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Capillary-wave effects at critical wetting in type-I superconductors

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We discuss the effect of fluctuations of the superconductor-normal (SC/N) interface on the (short-range) critical wetting transition in type-I superconductors. Functional renormalization of a standard effective interface Hamiltonian shows that the fluctuation regimes found for short-range critical wetting in conventional fluid systems appear in superconductors with slight modifications. Because the fluctuation parameter $\omega \sim 1/(1-\sqrt{2}\kappa)$ depends on the Ginzburg-Landau parameter κ, strong fluctuation effects would be expected in the limit $\kappa \to 1/\sqrt{2}$. However, the capillary-wave spectrum of the SC/N interface has an unusual form due to a relevant magnetic field contribution which suppresses long wavelength fluctuations, invalidating conclusions drawn from the standard effective interface Hamiltonian, and validating the results of mean-field theory.

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Short-range critical wetting was an elusive goal for the wetting community until recently. Because long-ranged forces are present in most fluid systems, this wetting transition can only be seen under very special circumstances [1]. However, type-I superconductors are potentially a whole class of systems in which the thickness of a wetting layer is controlled by exponentially decaying, short-range forces [2–4]. As was shown on the basis of Ginzburg-Landau theory [2], the interface between coexisting superconducting (SC) and normal (N) phases in type-I superconductors with surface enhancement can become delocalized from the boundary between the metal and the vacuum (the wall), forming a superconducting sheath of arbitrary thickness, analogous to a wetting transition in a fluid system. First-order or critical wetting transitions occur, depending on the Ginzburg-Landau parameter κ, which is the ratio of the magnetic penetration depth λ to the (bulk) superconducting coherence length ξ. For $\kappa < 0.374$, wetting transitions are first order; for $0.374 < \kappa < 1/\sqrt{2}$, they are critical [2]. In type-II superconductors, $\kappa > 1/\sqrt{2}$, wetting by the SC phase is preempted by nucleation of superconductivity in the bulk of the sample, and we do not consider these cases.

Mean-field theory for the critical wetting transition reveals a rich, nonuniversal behavior of the surface specific heat [4,5]. However, in order to be certain of the validity of mean-field theory, we must consider the effect of thermal fluctuations. In analogy with fluid systems, the lowest-lying excitations can be identified as capillary waves on the SC/N interface. These can be studied with an effective Hamiltonian [3,6], for which the standard form is

$$\mathcal{H}[f] = \int d^2\mathbf{r} \left[\frac{\gamma_{SC/N}}{2} (\nabla_{\mathbf{r}} f)^2 + V(f) \right], \tag{1}$$

where $f(\mathbf{r})$ is the thickness of the superconducting sheath. The first term in Eq. (1) represents the capillary effect of the surface tension $\gamma_{SC/N}$ between the SC and N phases. The interface potential $V(f)$ accounts for the effective interaction between the SC/N interface and the wall, and is given by [3,4]

$$V(f) = a \exp(-\sqrt{2} f/\xi) + b \exp(-f/\lambda) + \mathcal{O}(\exp(-2\sqrt{2} f/\xi)). \tag{2}$$

For systems undergoing critical wetting, $b > 0$ and $a < 0$ for temperatures T below the wetting transition T_w, so that $V(f)$ has a single minimum at $f = f_0$. Approaching T_w, the leading coefficient vanishes, $a \propto -(T_w - T)$, and f_0 diverges.

The applicability of the effective Hamiltonian (1) to critical wetting in fluids in three dimensions has been questioned, since it is unable to reproduce the structure of certain correlation functions in the vicinity of the wall [7]. This problem arises because the model does not properly describe fluctuations which carry the interface close to the wall. As we shall see later, such large fluctuations do not occur for the SC/N interface, so that we can, at least as a start, employ a model similar to that given by Eqs. (1) and (2) for a discussion of the critical wetting transition in superconductors.

If we naively apply the results of a linear functional renormalization scheme [8,9] to Eqs. (1) and (2), we find that there are three different fluctuation regimes for critical wetting in type-I superconductors. In analogy to fluid systems, these can be classified according to the magnitude of the fluctuation parameter $\omega \equiv k_B T / [2\pi \xi^2 \gamma_{SC/N}]$. One finds a weak fluctuation regime for $0 < \omega < 4\kappa^2$ in which the interfacial correlation length $\xi_i \sim |T - T_w|^{-\nu}$ diverges at the wetting transition $T = T_w$ with

$$\nu = ([2\sqrt{2}\kappa - \omega][1/(\sqrt{2}\kappa) - 1])^{-1}. \tag{3}$$

At the upper limit, $\omega = 4\kappa^2, \nu = (2[1/\sqrt{2} - \kappa])^{-2}$. For $\omega > 4\kappa^2$, the old results for short-range critical wetting apply [8], i.e., there is an intermediate and a strong fluctuation regime. Approaching the type-II regime, $\kappa \to 1/\sqrt{2}$, the fluctuation parameter diverges, $\omega \to \infty$, since $\gamma_{SC/N} \sim (1 - \sqrt{2}\kappa) \to 0$. Strong fluctuation effects would therefore appear to

