Threshold of induced transparency in the relativistic interaction of an electromagnetic wave with overdense plasmas

F. Cattani,[1] A. Kim,[2] D. Anderson,[1] and M. Lisak[1]
[1]Department of Electromagnetics, Chalmers University of Technology, S-412 96 Göteborg, Sweden
[2]Institute of Applied Physics, Russian Academy of Sciences, 603600 Nizhny Novgorod, Russian Federation
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An exact analytical investigation of the stationary solutions describing the interaction between high-intensity laser radiation and an overdense plasma is presented. Both the relativistic and striction nonlinearities are taken into account, and their joint action gives rise to a solitary solution. This solution clearly shows that there exists an inherent limit of the induced transparency on the density of the overdense plasma in order to obtain a stationary physical solution. Furthermore, it is found that the striction nonlinearity tends to create a strong peaking of the plasma electron density, which suppresses the laser penetration and significantly enhances the threshold intensity for induced transparency.

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I. INTRODUCTION

Modern laser technology offers a wide range of possibilities for exploring those regimes of interaction between a laser pulse and a plasma where effects such as relativistic and striction nonlinearities play a fundamental role in determining the dynamical evolution of the system (see, for example, [1]). These effects become very important in the proposed scientific and technical applications, which range from plasma-based particle and photon accelerators [2] to inertial confinement fusion [3,4]. Since in this new regime of high laser intensity the quiver velocity of electrons is relativistic, one of the main effects in laser-plasma interaction is associated with the relativistic increase of the inertial electron mass and the consequent lowering of the natural plasma frequency that may crucially modify the optical properties of a plasma. In the 1970's, it was shown that superintense electromagnetic radiation would be able to propagate through a classically overdense plasma due to the relativistic correction to the electron mass, the so-called induced transparency effect [5–9]. By means of an analytical approach, specific self-consistent solutions describing the penetration of relativistically strong waves into inhomogeneous plasmas were found. Recent numerical simulations based on relativistic particle-in-cell codes [4,10–13], multifluid plasma codes [14], and Vlasov simulations [15], as well as recent experiments [16,17], have revealed a number of new features of the interaction dynamics, such as laser hole boring, enhanced incident energy absorption, multi-MeV electron beams production and strong magnetic field generation. Nevertheless, important characteristics such as the threshold of self-induced transparency in the relativistic interaction of a laser with sharp-boundary overdense plasmas were not considered in detail. The commonly presented definition of this threshold is derived from a traveling-wave approach for homogeneous plasmas, but, as it is shown in this work, this same threshold should be essentially modified in the case of non-uniform plasmas due to the action of the ponderomotive force, which pushes electrons into the plasma thus creating a strong peaking of the plasma electron density. This results in a suppression of the laser penetration and a significant enhancement of the threshold of penetration.

In order to define the threshold of induced transparency, it is enough to consider a stationary model to describe the interaction between circularly polarized, superintense electromagnetic radiation normally incident on a sharp boundary separating vacuum and a cold, overdense plasma. It follows that, in order to obtain a physical solution, there is an inherent limit for the penetration of the laser radiation into the plasma that depends on the supercritical parameter $n_0 = \omega_p^2/\omega^2$, where ω is the carrier frequency of the laser radiation and ω_p is the plasma frequency of the initial unperturbed plasma. The regime of induced transparency for laser energy penetration through the plasma will take place when the incident intensity exceeds this threshold.

