RAPID COMMUNICATIONS

PHYSICAL REVIEW E VOLUME 61, NUMBER 3 MARCH 2000

Model for urban and indoor cellular propagation using percolation theory

G. Franceschetti,[1] S. Marano,[2] N. Pasquino,[1] and I. M. Pinto[3]

[1]*Dipartimento di Ingegneria Elettronica e delle Telecomunicazioni, Università degli Studi di Napoli "Federico II," Napoli I-80125, Italy*

[2]*Dipartimento di Ingegneria dell'Informazione e Ingegneria Elettrica, Università degli Studi di Salerno, Fisciano (SA) I-84084, Italy*

[3] *Università degli Studi del Sannio a Benevento, Benevento I-82100, Italy*

(Received 30 September 1999)

A method for the analysis and statistical characterization of wave propagation in indoor and urban cellular radio channels is presented, based on a *percolation model*. Pertinent principles of the theory are briefly reviewed, and applied to the problem of interest. Relevant quantities, such as pulsed-signal arrival rate, number of reflections against obstacles, and path lengths are deduced and related to basic environment parameters such as obstacle density and transmitter-receiver separation. Results are found to be in good agreement with alternative simulations and measurements.

PACS number(s): 42.25.Dd,41.20.Jb

I. INTRODUCTION

Telecommunications have experienced, in recent years, unprecedented development, mainly stimulated by widespread demand for fast and reliable access to information with steadily improving quality of service. When terrain profiles, customer distribution, absence of existing facilities, and allowance for user mobility make wired systems unaffordable and not worth the cost, wireless cellular systems are the choice. However, in order to get reliable estimates of the coverage and therefore mean extension of the cells, we need a working knowledge of the propagation properties of the indoor and outdoor radio channels involved.

Most widely used techniques devised for this goal adopt an empirical approach whereby measured electromagnetic fields are used to build some statistical characterization of the channel under observation [1–6]. On the one hand, these methods are time and cost demanding, since they need extensive measurement campaigns at every location to be characterized. On the other hand, they are quite reliable in terms of accuracy.

An alternative procedure which is viable when dealing with microcellular environments is referred to as the "ray tracing" [7] method. This basically consists of tracing the trajectories of the electromagnetic energy flux from the transmitter to the receiver, making some judicious use of the diffraction theory, whenever needed, to compute the principal part of the field in an (asymptotic) short-wave expansion. To accomplish this, some *a priori* knowledge of electromagnetic and physical properties of the propagation environment and an adequate amount of processing power is required.

The method proposed here is based on a model of the propagation channel borrowed from the percolation theory. This will allow for a remarkably simple description of the channel itself and the propagation phenomena in it.

This Rapid Communication is organized as follows. In Sec. II the proposed model is outlined, and the relevant assumptions and simplifications are discussed. In Sec. III simulations are presented for the propagation of an ideal pulse through a channel within the framework of the proposed model.

II. PERCOLATIVE APPROACH

Percolation theory (PT) describes diffusion phenomena in a random lattice, as a function of the interconnection density. It has been widely applied to such diverse fields as biophysics, polymer science, and electrical engineering, to name a few [8]. By properly tailoring PT concepts and analysis tools, one could study the propagation of an electromagnetic (EM) wave, e.g., in urban environments, where open paths take the role of the percolative clusters (a cluster is defined as a sequence of neighboring empty cells), and EM energy flux plays the role of the diffusing fluid [9,10].

As a key feature, PT allows us to describe relatively complex phenomena using a relatively *small* number of *simple* parameters. For urban channel propagation, the relevant quantities are the scatterer density, the (wavelength-scaled) cell-side length, and the transmitter-receiver distance. We shall accordingly model a city as a two-dimensional square-cell lattice, where occupied cells represent buildings whose known density is q. Cell-side length will be assumed as fixed and denoted by a.

