Hybrid Čerenkov mode in a resonant medium

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Energy stored in a resonant medium may be used to amplify the Čerenkov radiation generated by a small driving bunch. The hybrid eigenmode of the system is a combination of Čerenkov radiation and eigenmode of the resonant medium. In the vicinity of the resonant frequency of the medium and when Čerenkov condition is satisfied, the eigenfrequency has an imaginary part. The latter occurs in a relatively limited range of energies of the driving bunch and it depends on the guiding geometry and on the population inversion. Simulation results of electron acceleration using this eigenmode are presented.

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The present generation of accelerators is driven by microwave radiation that is injected either in cavities or in a periodic structure where they accelerate the particles. Requirements of a future system, namely, high-energy, compactness, and cost, impose gradients that are at least one order of magnitude higher than today's gradients. This translates into a surface electric field approaching the 1-GV/m level that, in turn, is very difficult to sustain without occurrence of breakdown.

In order to overcome this limitation it was suggested to utilize a concept introduced first by Tajima and Dawson [1], that is, to accelerate electrons by a space-charge wave that propagates in plasma. The various schemes being investigated may be divided into several categories according to the driver. The latter is the energy supplier: in the case of a laser wake-field acceleration (LWFA) [2–11] this is a short and intense laser pulse, for a plasma beat-wave accelerator (PBWA) [12] these are two long laser pulses oscillating at slightly different frequencies, and in the case of plasma wake-field acceleration (PWFA) [13–16] the driver is an intense electron pulse.

Another acceleration scheme that is related to that considered here, and to some extent to those previously discussed, is the Čerenkov wake-field acceleration (CWFA). In this case a driving bunch is injected in a vacuum tunnel of a dielectric loaded waveguide. The Čerenkov radiation generated by this bunch may be used to accelerate [17] another bunch that trails behind. All the initial energy is initially stored in the driving bunch and is converted into an accelerating field by the *passive* dielectric structure.

In the past we suggested [18,19] to combine the concept of a wake-field associated with a bunch moving in a dielectric medium with the principles that enable the operation of a laser. Its essence was to inject a bunch of charged particles in an *active* medium and, in this way, energy stored in the medium can be transferred directly to the moving particles. This scheme was called particle acceleration by stimulated emission of radiation (PASER) and it may be conceived as the inverse of Frank-Hertz effect. For the regime corresponding to the Frank-Hertz experiment, namely, a single electron-atom collision (in average), the phenomenon was demonstrated experimentally a long time ago [20]; the efficiency of the accumulative process is yet to be proven experimentally. More recently, we have shown theoretically [21] that the wake generated by a (driving) bunch moving in an active medium may be amplified by the medium and then used for acceleration of another bunch that trails many wavelengths behind. This mechanism has the advantage that the longitudinal electric field is inherent to the excitation mode contrary to the regular ''vacuum'' mode that its longitudinal electric field is limited by the Rayleigh diffraction length. This last problem is partially solved in LWFA by generating plasma channels that facilitate focusing of the laser field over several Rayleigh lengths [2,3,5,9].

Using an active medium for particle acceleration suggests that this mechanism may be conceived as the *inverse laser* effect. In the past, a variety of mechanisms, which in ''natural'' conditions generate radiation, were considered for acceleration of particles. For example, Palmer [22] suggested what is today known as the inverse free-electron laser (I-FEL) [23]. A similar scheme was suggested by Pantell and co-workers [24] but in this case with Čerenkov radiation and finally, the Smith-Purcell effect [25] may be used to accelerate particles [26]. An overview of many of the schemes mentioned here may be found in Ref. [27].

In this study we analyze the hybrid eigenmode that leads to the amplification of Čerenkov radiation. This mode is used for electron acceleration since inherently [21] it has a longitudinal electric field; the characteristics of the acceleration are also investigated.