II. MODEL EQUATIONS

The ultrahigh-intensity, laser-plasma interaction is described by the relativistic equation of motion for the electrons, the equation of continuity, and Maxwell's equations, as expressed by Poisson's equation and the wave equation for the vector potential. For the short-time scale of interest, we assume that ions are inertially frozen in space and treated as a uniform neutralizing background. We will consider a one-dimensional case, with all the physical quantities depending only on the coordinate x, along the direction of propagation. Then, we have the following governing set of self-consistent equations in the Coulomb gauge (see, for example, [18,19]):

$$\frac{\partial^2 \mathbf{A}}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = \frac{4\pi e^2}{mc^2\gamma} N_e \mathbf{A}, \tag{1}$$

$$\frac{\partial^2 \phi}{\partial x^2} = -4\pi e (N_0 - N_e), \tag{2}$$

$$\frac{\partial p_\parallel}{\partial t} = e\frac{\partial \phi}{\partial x} - mc^2\frac{\partial \gamma}{\partial x}, \tag{3}$$

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$$\frac{\partial N_e}{\partial t}+\frac{\partial}{\partial x}\left(N_e\frac{p_\parallel}{m\gamma}\right)=0. \tag{4}$$

Here, $p_\parallel$ is the longitudinal momentum of the electrons, $\gamma=(1+p_\parallel^2/m^2c^2+e^2|\mathbf{A}|^2/m^2c^4)^{1/2}$ is the relativistic factor, N_e is the local electron density (N_0 is the background ion density), $\mathbf{A}$ and ϕ are vector and scalar potentials of the electromagnetic fields, respectively, and m is the rest electron mass.

It is convenient as usual to introduce the dimensionless variables in relativistic units:

$$\mathbf{a}=\frac{e\mathbf{A}}{mc^2},\quad \varphi=\frac{e\phi}{mc^2},\quad p_\parallel=\frac{p_\parallel}{mc},\quad t=\omega t,\quad x=\frac{\omega}{c}x. \tag{5}$$

We also assume that circularly polarized laser radiation with vector potential $\mathbf{A}=A(x,t)\mathrm{Re}[(\mathbf{y}+i\mathbf{z})\exp(i\omega t)]/\sqrt{2}$ is normally incident from vacuum ($x<0$) onto a semi-infinite plasma ($x\geqslant 0$). In the stationary regime, the basic equations read [8]

$$\frac{d\varphi}{dx}=\frac{d\gamma}{dx}, \tag{6}$$

$$\frac{d^2\varphi}{dx^2}=n_0(n-1), \tag{7}$$

$$\frac{d^2a}{dx^2}+\left(1-\frac{n_0}{\gamma}n\right)a=0. \tag{8}$$

Here, $\gamma=(1+a^2)^{1/2}$, $a=eA(x)/mc^2$ is the normalized amplitude of the vector potential, n is the electron density normalized on the unperturbed density $n=N_e/N_0$, and $n_0=N_0/N_c>1$ is the overdense plasma parameter where $N_c=m\omega^2/4\pi e^2$ is the critical plasma density.

Equation (6) describes how, in the region where electron density $n(x)\neq 0$, the ponderomotive force $d\gamma/dx$ must be compensated by the force of the longitudinal field due to space-charge separation. Although a similar model has already been discussed [8], we will emphasize the analysis of the physical problem applicable to the interaction of high-intensity laser with sharp-boundary overdense plasmas and present an exact expression of the threshold intensity for the phenomenon of self-induced transparency.

III. THRESHOLD OF INDUCED TRANSPARENCY

Let us start our analysis by discussing the effect of longitudinal field production at the plasma-vacuum boundary due to charge separation caused by the action of the ponderomotive force. As was mentioned above, Eq. (6) indicates that in the electron plasma region the ponderomotive force must be compensated at the boundary $x=0$ by a force due to the charge-separation field if $n(x=0)\neq 0$.

On the other hand, integrating Poisson's equation over the whole plasma half space, we obtain from the condition of charge neutrality that $d\varphi(0)/dx=0$. However, this implies that $d\gamma(0)/dx=0$, a condition that does not necessarily have to be fulfilled. We conclude that a possible stationary solution of the considered model equations may include a depletion region at the vacuum-plasma boundary, where ion charges are uncompensated. In this region, the ponderomotive force is unbalanced and pushes all electrons forward inside the plasma, thus shifting the actual electron plasma boundary from $x_d=0$ to a new position $x_d>0$. In this situation we have a region depleted of its electrons ($n=0$ for $0<x<x_d$) where the longitudinal field varies linearly with x as $d\varphi/dx=-n_0x$, reaching a maximum electric field E_m at the new boundary position $x=x_d$,

$$E_d=n_0x_d. \tag{9}$$

In the region $x<x_d$, including the pure ion layer ($0<x<x_d$), the electromagnetic field $a(x)$ corresponds to a vacuum solution.

