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Experimental observation of localization in the spatial frequency domain of a kicked optical system

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An experimental realization of an optical ''kicked'' system is presented. It exhibits localization analogous to that of the quantum ''kicked-rotor.'' In the experiment, free space propagating light is periodically kicked by thin sinusoidal phase gratings, which produce high order diffractions and tend to increase the spatial frequency band. The wave property suppresses this diffusive spread. The localization is realized in a regime near anti-resonance of the system, which is also studied theoretically. The behavior in this regime is similar to that of electronic motion in incommensurate potentials. A crucial part of the experimental system is the grating in-phase positioning, which is done by using the Talbot effect.

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The investigation of dynamical localization is part of the field of ''Quantum Chaos,'' where the quantal behavior of systems which are chaotic in the classical limit is studied [1–3]. The classical dynamics of chaotic systems resembles random motion, although it is generated by deterministic equations, and if the phase space is unbounded the random-like motion leads to diffusion in this space. In the quantum case, however, it is suppressed by quantum interference leading to dynamical localization [4], which is similar to Anderson localization in disordered solids [5]. So far, only one type of direct experimental observation of localization for the quantum kicked-rotor, which is the standard system for the exploration of the quantal suppression of classical chaos, has been published using laser cooled Na and Cs atoms in a magneto-optic trap [6].

Since localization is actually a wave phenomenon it is expected to occur also for ''classical'' electromagnetic waves [7,8]. In this work we present an experimental realization of an optical ''kicked'' system. We examine the localization properties in a specific regime near ''antiresonances'' of the system. Resonances and antiresonances in our optical system are shown to be related to the Talbot effect, and were used by us to adjust critical parameters in the experimental system.

In the experimental system, schematically described in Fig. 1, a free space propagating light beam along the z axis is successively ''kicked'' by identical thin sinusoidal phase gratings. The gratings are parallel and have an identical spacing z_0 between them and aligned phases. In the process, the successive kicks produce high order diffractions which tend to increase the beam's spatial frequency band. Nevertheless, as we see below, localization in the spatial frequency domain, with a characteristic exponential confinement, occurs after several kicks. In the ''classical'' regime, the diffraction leads to nonlocalized diffusive behavior. This corresponds to the case where the light intensities, instead of the electric field amplitudes, are added up. In our experiment, a regime of similar behavior is obtained when the grating phases are randomly positioned in the system, resulting in a destructive interference that behaves as the destruction of wave coherence.

The transverse (x coordinate) dependence of the light electric field envelope ψ, is given, in the slowly varying amplitude (paraxial) approximation, by the following Schrödinger-like propagation equation [9],

$$i\frac{\partial\psi}{\partial z} = -\frac{\lambda}{4\pi}\frac{\partial^2\psi}{\partial x^2} + \kappa\cos(k_g x)\sum_N \delta(z - Nz_0)\psi. \qquad (1)$$

It includes the ''kick'' δ functions potential, resulting from the sinusoidal phase gratings with a wave vector k_g and an amplitude κ, where λ is the light wavelength. Here the coordinate z along the direction of propagation plays the role of time for the kicked rotor. The light intensity is proportional to $|\psi|^2$.

Unlike the quantum kicked-rotor, our optical system lacks inherent discrete ''energy'' levels, in the light transverse spatial frequency domain. However, when the input light is a plane wave or a broad Gaussian beam, the sinusoidal grating kicks confine the dynamics to discrete spatial frequency modes of the gratings diffraction orders n (corresponding to the angular momentum of the kicked-rotor), which are coupled between themselves. In the present work the initial mode is $n=0$, and therefore only modes with integer n are involved. In this process, the repeated kicks tend to increase the number of the diffraction orders. The propagation between the kicks adds extra phases, γn^2 quadratically depen-

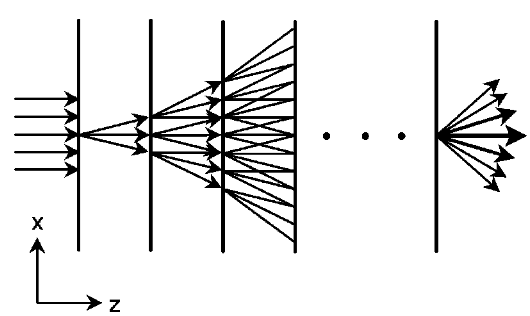

FIG. 1. Schematic description of the free space ''kicked'' optical system with the array of phase gratings having an identical spacing between them. Some of the light paths are shown.