Without loss of generality we can imagine a uniform distribution of buildings, and consider the status of each cell as being totally independent of that of the surrounding ones. An EM wave radiator in the channel is modeled as the source of an isotropic set of rays (EM energy-flux lines), and can always be assumed to be located at the center of the lattice if this latter has infinite extent. The *ray model* of EM wave propagation holds in the short-wave limit, where the EM field characteristic wavelength $\lambda = c/f$ is much smaller than the scatterer size $(\lambda/a \ll 1)$.

Electromagnetic energy-flux lines (rays) *percolate* through the empty cells from transmitter to receiver, obeying the laws of geometrical optics. Obstacles are taken as opaque, with an average (e.g., at normal incidence) reflection coefficient R. Refracted waves are supposed to be completely absorbed, so that no ray can traverse an obstacle and reemerge from it. We ignore diffraction effects (contributing terms of higher order in powers of λ/a), and assume locally plane-wave fronts and scatterer surfaces.

It is clear that if the density of occupied cells is too high, no ray is likely to reach the cell where the receiver is located, and the radiated energy will be absorbed by the surrounding obstacles. By allowing the occupied-cell density to decrease, a ''spanning'' or percolating cluster may form, allowing rays to reach the receiver.

The critical value $q=q_c$ at which the above happens is referred to as the *percolation threshold*. In a number of physical problems (e.g., spin diffusion in ferromagnets), this corresponds to a *phase transition* of the structure. The percolation threshold depends on the geometry and topology of the problem. For two-dimensional (2D) square-cell lattices $q_c \approx 0.40\,725$. As a result, the occupied-cell density value in all subsequent simulations will be chosen such that $q<q_c$, so as to allow the existence of percolating cluster(s).

Before discussing the software implementation of the model and presenting the main results of our simulations, a preliminary discussion of the above approximations is in order. Free- and occupied-cell densities in a realistic city-modeling lattice will usually be nonuniform. Urban areas will likewise resemble clusters of clusters, separated by large empty areas. Furthermore, in a real city there are usually *highly connected* structures (i.e., streets and squares) that introduce some degree of correlation in the statistical distribution of occupied cells. Nonetheless, our simple uniform-uncorrelated building distribution could still apply to many European historical city centers, in view of the peculiar way they grew [9].

Our model assumes flat obstacles affecting propagation through simple reflection. Because of the large amount of fine details in the external structure of buildings, this is a crude assumption. Indeed, diffraction phenomena may not only be present, but occasionally play a major role in determining urban channel characteristics [11]. These could be included, in principle, e.g., along the lines sketched in [12], at the expense of an additional computational burden.

We consider a two-dimensional propagation problem. As a matter of fact, transmitters are usually placed on top of buildings, and therefore reflection, absorption, and possible diffraction from floors and roofs should be taken into account. This is possible, in principle, by appropriate 3D generalization of the model, resulting in a different percolation threshold q_c.

Finally, no information on possible time changes of the scenario have been considered. In urban environments, moving obstacles such as buses, cars, and motorcycles can take part in the determination of the propagation characteristics. This could be taken into account, in principle, by averaging over several different lattices with the same density.

Averaging over a large number of different lattices is, however, implied when dealing with a statistical characterization. This allows us to apply the general results obtained, paying no attention to the effective building distribution but simply referring to an overall density parameter.

III. SIMULATIONS RESULTS

Simulation procedures have been implemented in MATLAB. Uniformly distributed random lattices modeling the cellular channel have been generated using the RANDOM built-in function, which generates a (pseudo)random number uniformly distributed in [0,1]. For each cell such a random number is generated and compared to a fixed threshold $q<q_c$ in order to decide whether or not the cell is occupied. Applying the same procedure to every cell results in a lattice whose average occupation density is q, i.e., if N is the total number of cells, there will be $\sim qN$ occupied cells. It is apparent that only a *finite* lattice can be simulated in practice. In our simulations we used a square lattice of 81×81 cells.