Envision a gas that its dielectric characteristics may be represented by the following frequency-dependent dielectric coefficient:

$$\epsilon(\omega)=1+\sum_{\nu}\frac{\omega_{p,\nu}^2}{\omega_{0,\nu}^2-\omega^2+2j\omega\omega_{1,\nu}}, \qquad (1)$$

where $\omega_{0,\nu}$ are the resonance frequencies of the medium, $\omega_{1,\nu}$ is associated with the loss parameter in the medium and it is responsible to the attenuation of an electromagnetic wave when the medium is not inverted. The numerator $\omega_{p,\nu}$ is the angular plasma frequency associated with the density of atoms that have this specific resonance (ν); in the case of an inverted medium $\omega_{p,\nu}^2$ is negative. Several comments are

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in place before we proceed: first, it is tacitly assumed in this expression that the resonances are sufficiently apart from one another, i.e., $\omega_{0,\nu} \gg \omega_{1,\nu}$. Second, only linear effects are considered and third, the time dependence of the field is according to $e^{j\omega t}$. As a typical medium we may consider the ammonia that was the medium used for the first maser; it has a resonance at 23.87 GHz and its typical linewidth [28] is less than 10 kHz. In what follows it will be assumed that this gas fills uniformly an azimuthally symmetric waveguide of radius R. The assumption that the electromagnetic wave is confined by a waveguide does not affect significantly the results to be presented here but it simplifies the analysis. Conceptually similar results may be anticipated for CO_2, argon, or neon. Moreover, the energy may be stored in a solid-state medium and the driving bunch may move in vacuum, in the vicinity of the medium, and still the wake will be amplified; thus eventually it may be used for acceleration. In fact the population inversion may be substantially easier in solid-state systems with current diode-laser technology comparing to flash-lamp or discharge excitation in the case of gaseous medium.

In the center of the cylindrical waveguide a charged bunch (q) is moving at a constant velocity v. For simplicity we shall assume that the size of the bunch is much smaller than the wavelength at resonance and smaller than the radius of the waveguide; therefore, the current density associated with its motion may be approximated by $J_z(r,z,t) = -q\delta(z - vt)\delta(r)/2\pi r$. This current density generates an electromagnetic field that may be derived from the magnetic vector potential given by

$$A_z(r,z,t) = \frac{q}{4\pi\epsilon_0 R^2}\sum_{s=1}^{\infty}\frac{4J_0\left(p_s\frac{r}{R}\right)}{J_1^2(p_s)} \times\left\{\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega\,\frac{e^{j\omega(t-z/v)}}{\omega^2[\epsilon(\omega)-1/\beta^2]-\omega_{c,s}^2}\right\}, \tag{2}$$

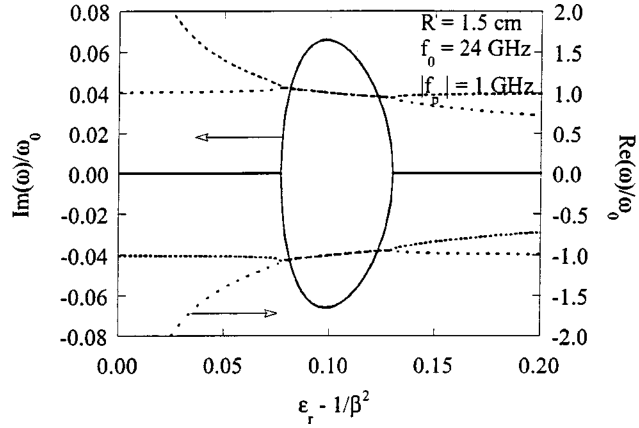

FIG. 1. Imaginary and real part of the eigenfrequency of each one of the four eigenmodes.

where p_s are the zeros of the Bessel function of the zero order and first kind, i.e., $J_0(p_s) \equiv 0$, $\omega_{c,s} \equiv cp_s/R$ is the cutoff angular frequency of the sth mode and $\beta = v/c$. For evaluation of this expression we assume that the radius of the waveguide is chosen such that at the resonance frequency of the medium there is only one propagating mode, i.e., $s=1$. In addition we assume that there is only one inverted state (ν_0) in the system therefore the dielectric coefficient may be written as

$$\epsilon(\omega) \simeq \epsilon_r + \frac{\omega_{p,\nu_0}^2}{\omega_{0,\nu_0}^2 - \omega^2 + 2j\omega\omega_{1,\nu_0}}, \tag{3}$$