dent on the diffraction order n, where $\gamma=\pi\lambda z_0/\lambda_g^2$, and $\lambda_g=2\pi/k_g$ is the grating period. For large n, this factor behaves as a random number. The resulting one period evolution operator is $\hat{U}=\exp(-i\gamma\hat{n}^2)\ \exp[-i\kappa\cos(k_g x)]$, where $\hat{n}=-i[\partial/\partial(k_g x)]$. As a result of the effective randomness of the factors $\exp(-i\gamma n^2)$, it turns out that the overall contribution is weakened, resulting in exponential localization [4,5,1]. Consequently, low n are mostly composed of former low order diffractions (spatial frequencies), which add constructively. It is crucial that the propagator $\hat{U}$ is identical for all kicks. Although $\exp(-i\gamma n^2)$ behave as random numbers, these are identical for all intervals of free motion. As described below, our experiment was carried out in a special regime of the kicked optical system. It is the vicinity of antiresonance of which description and properties touch the two-sided kicked-rotor [10]. There, the localization behavior can be approximated by the exactly solvable ''linear-kicked-rotor'' (shown to be equivalent to an incommensurate potential of a tight-binding model for electrons in solids) [11], with a linear, rather than quadratic, n dependent phases in the free space propagation term $\exp(-i\tau n)$.

In the experimental setup, the exact locations of the gratings with aligned phases is critical. For that matter, we developed a special technique which brought us to a specific regime of localization. This is the vicinity of ''antiresonances'' of our optical system, which are related to half Talbot distances. The Talbot effect [12] is the occurrence of optical self-imaging of periodic images along propagation in free space, at multiple distances of the Talbot length, $z_T=2\lambda_g^2/\lambda$ $(\gamma=2\pi)$. Thus, when the gratings spacing in our system equals the Talbot length, it is equivalent to the case where the free space propagation phase shrinks to zero (mod 2π), and the effect of all gratings is coherently added. This corresponds to a resonancelike state. The system with grating locations at half Talbot distances (odd multiples of $z_T/2$), corresponds to antiresonance, where any two successive (aligned) gratings cancel each other. This effect of cancellation served us to accurately adjust the grating locations and align their phases. The alignment was done in a reverse way, from the last (output) grating towards the first, by checking that each added grating cancelled the diffractions due the former grating. After the alignment was *completed*, we *changed* the laser wavelength, λ, *to move away* of the trivial antiresonance state. In the experiment, we aligned the system with the Argon-ion laser line $\lambda=501$ nm, with a spacing of $z_0=(3/2)z_T=3.832$ cm $(\gamma=3\pi)$ for $\lambda_g=80$ μm, and made the measurement at $\lambda=496$ nm, where $\gamma=2.97\pi$. The strength of the kick by the grating was $\kappa=5.94$. These parameters are used in the numerical calculations (unless stated otherwise). It was important in our experiment *to stay near antiresonance* because of the limited number of gratings, which was 9, due to absorption and finite sizes of the apertures and the optical elements' cross section. Near antiresonance, the transition from the diffusion regime to localization can occur after a few kicks, as can be seen from Fig. 2, where $\sigma\equiv\sqrt{\langle(n-\langle n\rangle)^2\rangle}$ is plotted. A large number of kicks, which is experimentally inaccessible in our system, is required to observe localization away from antiresonance. In the calculations of Fig. 2, as well as in all other numerical calculations, the initial state is $n=0$, corresponding to the experiment, resulting in $\langle n\rangle=0$.

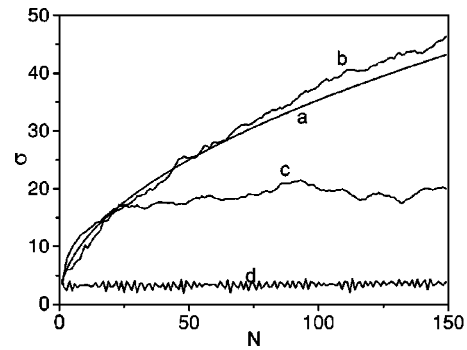

FIG. 2. Numerical simulation of the evolution of the spatial frequency width σ as a function of the number of kicks N, for a ''classical'' system (without interference) that exhibits diffusive behavior is shown in curve *a*, and for phase disordered gratings (where we added for each grating N a random phase, φ_N, such that $\gamma n^2+\varphi_N n$, in curve *b*. A similar diffusive behavior was obtained for random γ. Confinement behavior is obtained for ordered gratings, far from resonance or antiresonance in curve *c* (with $\gamma=0.74\pi$), and near antiresonance in curve *d* (with $\gamma=2.97\pi$, or $\delta=-0.03$). For all graphs we used $\kappa=5.94$, which matches the experiment.