At the boundary of the depletion region $x=x_d$, we obtain $d\gamma(x_d)/dx=-n_0x_d$, which can be written as

$$x_d=-\frac{1}{n_0}\left[\frac{d(\sqrt{1+a^2})}{dx}\right]_{x=x_d}. \tag{10}$$

Finally, for $x>x_d$, making use of the Poisson and continuity equations, we can write the wave equation as

$$\frac{d^2a}{dx^2}-\frac{a}{1+a^2}\left(\frac{da}{dx}\right)^2+(1+a^2-n_0\sqrt{1+a^2})a=0. \tag{11}$$

At the boundary $x=x_d$, the solution of Eq. (11) must be matched to the vacuum solution by the continuity conditions for the electric and magnetic components of the laser field. Representing the vacuum solution as a sum of incident and reflected electromagnetic waves, the incident amplitude has to obey the following condition:

$$a_i=\frac{1}{2}\sqrt{a_d^2+\left[\frac{da}{dx}\right]^2_{x=x_d}} \tag{12}$$

in which the boundary position x_d must be defined self-consistently.

For homogeneous ion density, the system described by Eq. (11) has the following Hamiltonian:

$$\mathcal{H}=\frac{1}{2(1+a^2)}\left(\frac{da}{dx}\right)^2-\frac{1}{2}(2n_0\sqrt{1+a^2}-a^2). \tag{13}$$

A phase portrait and a class of possible solutions described by the Hamiltonian (13) were presented in [8]. Here we will consider in more detail the case where $n(x)\to 1$ and both $a(x)$ and da/dx vanish as $x\to\infty$, that is, of all the stationary solutions, we only consider localized field distributions, in which case the integral of motion becomes $\mathcal{H}=-n_0$. It should be emphasized that only this kind of solution, which is consistent with the semi-infinite plasma configuration, allows us to define stationary field-plasma distributions and gives conclusions related to the induced transparency effect for comparatively thick plasma layers. In this case, Eq. (13) can easily be integrated to yield the following single-parameter solitary solution:

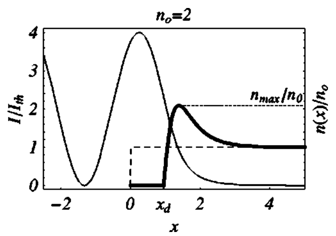

FIG. 1. Relative intensity of the field distribution (continuous line) along with the electron density profile (thick continuous line) and the ion-density distribution (dashed line) for $n_0=2$ and maximum incident intensity. The electron density vanishes at the actual boundary position x_d. All the quantities are dimensionless.

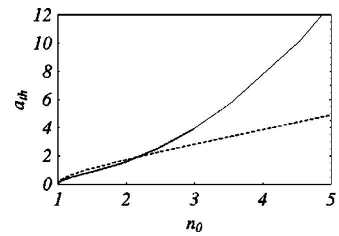

FIG. 2. Maximum incident amplitude as a function of the critical parameter n_0. The continuous line represents the threshold given by Eq. (15) valid for $n_0>1.5$, and for $n_0<1.5$ as given by Eq. (11). The dashed line represents the commonly used threshold as given by Eq. (16). All amplitudes are dimensionless.

$$a(x)=\frac{a_m \cosh[(n_0-1)^{1/2}(x-x_0)]}{n_0 \cosh^2[(n_0-1)^{1/2}(x-x_0)]-(n_0-1)}. \quad (14)$$

The parameter x_0 defines the peak position of the function $a(x)$, where the amplitude is equal to $a_m=2[n_0(n_0-1)]^{1/2}$. Matching this solution to the vacuum one, we can write the full solution of the stationary problem. However, it should be noted that for constructing the full solution we can only use the part of the function (14) where its first derivative, defining the ponderomotive force, is positive and pushes electrons into the plasma. Only in this case can the ponderomotive force be balanced by the electric force due to charge separation. In the region where ponderomotive force pulls electrons out of plasma, the condition of charge quasineutrality cannot be fulfilled.