where $\epsilon_r \equiv \Sigma_{\mu\neq\nu_0}\omega_{p,\mu}^2/(\omega_{0,\mu}^2 - \omega_{0,\nu_0}^2)$ and we ignored losses at other resonant frequencies. In what follows we shall drop the index of the resonant frequency. Based on these assumptions we may simplify the integral in Eq. (2) to read

$$\mathcal{F}(\tau) \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega\,\frac{e^{j\omega\tau}}{\omega^2[\epsilon(\omega)-1/\beta^2]-\omega_c^2} \simeq \frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega\,\frac{e^{j\omega\tau}[\omega^2 - 2j\omega\omega_1 - \omega_0^2]\frac{1}{\overline{\epsilon_r}}}{\omega^4 - 2j\omega_1\omega^3 - \omega^2\left[\omega_0^2 + \frac{\omega_p^2+\omega_c^2}{\overline{\epsilon_r}}\right] + 2j\frac{\omega_1\omega_c^2}{\overline{\epsilon_r}}\omega + \frac{\omega_c^2\omega_0^2}{\overline{\epsilon_r}}}$$
$$\simeq \frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega e^{j\omega\tau}\frac{1}{\overline{\epsilon_r}}\sum_{n=1}^{4}\frac{U_n}{\omega-\Omega_n}, \tag{4}$$

where $\overline{\epsilon_r} \equiv \epsilon_r - 1/\beta^2$, Ω_n are the four zeros of the polynomial at the denominator, and $U_n = (\Omega_n^2 - 2j\Omega_n\omega_1 - \omega_0^2)/[\Pi_{m\neq n}(\Omega_n - \Omega_m)]$. This expression [Eq. (4)] is one of the important results of this analysis since it indicates that the eigenmodes of the system are a combination of Čerenkov radiation and resonant mode. They are determined by the zeros of the fourth-order polynomial at the denominator of the integral that defines $\mathcal{F}(\tau)$.

We shall focus here on the case when the Čerenkov condition is satisfied, i.e., $\epsilon_r > 1/\beta^2$. Figure 1 illustrates the four complex eigenfrequencies as a function of the Čerenkov parameter $\overline{\epsilon_r}$ for a radius of $R = 1.5$ cm, resonant frequency of