The ''long term'' localized spectrum calculated numerically from Eq. (1) near antiresonance is presented in Fig. 3 It has a ''fir-tree'' shape with a slightly faster than exponential decay, as the analysis gives for localization in this regime which corresponds to the linear-kicked-rotor [11] (see discussion below and in [13]). The ''plateaus'' have a more moderate exponential decay.

The results for the spreading near antiresonance (presented in Fig. 3) can be understood from the correspondence with the linear kicked rotor. Near antiresonance $\gamma=(2M+1)\pi+\delta\pi$, where M is an integer ($M=1$ in the present experiment) and $\delta\ll 1$. For $n\ll n_0$ the local approximation $\frac{1}{2}\delta(n_0+n)^2\approx\delta n_0 n+C$ can be made when $\frac{1}{2}\delta n^2\ll 1$, where the constant $C=\frac{1}{2}\delta n_0^2$. Using the fact that $\exp[-i\pi(2M$

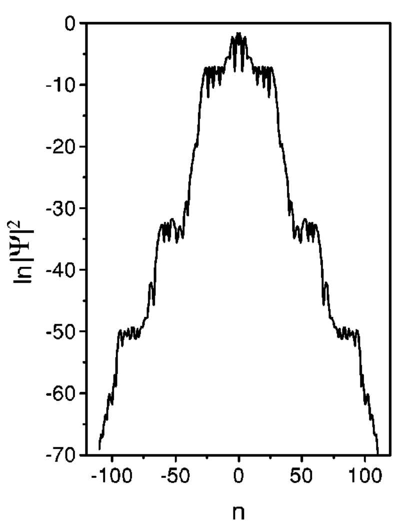

FIG. 3. Typical numerically calculated confined spatial frequency spectrum $|\psi|^2$ near antiresonance, with $\gamma=2.97\pi$ (or $\delta=-0.03$), after $N=200$ kicks, having a ''fir-tree'' shape.

$+1)n^2]=\exp(-i\pi n)$, the free propagation part of $\hat{U}$ can be written approximately as $\exp[-i\gamma(n_0+n)^2]\approx\exp\{-i[(\pi+\tau)n+\text{const}]\}$, where $\tau=2\pi\delta n_0$. Because of the smallness of δ, the parameter τ varies slowly with the center of the expansion n_0 and will be assumed constant. The resulting local model can be defined by the one step evolution operator

$$\hat{U}=e^{-i\kappa\cos(k_g x)}e^{-i(\tau+\pi)\hat{n}}. \tag{2}$$

The ''quasienergy'' states of this evolution operator, which were obtained in [13] following [11], are

$$\psi_\nu(n)=e^{i(\tau/2)(n-\nu)}(-i)^{|n-\nu|}J_{|n-\nu|}\left(\frac{\kappa}{2\sin[(\tau+\pi)/2]}\right), \tag{3}$$

where ν is the center of localization and J_n are Bessel functions. If the Bessel functions decay rapidly, as is the case when the index is much larger than the argument in magnitude, the linear approximation holds since τ can be considered constant in a range where the Bessel function varies considerably. Therefore the approximation is expected to fail where this argument is large, i.e., at the points where $\sin[(\tau+\pi)/2]=0$ or $n_0^{(l)}=(2l-1)/2\delta$, where l is an integer. In the regions that are far from the $n_0^{(l)}$ the eigenfunctions fall off locally as Bessel functions resulting in the rapid fall off of intensity in Fig. 3. The ''plateaus'' in Fig. 3 are found starting from $n_0^{(l)}=17,50,83\dots$ ($n_0^{(l)}$ is the point on the ''plateau'' that is the closest to the origin), corresponding to $l=1,2,3\dots$ for $\delta=-3(5/501)\approx-0.03$. The width of the ''plateaus,'' Δn_0 can be estimated from the requirement for the linear approximation to hold for some distances (somewhat larger than Δn_0) from $n_0^{(l)}$ leading to $\Delta n_0\approx a\sqrt{(\kappa/\delta)}$ where a is a numerical constant [13]. This relation was tested numerically and the value $a=\sqrt{3/\pi}$ was obtained [13].