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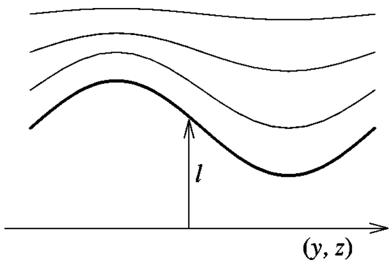

FIG. 1. Fluctuations $l(y,z)$ of the SC/N interface (bold line) away from the (y,z) plane bend the lines of magnetic flux (thin lines), giving rise to an anisotropic $|\mathbf{q}|$ contribution to the capillary wave energy [Eq. (5)]. For long wavelength fluctuations, this effect dominates the usual $\mathbf{q}^2$ term that is generated by the surface tension.

dominate the physics of critical wetting in type-I superconductors with a sufficiently high value of κ.

However, this result is based on Eq. (1), which neglects the influence of the magnetic field on the capillary-wave spectrum of the interface, as we now demonstrate. We consider a free SC/N interface lying in the (y,z) plane, and a uniform magnetic field applied in the y direction, $\overline{H}=\mathbf{H}=(0,H_c,0)$, equal in magnitude to the bulk critical field H_c to ensure the coexistence of the SC $(x<0)$ and $N(x>0)$ phases. We use an overbar for a general vector, and a bold typeface for vectors lying in the (y,z) plane. As shown schematically in Fig. 1, if the interface is distorted away from the (y,z) plane, the lines of magnetic flux, which do not penetrate the SC phase, are also distorted.

In the limit $\kappa=0$, there is no overlap of the superconducting order parameter and the magnetic flux density, and the effect of a surface fluctuation $l(\mathbf{r})=l_{\mathbf{q}}\cos(\mathbf{q}\cdot\mathbf{r})$ is found by solving the field equations in the N phase $\overline{\nabla}\cdot\overline{B}=0$ and $\overline{\nabla}\times\overline{H}=0$, with the boundary condition $\overline{B}\cdot\overline{n}=0$, where $\overline{n}$ is the normal to the distorted surface (i.e., that surface where the order parameter vanishes). To first order in $l_{\mathbf{q}}$, the change in the flux density $\overline{B}=\mathbf{H}+\delta\overline{B}$ is

$$\delta\overline{B}=(\mathbf{q}\cdot\mathbf{H})[\mathbf{e}_{\mathbf{q}}\cos(\mathbf{q}\cdot\mathbf{r})-\overline{e}_x\sin(\mathbf{q}\cdot\mathbf{r})]e^{-|\mathbf{q}|x}l_{\mathbf{q}}, \quad (4)$$

where $\mathbf{e}_{\mathbf{q}}$ and $\overline{e}_x$ are unit vectors in the $\mathbf{q}$ and x directions, and we have assumed that the relative permeability of the N phase is equal to 1. The effect of a general surface distortion can be found by superposition. For nonzero κ, Eq. (4) is valid for wavelengths longer than the magnetic penetration length λ.

The contribution to the capillary wave energy from this distortion follows from the change in the energy of the magnetic field, $1/2\int\overline{B}\cdot\overline{H}d^3r$. Including the usual $\mathbf{q}^2$ from the capillary effect of the SC/N interface, which is equivalent to the gradient-squared term in Eq. (1), and a ''mass'' term m^2 for later discussion, we find that the effective Hamiltonian for the SC/N interface to second order in the fluctuation $l_{\mathbf{q}}$ is

$$\mathcal{H}[l]=\int_{|\mathbf{q}|<\Lambda}\frac{d^2\mathbf{q}}{(2\pi)^2}\frac{1}{2}\{(\mathbf{q}\cdot\mathbf{H})^2|\mathbf{q}|^{-1}+\gamma_{\mathrm{SC}/N}\mathbf{q}^2+m^2\}|l_{\mathbf{q}}|^2. \quad (5)$$

In writing down this expression, we neglect variations of the structure of the SC/N interface caused by the surface fluctuations, apart from simple translational shifts. For example, the possible appearance of regions within the interface in which both the magnetic flux and the superconducting order parameter vanish [3] is ignored. This approximation is permissible if we are only interested in the spectrum of the lowest-lying capillary waves at wavelengths which are larger than the coherence length ξ. To this purpose we have included a high wave-vector cutoff $\Lambda\approx\xi^{-1}$. Similarly, Eq. (5) is applicable at long wavelengths to the cases of nonzero κ which show a critical wetting transition, since for these the penetration depth λ is less than ξ.