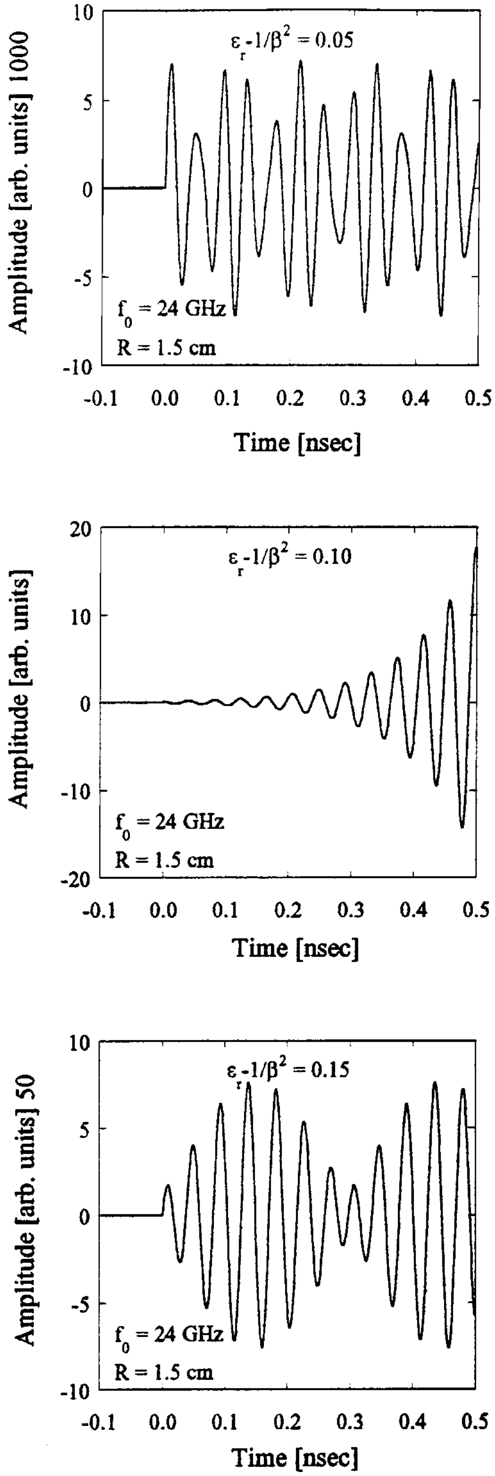

FIG. 2. The electromagnetic field as a function of the delay parameter (τ) with $\overline{\epsilon_r}$ as a parameter; the wake grows exponentially when the imaginary part of the eigenfrequency has a negative component. In all three cases the population is inverted, yet only in one specific condition is the Čerenkov radiation amplified; this corresponds to $\overline{\epsilon_r}=0.1$ (see Fig. 1).

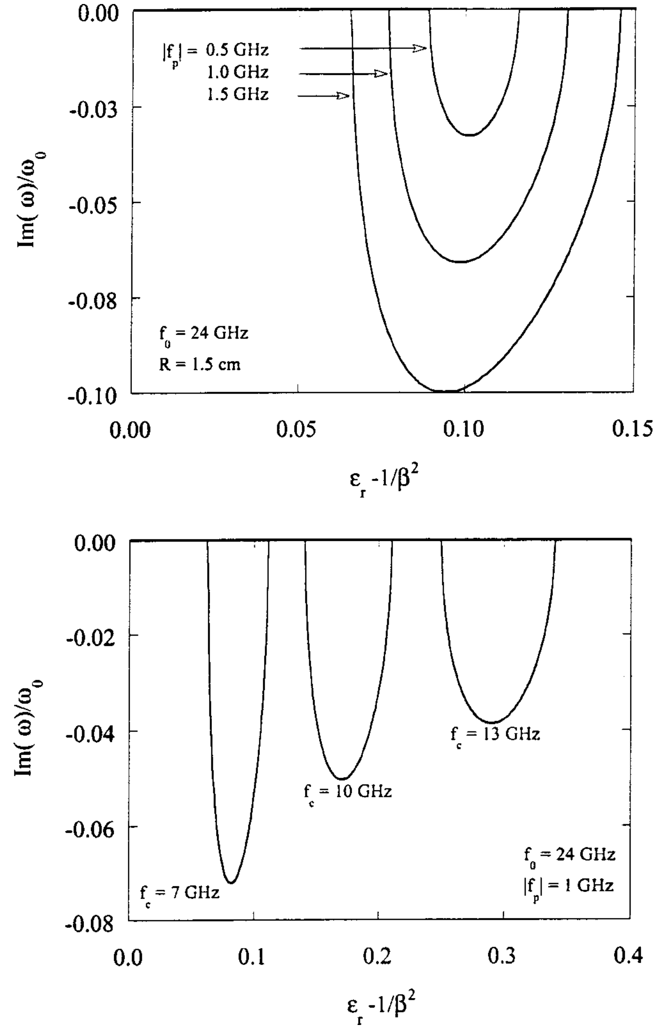

FIG. 3. Left: Imaginary part of the eigenfrequency as a function of the ''slippage'' ($\overline{\epsilon_r}=\epsilon_r-\beta^{-2}$). The corresponding population inversion, expressed in terms of the plasma frequency, is a parameter. Note that for different population inversions, $\overline{\epsilon_r}$ practically does not change; however the growth rate does increase—as expected. An increase by a factor of 9 in the population inversion density leads to an increase by a factor of 3 in the growth rate. Right: Imaginary part of the eigenfrequency as a function of the ''slippage'' ($\overline{\epsilon_r}=\epsilon_r-\beta^{-2}$). The cutoff frequency of the waveguide is a parameter. At large-waveguide radius (low cutoff) the growth rate is large and it occurs in a narrow range of velocities.