What is the form of the eigenfunctions of the model (1) predicted by the linear approximation (2)? For small δ the linear approximation holds for most n, since the distance between the regions where it fails grows like $1/\delta$, while their width is $\Delta n_0\sim\sqrt{1/\delta}$. In the regions where the linear approximation holds the eigenfunctions of model (1) are superpositions of few eigenfunctions of model (2) centered in regions where the approximation fails [the values of ν of Eq. (3) are in such regions]. The reason for this form is that the rate of decay is determined by the local properties of the equation, while the position of the center and the value of the quasienergy are determined by the normalization condition, and the matching between the regions where the linear approximation with different τ holds. The local approximation of the eigenfunctions of Eq. (1) by functions of the form (3) was tested in [13].

The experimental data in Fig. 4 show the confinement after the eighth and ninth gratings for the ordered grating system, and also the spread spectrum without localization for a disordered system, where the gratings phases were not aligned. The difference between the results in the ordered and disordered systems demonstrates that an effect of interference was observed here. Note that the experimental range

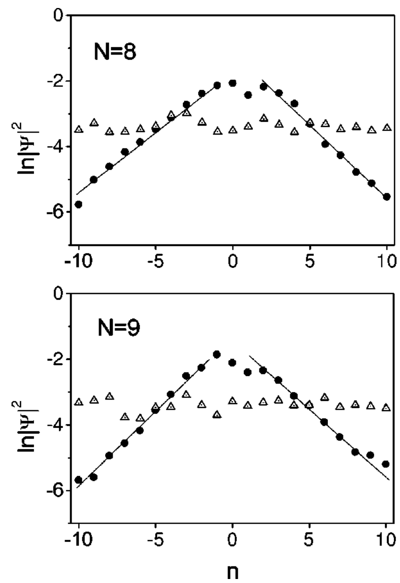

FIG. 4. Experimental confined spatial frequency spectrum intensity after the eigth and ninth gratings for the ordered (closed circles) and disordered (open triangles) gratings. The initial state (direction) is $n=0$, within experimental accuracy.

presented in Fig. 4 corresponds to a narrow range in the center of Fig. 3. The straight line fit shows an exponential behavior. This was repeatedly seen in our other measurments we made, which are not given here, with slightly different experimental parameters and for the various kick numbers. The experimental profiles are confined with a localization width of $\sigma\approx3-4$. The spatial frequency width found in the experiment after each of the nine kicks is compared with the theoretical predictions in Fig. 5. It shows a good agreement, even in the oscillating behavior, characteristic of the vicinity of the antiresonance. Exacly at antiresonance, where each even grating cancels the spreading due to its former odd grating, the oscillation of the width as a function of the grating number has a period of 2 (with values $\sigma_a,0,\sigma_a,0,\sigma_a,0\dots$) presenting a trivial confinement. Off antiresonance, but still in its vicinity, there is a more complex variation of σ, as can be seen in the example of Fig. 5,

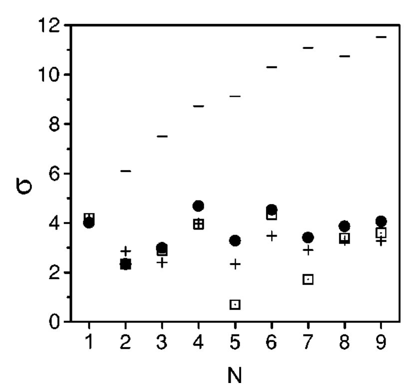

FIG. 5. Experimental spatial frequency intensity width σ after each of the nine gratings (closed circles) compared with the theoretical results of Eq. (1) near antiresonance (+), with $\gamma=2.97\pi$, and away from antiresonance (–), where $\gamma=\sqrt{5}-1$. The results of the linear model (2), marked by squares, are presented as well.

and in curve *d* of Fig. 2. We also checked in the experiment the possible influence of the *finite sizes* of the input light and the gratings, that are idealized in the theory, and found no significant effect on our data. More details of this aspect will be given elsewhere [13].

In conclusion, we have presented an experimental realization of an optical ''kicked'' system exhibiting localization in spatial frequency mode space. Free space propagating light is periodically kicked by thin sinusoidal phase gratings. The study concentrated on the near antiresonance regime, where various theoretical properties and experimental observations were discovered. This behavior differs from localiza-tion far from antiresonance and in random systems and is similar to localization found for incommensurate potentials.

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