At the symmetry point, where the amplitude of the solution (14) reaches its maximum value of a_m the density has a corresponding minimum given by

$$n_m=1-4(n_0-1)^2. \quad (15)$$

The condition that this value has to be positive, $n_m \geqslant 0$, gives us a condition for the background plasma density and an additional argument in favor of the depletion region. We may use a localized solution of the form given by Eq. (14) only if $n_0 \leqslant 1.5$, whereas, for $n_0>1.5$ we have to take into account only that part of the solution where the corresponding electron density function is positive, $n(x) \geqslant 0$.

The field structure together with the corresponding electron-density profile are shown in Fig. 1 for a value of the parameter n_0 higher than the critical value. As follows from the boundary condition given by Eq. (12) and the field solution given by Eq.(14), for a given n_0, there is a maximum value of the incident intensity such that a stationary nonlinear skin-layer-like solution exists. This maximum incident intensity can be identified with the penetration threshold for induced transparency: at incident intensities exceeding this threshold, laser radiation can propagate through the overdense plasma.

For the case of $n_0 \leqslant 1.5$ (no depletion region), the threshold of induced transparency is easily calculated from Eq. (12) with a maximum of a_d equal to the maximum amplitude of the obtained solution, and it turns out to be

$$a_{th} \equiv (a_i)_{max}=\sqrt{n_0(n_0-1)}. \quad (16)$$

As for the intensity threshold at overcritical parameters $n_0>1.5$, the calculation is slightly more complicated. Since in this case, as can be seen in Fig. 1, a depletion layer in the plasma region near the boundary to the vacuum is a necessary condition, let us calculate the field amplitude, a_{d*}, at the point where the electron density vanishes. Combining Eqs. (8), (11), and (13), we find that this amplitude satisfies the following relation:

$$\frac{3}{2} n_0 \sqrt{1+a_{d*}^2}=n_0+a_{d*}^2, \quad (17)$$

i.e.,

$$a_{d*}^2=n_0\left(\frac{9}{8}n_0-1+\frac{3}{2}\sqrt{\frac{9}{16}n_0^2-n_0+1}\right). \quad (18)$$

As was mentioned above, the boundary problem requires that we match the solution inside the plasma region $x \geqslant x_d$ to the vacuum solution for $x \leqslant x_d$ at the point where the electron density vanishes. On the other hand, an expression for the incident amplitude as a function of the amplitude at the boundary of the depleted region is obtained from Eq. (13):

$$a_i^2=\frac{1}{4}[2n_0(1+a_d^2)(\sqrt{1+a_d^2}-1)-a_d^4]. \quad (19)$$

It is recognized that when the incident amplitude assumes its maximum value, the corresponding boundary amplitude $(a_d)_{max}$ has exactly the same value at which the electron density vanishes, i.e., $(a_d)_{max}=a_{d*}$, and finally we obtain the penetration threshold as

$$a_{th} \equiv (a_i)_{max}=\frac{1}{2}\left[\frac{2}{3}(1+a_{d*}^2)(2a_{d*}^2-n_0)-a_{d*}^4\right]^{1/2}. \quad (20)$$

The threshold value is thus defined as the maximum value for the incident amplitude that can be matched to the field structure inside the plasma at the point where the electron density vanishes, i.e., at the actual boundary.