24 GHz, and the absolute value of the plasma frequency is $f_p=1$ GHz (recall that the population is inverted thus $\omega_p^2<0$). We observe that there is a region where the eigenfrequencies have both positive and negative imaginary components (solid lines); therefore the ''hybrid mode'' that is a combination of the Čerenkov radiation and the active medium mode may grow in space. In other words, Čerenkov radiation may be amplified by the active medium. Using Cauchy residue theorem and bearing in mind that causality implies that there is no electromagnetic signal in front of the particle we obtain

$$A_z(r,\tau\equiv t-z/v)\simeq\frac{q}{4\pi\epsilon_0\overline{\epsilon_r}R^2}\left[\frac{2}{J_1(p_1)}\right]^2 J_0\left(p_1\frac{r}{R}\right)\times\sum_{n=1}^{4}\mathrm{Re}[jU_n e^{j\Omega_n\tau}]h(\tau), \qquad (5)$$

where $h(\tau)$ is the step function. Figure 2 illustrates this last quantity as a function of the delayed time variable ($\tau\equiv t-z/v$) with $\overline{\epsilon_r}$ as a parameter; the wake grows exponentially

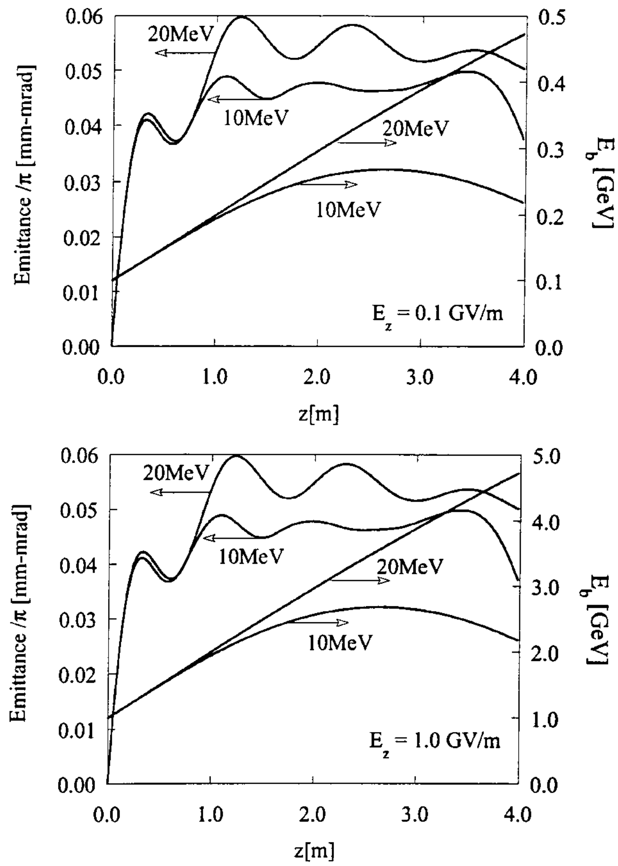

FIG. 4. The emittance and the average energy of the bunch along the interaction region. The parameters of the simulation are presented in the Table I. Note the dependence on the energy of the driving bunch in 4 m of interaction length.

when the imaginary part of the eigenfrequency has a negative component. In all three cases the population is inverted, yet only in a specific regime ($\overline{\epsilon_r}=0.1$), indicated by Fig. 1 that the frequency has an imaginary part, the Čerenkov radiation is indeed amplified.

The two frames in Fig. 3 illustrate the dependence of the imaginary part of the eigenfrequency as a function of $\overline{\epsilon_r}$ with the cutoff frequency and the population inversion density as a parameter. The former shifts the velocity where maximum growth occurs—the higher the cutoff frequency, the higher the particle's velocity required. Varying the population inversion, amplification may occur for a wider range of energies of the driving particle: when the population inversion is small, the growth rate is both narrow and low; its increase causes both larger growth in a wider range of driving bunch energies.

After presenting the main characteristics of a wake field as it propagates in an active medium, we shall examine the interaction of a bunch of particles that are accelerated by such a wake field. Specifically we examine the acceleration process, emittance variations, and phase-space distribution. For the purpose of this model, several assumptions are in place: first, we ignore the accelerated particles effect on the local electromagnetic field. Second, the bunch is small on scale of one wavelength and variations of the field due to the active medium at the location of the accelerated bunch are negligible; therefore, the electromagnetic field components are given by

TABLE I. Parameters for the simulation.