For fluctuations exactly perpendicular to the applied field $\mathbf{H}$, the magnetic field lines are not distorted and the capillary energy (5) has a conventional $|\mathbf{q}|^2$ dependence on the wave vector. This means that the interface Hamiltonian (1) *is* valid for SC/N interface configurations that vary only in the direction transverse to the applied field, for example, in calculating droplet shapes in Ref. [3] and also in calculating the capillary contribution to the free energy for the intermediate state in a thin slab [10]. However, in all other directions, the dispersion law has an unusual $|\mathbf{q}|$ dependence at long wavelength, as a result of the magnetic field contribution.

The amplitude of thermally excited fluctuations of the SC/N interface is found by applying equipartition to the Hamiltonian (5), and the height-height correlation function $C(\mathbf{r})=1/2\langle[l(\mathbf{r})-l(\mathbf{0})]^2\rangle$ is

$$\frac{C(\mathbf{r})}{k_BT}=\int_{|\mathbf{q}|<\Lambda}\frac{d^2\mathbf{q}}{(2\pi)^2}\frac{[1-\cos(\mathbf{q}\cdot\mathbf{r})]}{(\mathbf{q}\cdot\mathbf{H})^2|\mathbf{q}|^{-1}+\gamma_{\mathrm{SC}/N}\mathbf{q}^2+m^2}. \quad (6)$$

Let us first consider the free interface, $m^2=0$. The integral in Eq. (6) is convergent at long wavelengths, and the (squared) interface width $W^2=\lim_{|\mathbf{r}|\to\infty}C(\mathbf{r})$ is finite,

$$W^2=\frac{k_BT}{\pi\gamma_{\mathrm{SC},N}}\log\left[\left(1+\frac{\gamma_{\mathrm{SC},N}\Lambda}{H_c^2}\right)^{1/2}+\left(\frac{\gamma_{\mathrm{SC}/N}\Lambda}{H_c^2}\right)^{1/2}\right]. \quad (7)$$

Thus, the free SC/N interface does not make unbounded fluctuations and is not ''rough.'' Long wavelength capillary waves are suppressed by the $(\mathbf{H}\cdot\mathbf{q})^2|\mathbf{q}|^{-1}$ term in the effective Hamiltonian. This is in contrast to a fluid interface in three dimensions for which the width W is divergent and the interface is rough, because the amplitude of long wavelength fluctuations is determined by a less suppressive $\mathbf{q}^2$ term. Indeed, the SC/N interface is more like a conventional fluid interface in dimensions greater than 3, which is also not rough.

The approach of the correlation function $C(\mathbf{r})$ for the free interface to the limiting value W^2 for large $|\mathbf{r}|$ also follows from Eq. (6). We find a power-law behavior, with an amplitude that depends on the angle θ between $\mathbf{r}$ and $\mathbf{H}$, $W^2-C(\mathbf{r})\sim(|\sin\theta||\mathbf{r}|)^{-1/2}$. In the direction of the field, $\theta=0$, the power law is modified and becomes $|\mathbf{r}|^{-1/3}$.

To examine the influence of thermally excited capillary waves on the mean-field wetting behavior, we expand the interface potential $V(f=f_0+l)$ about its minimum f_0 to second order in l. This gives a mass term in the effective Hamiltonian (5), $m^2=(d^2V/df^2)_{f=f_0}$, which vanishes approaching the critical wetting transition, $m^2\sim(T_w-T)^{1/(1-\sqrt{2}\kappa)}$.

For nonzero m, the integral in Eq. (6) is still convergent at long wavelengths, and the interface width W^2 is finite, but slightly smaller than for the free interface, as fluctuations are suppressed by the additional m^2 term. Approaching wetting, $m^2 \rightarrow 0$, and W^2 increases smoothly to the finite value (7). At large distances, the correlation function $C(\mathbf{r})$ approaches W^2 in an damped-oscillatory fashion

$$W^2 - C(\mathbf{r}) \sim \mathrm{Re}\exp[-|\mathbf{r}|(1/\xi_d + i/\xi_o)]. \tag{8}$$

Both the decay length ξ_d and the oscillatory length ξ_o diverge approaching the wetting transition, $\xi_d \sim |\csc\theta|(m^2/\gamma_{SC/N})^{-1/2}$ and $\xi_o \sim |\sin\theta| \sec^2\theta (m^2/H_c^2)^{-1}$. In the direction of the field $\theta = 0$, both lengths have the same divergence at wetting $\sim (\gamma_{SC/N}/H_c)^{-1/2}(m^2/\gamma_{SC/N})^{-3/4}$.

Thus, approaching a critical wetting transition, the SC/N interface unbinds, but it does not simultaneously roughen. Despite the emergence of diverging lengthscales for the (in plane) correlation function, the interface width saturates as it becomes free. These finite fluctuations do not alter the nature of the singularities at the critical wetting transition or alter the temperature of the wetting transition, just as is the case for fluid interfaces in dimensions greater than three [8]. We conclude that the mean-field results for the divergence of the wetting layer thickness and the singularity in the surface specific heat for the short-range critical wetting transition in superconductors [4,5] are valid.

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