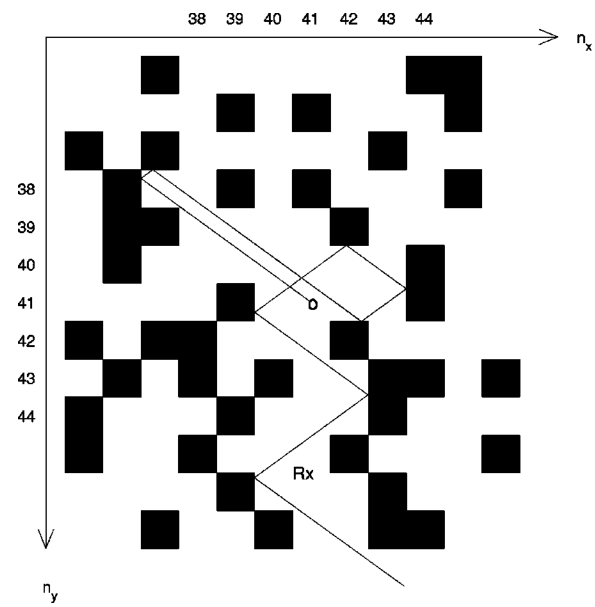

FIG. 1. Example of ray propagation in a random lattice.

An isotropic set of rays emanating from the source has been taken as a suitable source model. The set has been generated simply taking a reasonably large number of equispaced angular directions, representing different rays. The total number N_r of rays/directions included in the simulations is set by software and hardware limitations and was taken as $N_r=2^{14}$ in our investigation.

Ray trajectories are traced using Snell's law. Path lengths are deduced from obvious trigonometric formulas, and reflection coefficients through Fresnel formulas. Rays reaching the lattice boundary are considered to be irreversibly lost. Rays are also discarded if their amplitude falls below the assumed ambient noise floor.

For each ray reaching the cell where the receiver is located (see Fig. 1) the overall path length, the arrival time (pulsed source), the number of reflections undergone, and the number of cells crossed are stored for subsequent use. The ray tracing procedure is stopped as soon as all rays emitted have reached the receiver, or been lost through the lattice boundary, or attenuated below the assumed ambient noise floor. The distribution of the main features (arrival times, number of reflections undergone, and overall path length) over several (typically 10^2) realizations of the lattice are then computed for different values of lattice density q, and distance between source and observation points d.

Note that, according to [10] the proper distance to be considered here is the *city-block* distance $d=\Delta n_x+\Delta n_y$, where Δn_x and Δn_y are the number of horizontal and vertical cells between transmitter and receiver, respectively, instead of the Euclidean one. The main results of our simulations are summarized in Figs. 2–4.

As usual, when dealing with counting processes, assum-

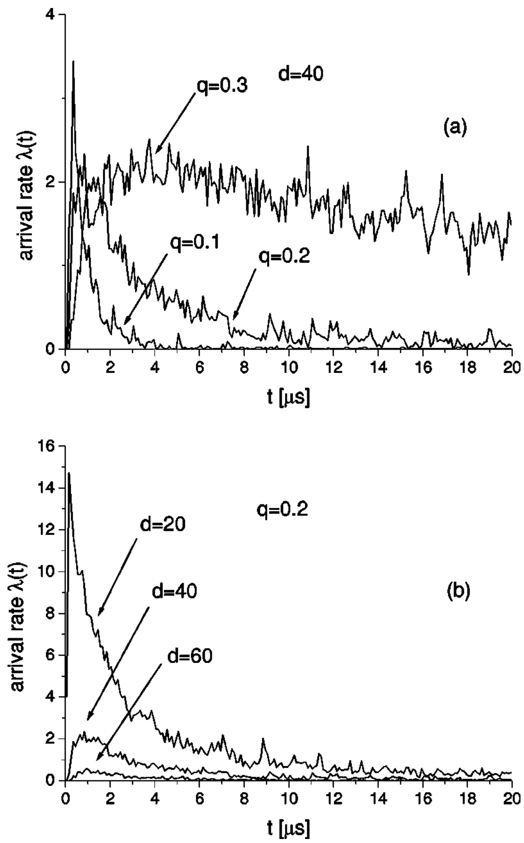

FIG. 2. Arrival rate $\lambda(t)$ vs arrival times t.