Number of electrons in the bunch	$N_b=10^9$
Radius of the bunch	$R_b=1$ mm
Simulation duration	$\zeta_{\max}=200$
Intial phase distribution	$\lvert\chi_i-\pi\rvert<\Delta\chi/2=\pi/36$
Number of macroparticles in the simulation	$N_p=1000$
Accelerating gradient	$E_z=100$ MV/m, 1 GV/m
Initial electrons energy	$\mathcal{E}_b=100$ MeV, 1 GeV
Driving bunch energy	$\mathcal{E}_d=10$ MeV 20 MeV
Resonant frequency	$f_R=24$ GHz

$$E_z(r,z,t)=E_0\cos[\omega_R(t-z/c\beta_d)],$$

$$E_r(r,z,t)=-\frac{1}{2\beta_d}\left(\frac{\omega_R}{c}r\right)E_0\sin[\omega_R(t-z/c\beta_d)],$$

$$H_\phi(r,z,t)=-\frac{1}{2\beta_d}\left(\frac{\omega_R}{c}r\right)\frac{E_0}{\eta_0}\{\sin[\omega_R(t-z/c\beta_d)] -\delta\cos[\omega_R(t-z/c\beta_d)]\}, \tag{6}$$

where ω_R is the real part of the eigenfrequency corresponding to the growing wave and β_d is the normalized velocity of the driving bunch; $\eta_0\equiv\sqrt{\mu_0/\epsilon_0}$, $\delta\equiv\omega_I/\omega_R+\omega_p^2/(2\epsilon_r\omega_I\omega_R)$, and ω_I is the imaginary part of the eigenfrequency.

We may now determine the dynamics of electrons as they move in an active medium,

$$\frac{d}{d\zeta}P_{x,i}=\frac{1}{2}\mathcal{E}_gX_i\left[\left(\frac{1}{\beta_g}-\epsilon_r\frac{P_{z,i}}{\gamma_i}\right)\sin\chi_i+\epsilon_r\frac{P_{z,i}}{\gamma_i}\delta\cos\chi_i\right] +\mathcal{E}_{\rm sc}X_i\left(\frac{1}{\langle P_{z,i}/\gamma_i\rangle}-\frac{P_{z,i}}{\gamma_i}\right),$$

$$\frac{d}{d\zeta}P_{y,i}=\frac{1}{2}\mathcal{E}_gY_i\left[\left(\frac{1}{\beta_d}-\epsilon_r\frac{P_{z,i}}{\gamma_i}\right)\sin\chi_i+\epsilon_r\frac{P_{z,i}}{\gamma_i}\delta\cos\chi_i\right] +\mathcal{E}_{\rm sc}Y_i\left(\frac{1}{\langle P_{z,i}/\gamma_i\rangle}-\frac{P_{z,i}}{\gamma_i}\right),$$

$$\frac{d}{d\zeta}P_{z,i}=-\mathcal{E}_g\left[\cos\chi_i-\frac{X_i+Y_i}{\beta_d}\sin\chi_i\right],$$

$$\frac{d}{d\zeta}\chi_i=1-\frac{1}{\beta_d}\frac{P_{z,i}}{\gamma_i},$$

$$\frac{d}{d\zeta}X_i=\frac{P_{x,i}}{\gamma_i},\quad \frac{d}{d\zeta}Y_i=\frac{P_{y,i}}{\gamma_i},\quad \frac{d}{d\zeta}Z_i=\frac{P_{z,i}}{\gamma_i}. \tag{7}$$

Here $\gamma_i\equiv[1+P_{x,i}^2+P_{y,i}^2+P_{z,i}^2]^{1/2}$, $\mathcal{E}_g\equiv eE_0/mc\omega_R$, $\zeta\equiv\omega_Rt$, the location (x_i,y_i,z_i) of each particle is normalized to the radiation wavelength $X_i=\omega_Rx_i/c$, $Y_i=\omega_Ry_i/c$, and $Z_i=\omega_Rz_i/c$. The dc space-charge effect is taken into consideration assuming a bunch of radius R_b and current I; hence $\mathcal{E}_{\rm sc}\equiv eI\eta_0/2\pi mc^2(\omega_RR_b/c)^2$ and ignoring the medium.

