say that external disorder has significantly been transferred into internal order by the coupling? We believe that the phenomenon of coupling sustained propagation of the constructive effect of noise may be important in both nature and technology.

Note added. More recently, coherence resonance in coupled nonindentical exticatabel systems [21] and globally coupled Hodgkin-Huxley neurons [22] has been investigated.

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