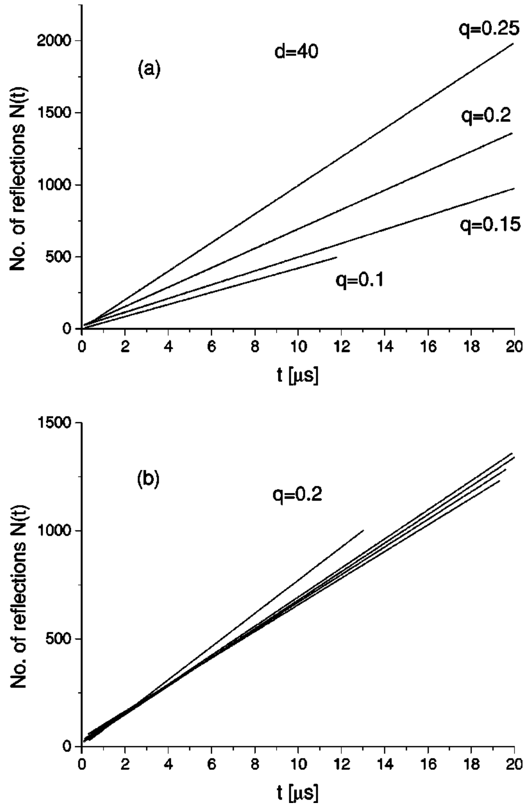

FIG. 3. Number of reflection $N(t)$ vs arrival times t.

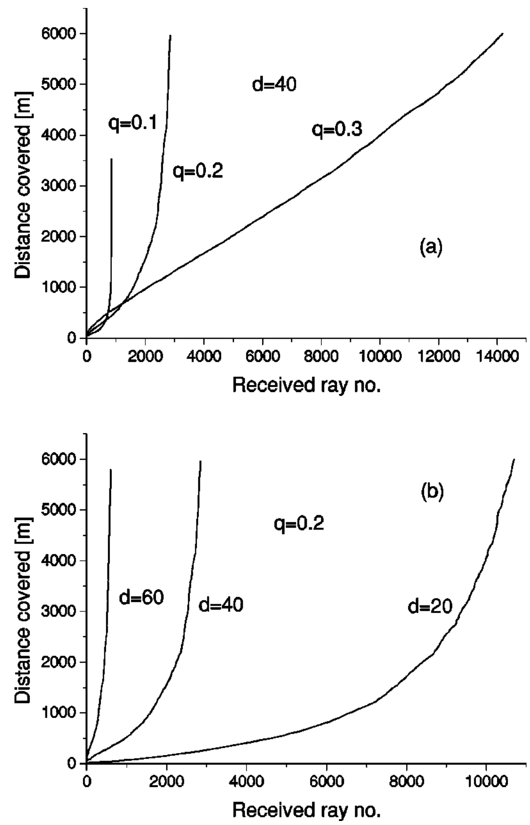

FIG. 4. Distances covered by rays.

ing a pulsed source, we can *a priori* assume a Poisson distribution for the arrivals. Under this hypothesis, the probability of counting m arrivals in the interval $[t_1,t_2]$ is

$$P[N(t_1,t_2)=m]=\frac{[\Lambda(t_1,t_2)]^m}{m!}e^{-\Lambda(t_1,t_2)},$$

where the *intensity function*

$$\Lambda(t_1,t_2)=E[N(t_1,t_2)]=\int_{t_1}^{t_2}\lambda(t)dt$$

with $\lambda(t)>0$ and $\Lambda(t_1,t_2)>0$ $(t_1>t_2)$.