Figure 4 illustrates results from a numerical solution of

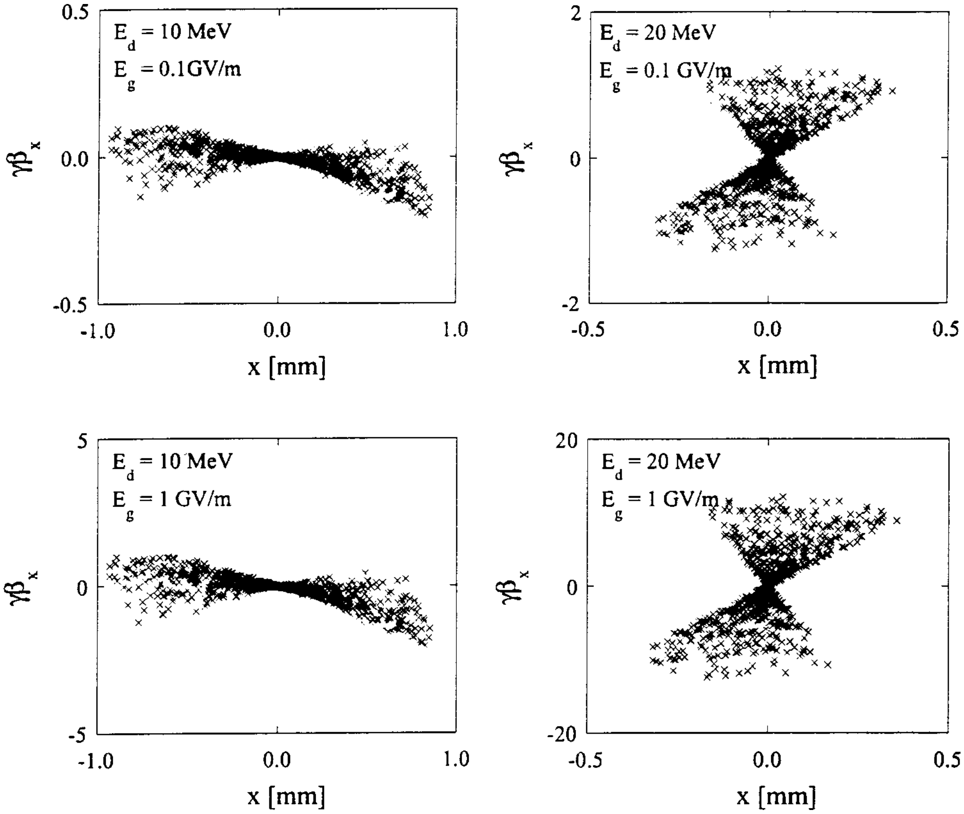

FIG. 5. Transverse phase space at the output for the four cases illustrated in Fig. 4. Note that for a given driver energy ($\mathcal{E}_d$), the shape of the phase space is conserved but the scale differs. When changing the energy of the driving bunch, both the shape and the scale vary.

this set of equations. We calculate the emittance and the average energy of the bunch along the interaction region; the parameters of the simulation are presented in Table I. At the input, the gradient ($\mathcal{E}_g$) is either 0.1 or 1.0 GV/m and it is assumed that the emittance is vanishingly small. As the interaction progresses we observe that beyond a "build-up" region, the emittance is virtually independent of the initial energy of the accelerated particles. However, a 100% increase in the energy of the driving bunch ($\mathcal{E}_d$), from 10 to 20 MeV, causes a 20% increase in the emittance namely, from 0.05π to 0.06π mm mrad. This fact was also observed when examining the transverse phase space $(p_{x,i}, X_i)$ at the output