The threshold for nonlinear penetration is presented in Fig. 2 as a function of the overcritical parameter for both $n_0 \leqslant 1.5$ and $n_0>1.5$. For comparison we present here also the commonly used threshold following from a traveling-plane-wave approach [5–7]:

$$(a_i)_{max}^*=\sqrt{n_0^2-1}. \quad (21)$$

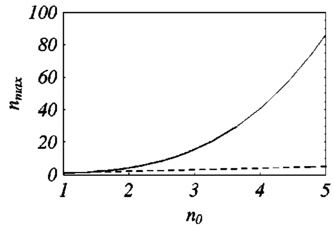

FIG. 3. Maximum electron density, normalized on the unperturbed density, as a function of the critical parameter n_0. The dashed line represents the unperturbed electron density $n=n_0$.

The difference between the threshold given by either Eqs. (16), or (20), calculated taking into account both the striction and the relativistic nonlinearities, and the one given by Eq. (21), becomes essential at higher background plasma densities. For example, at $n_0 \gg 1$ the threshold intensity depends on background density as $a_{th}^2 \simeq \frac{27}{64} n_0^4$, so that, for instance, for $n_0=10$ the threshold of penetration is more than forty times higher than the one defined by Eq. (21). The difference is due to the strong increase in the electron density caused by the action of the ponderomotive force. Figure (3) shows the dependence of the maximum electron density on the background plasma density and clearly demonstrates the existence of a suppressing effect on the penetration of the electromagnetic radiation into an overdense plasma.

For the maximum incident amplitude, we can calculate the width of the depletion region as

$$x_{d*} = a_{d*} \sqrt{\frac{2}{3n_0}\left(\frac{a_{d*}^2}{2n_0} - 1\right)}, \qquad (22)$$

and the associated maximum of the longitudinal electric field integrating Poisson equation between $x=0$ and x_b. Making use of the integral of motion and the expression for the maximum incident intensity, we find

$$E_d \equiv n_0 x_d = \left[4a_{th}^2 - \frac{2}{3} n_0 \left(\frac{a_{d*}^2}{n_0} - 1\right)\right]^{1/2}. \qquad (23)$$

The corresponding results are presented in Fig. 4. What is interesting is that at plasma densities in the range of $n_0 \simeq 1$–1.5, the longitudinal electric field is not an increasing function of the incident intensity: at the beginning it increases with incident intensity reaching its maximum value, but then decreases to zero [see Fig. 4(a)]. This is a direct consequence of the solitonlike solution presented by Eq. (14). At higher densities, the longitudinal field monotonously increases with laser intensity until its threshold value. This field, directly responsible for the ion dynamics and the formation of shock waves entering the matter and compressing it, reaches a maximum value about two times larger than the limit amplitude of the incident wave at $n_0 \gg 1$.

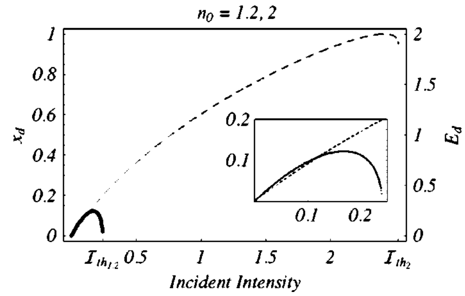

FIG. 4. Boundary position and boundary electric field [see Eq. (4)] as functions of the incident intensity, for $n_0=1.2$ (dotted line) and $n_0=2$ (dashed line). All quantities are dimensionless.

IV. CONCLUSIONS

In summary, we have used a stationary model to gain insight into the relativistic interaction problem of superintense laser radiation with an overdense plasma by considering circularly polarized incident laser radiation. The main conclusion is that for laser penetration into a rather thick overdense plasma, a traveling-plane-wave approach cannot be applied because in real situations this regime cannot be achieved, although, due to the relativistic increase of the electron mass and the associated decrease of the effective plasma frequency, the nonlinear refractive index may become positive. The nonlinear ponderomotive force generated in the laser-plasma interaction leads to a compression of the electron-density profile that counteracts the increased penetration due to the relativistic nonlinearity and plays a crucial role in the description of the interaction between an overdense plasma and high-intensity laser radiation.

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