The function $\lambda(t)$ (arrival rate at time t), also known as *parameter function* of the Poisson distribution [13], exhibits several maxima, each of which corresponds to a cluster of ray paths [Fig. 2(a)]. This is consistent with previous results discussed in [1]. Simulations at different density values show that longer paths occur at higher densities, since a larger amount of obstacles implies longer trajectories needed to reach the cell where the receiver is located. It should also be noted that arrivals exhibit a wider spread around the maxima, as the lattice density is increased. No sensible qualitative variations in the above picture are noted, on the other hand, for different values of the transmitter-receiver distance [Fig. 2(b)]. From Figs. 2(a) and 2(b) it is also deduced that the average amount of arrivals in the chosen observation window ($0\leqslant t\leqslant 20$ μs), obtained integrating $\lambda(\cdot)$ over the referred time interval, increases with density and decreases with separation.

Figure 3(a) shows the dependence of the total number of reflections $N(t)$ that rays undergo before reaching the receiver on the lattice density (linear interpolation among numerical results). Higher values of q result in a larger slope of $N(t)$. Figure 3(b) shows the dependence of $N(t)$ on the separation between transmitter and receiver. Again, no relevant qualititative changes occur for different transmitter-receiver distances.

The distribution of the overall path length of the rays contributing to the received signal for various values of the lattice density is shown in Fig. 4(a). Higher separations between transmitter and receiver imply longer paths, as expected, and this is shown in Fig. 4(b).

Figure 4(a) shows that two main regimes are possible. The early ray contributions cover comparable distances, which are weakly increasing with densities. Later ray-covered distances show a strong dependence on the lattice density q. Specifically, arrivals that immediately follow one another cover almost the same distance when the density is higher. This is quite easy to understand: a higher density and a larger number of obstacles do not allow a ray to go too far from the observation cell before redirecting it to the receiver again, so that the time interval between two successive ray transits through the same cell decreases with increasing q.

IV. CONCLUSIONS

A percolative model for indoor and urban channel propagation has been applied to a simple square-cell lattice 2D urban pulse propagation model. Arrival rates, number of reflections, and path lengths have been correlated with the density of occupied cells and the distance between transmitter and receiver.

As a next step, two main model refinements could be envisaged. The first one involves more accurate modeling of both the propagation channel in terms of cell geometry and distribution, and the EM interaction, e.g., by inclusion of diffraction. The second and more ambitious one seeks to obtain the urban-lattice EM impulse response (Green's function) in analytic closed form, on the basis of the above simplified percolative model. Work in both directions is underway.

ACKNOWLEDGMENT

Thanks are due to Professor Francesco Palmieri for stimulating the investigation and providing several interpretation hints.

[1] G.L. Turin, IEEE Trans. Veh. Tech. **21**, 1 (1972).
[2] G.L. Turin, IRE Nat. Conv. Rec., part 4, 154 (1956).
[3] H. Suzuki, IEEE Trans. Commun. **25**, 673 (1977).
[4] H. Suzuki, Doctoral Thesis, University of California, Berkeley, 1975 (unpublished).
[5] H. Hashemi, IEEE J. Sel. Areas Commun. **11**, 967 (1993).
[6] H. Hashemi, Proc. IEEE **81**, 943 (1993).
[7] G. James, *Geometrical Theory of Diffraction for Electromagnetic Waves* (P. Peregrinus, Stevenage, England, 1976).
[8] D. Stauffer and A. Aharony, *Introduction to Percolation Theory*, 2nd ed. (Taylor & Francis, London, 1992).
[9] G. Franceschetti, S. Marano, and F. Palmieri, IEEE Trans. Antennas Propag. **47**, 1393 (1999).
[10] S. Marano, F. Palmieri, and G. Franceschetti, J. Opt. Soc. Am. A **16**, 2459 (1999).
[11] R. Leubbers, IEEE Trans. Antennas Propag. **32**, 70 (1984).
[12] B. Bisceglia, G. Franceschetti, G. Mazzarella, I. Pinto, and C. Savarese, IEEE Trans. Antennas Propag. **36**, 1492 (1988).
[13] D. Snyder and M. Miller, *Random Point Processes in Time and Space*, 2nd ed. (Springer-Verlag, Berlin, 1991).