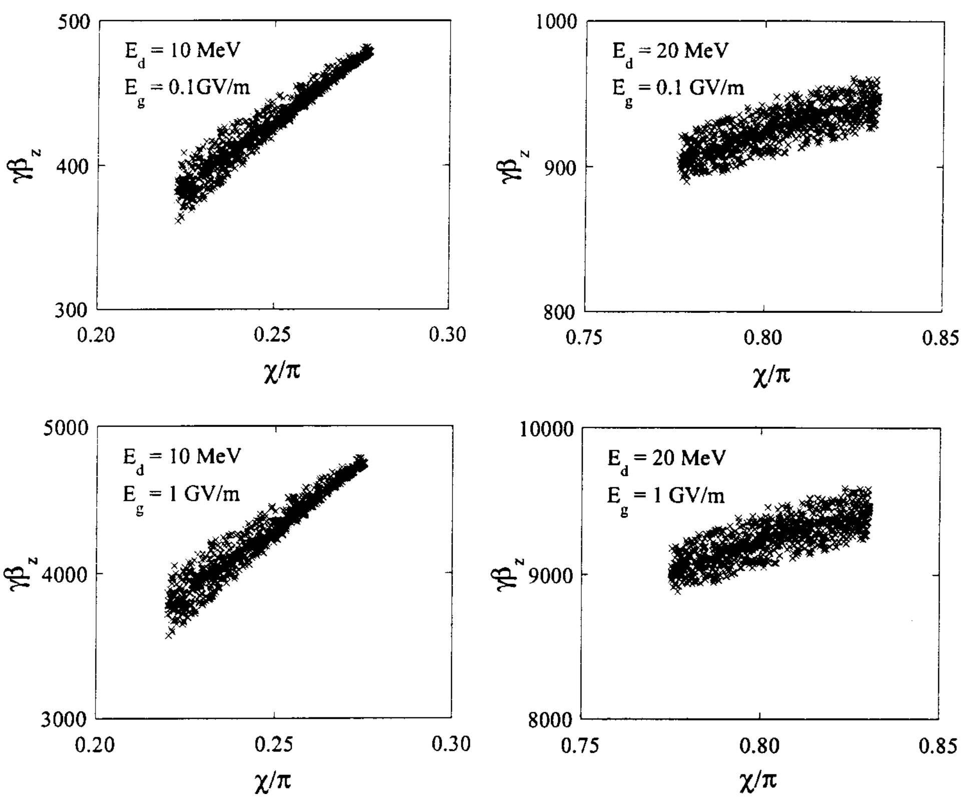

FIG. 6. Longitudinal phase space at the output for the four cases illustrated in Fig. 4. For a given driver energy, the shape of the phase space is conserved but the scale differs. When changing the energy of the driving bunch, both the shape and the scale vary. The deceleration revealed by Fig. 4 at low driver energy is readily understood here while observing the phase slippage of the bunch.

in all four cases—Fig. 5. For a given energy of the driving bunch ($\mathcal{E}_d$), the phase space is virtually identical, if the momentum of each particle is divided by γ_i. In the case of a given gradient ($\mathcal{E}_g$), but for different driving bunch energy ($\mathcal{E}_d$), there is a significant change in the phase space: doubling $\mathcal{E}_d$ causes an increase by a factor of 10 in the transverse momentum of the particles.

The longitudinal phase space, Fig. 6, reveals the reason that the average energy of the bunch (Fig. 4), for a driving bunch of 10 MeV, peaks after 2.5 m and afterwards drops: when $\mathcal{E}_d = 10\,\mathrm{MeV}$ the particles slip out of phase more rapidly and at the end of the interaction region their phase is 0.25π. That means that the particles are decelerated by the field. In the same distance the average phase of the accelerated bunch ($\mathcal{E}_b = 20\,\mathrm{MeV}$) has an average phase of 0.8π—in both cases the initial average phase of the particles was 1.0π.

In conclusion, energy stored in an resonant medium is used to amplify the Čerenkov radiation generated by a small driving bunch. It is shown that for a given resonant frequency, this amplification occurs in a relatively limited range of the driving beam energies; it is dependent on the guiding geometry and on the population inversion. A small test bunch is injected in the medium at a location where the amplified radiation has reached values on the order of 100 MV/m (or 1 GV/m) for the accelerating field. The emittance variation was examined, along the interaction of a 4-m-long section and for a beam of 1 nm radius. We found that starting from zero, it may increase to 0.05π–0.06π mm mrad independent of the initial energy of the test bunch and slowly dependent on the energy of the driving bunch. Simulation shows that the phase slip-page of the test bunch is significantly dependent on the energy of the driving bunch.

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