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Ponderomotive acceleration of electrons at the focus of high intensity lasers

Wei Yu,[1,2] M. Y. Yu,[1] J. X. Ma,[1,3] Z. M. Sheng,[4] J. Zhang,[5] H. Daido,[4] S. B. Liu,[5] Z. Z. Xu,[2] and R. X. Li[2]
[1]Institut für Theoretische Physik I, Ruhr-Universität Bochum, D-44780 Bochum, Germany
[2]Shanghai Institute of Optics and Fine Mechanics, Shanghai 201800, People's Republic of China
[3] University of Science and Technology of China, Hefei, Anhui 230027, People's Republic of China
[4]Institute of Laser Engineering, Osaka University, Osaka 565, Japan
[5]Institute of Physics, Beijing 100081, People's Republic of China

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Ponderomotive-force driven acceleration of an electron at the focus of a high-intensity short-pulse laser is considered using a model that accounts for its averaged drift motion but neglects its fast-varying quiver motion. It is shown that at relativistic laser intensities the ponderomotive acceleration mechanism can be significant even for an electron initially at rest. The latter can easily be driven out of the interaction region by the radial component of the ponderomotive force. For intensities above 10^{19} W μm^2/cm^2, energy gains in the range of MeV can be realized.

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In view of possible applications in the laser acceleration of electrons, the interaction of an electron with short-pulse tightly focused lasers has drawn much research interest [1–6] in the past few years. It is well known [7] that planar electromagnetic waves do not serve the purpose of electron acceleration. This is true even when light pressure effects are included, since when a wave overtakes an electron, the radiation pressure pushes the electron forward in the ascending (leading) front and backward in the descending (trailing) part of the laser pulse. As a result, the electron does not acquire net acceleration. However, if after being accelerated the electron leaves the interaction region before being decelerated, it will have gained energy. This scenario can occur in the focus region of a laser pulse. Hartemann *et al.* [1] proposed that if the electron quiver displacement imparted by the laser becomes comparable to the waist of the laser beam, the action of the laser field on the electron will be terminated when the accelerated electron leaves the focus region in the course of its quiver motion. As a result, the electron retains the energy gained during its encounter with the ascending front. The threshold power needed for this mechanism to operate can be estimated by equating the quiver displacement to the beam waist. One obtains

$$P \sim (0.21/\gamma_i^2)(R/\lambda)^4 \text{ TW}, \tag{1}$$

where γ_i is the initial electron energy in terms of the relativistic factor, and R and λ are the effective waist size and wavelength of the laser in the focus region, respectively. Equation (1) shows that the accelerated electron can escape from the interaction only if the laser is of very high power and/or is very tightly focused. That is, the field in the interaction region must be sufficiently high, especially if the electron is initially slow ($\gamma_i \sim 1$). For example, if $R/\lambda = 10$ and $\gamma_i = 1$ we have $P \sim 2100$ TW. Furthermore, according to this mechanism the accelerated electrons can escape from a linearly polarized laser pulse only in the direction of the polarization [1] as two oppositely directed jets.

On the other hand, experiments [2] and simulations [3,4] showed that rather effective acceleration of initially slow electrons can occur for laser powers well below the theoretical estimates [1,6]. Simulations [4] also indicated rather isotropic scattering: the electrons are expelled from the focus region mainly along the intensity gradient, with the polarization direction playing no particular role [4].

In the simulations [1,3,4] based on test particles, the trajectories of the individual electrons are followed. The computation is voluminous since both the high-frequency electron oscillations in the wave field and the relatively slowly varying electron response to the light pressure are included. Moreover, there is evidence [1,3,4] that at least for the initially slow electrons the energy gain is mainly in the averaged motion, with the high-frequency quiver motion nearly unaffected. By averaged motion we refer to the slowly varying (relative to the quiver motion) electron drift driven by the gradients in the radiation pressure, or the time-independent component of the ponderomotive force, in the focus region of a laser pulse [4,5]. There the radiation pressure on the electrons is much enhanced and is fairly isotropic. In particular, it contains a component that can accelerate electrons radially outward from the propagation axis. If the acceleration is sufficiently strong an electron can be driven out of the focus region and thus escape in nonaxial directions before it experiences the decelerating phase of the pulse. The question is whether the ponderomotive force is sufficiently strong to expel an electron from the focus region within a half-width of the laser pulse.

In this Rapid Communication we consider the ponderomotive-force driven acceleration mechanism by invoking a simple interaction model. Instead of following the exact electron trajectory, we follow only its drift motion by precluding the fast-varying quiver oscillations. An analysis based on the electron equation of motion in cylindrical coordinates near the focus of a laser pulse is presented. It is shown that the proposed acceleration mechanism is indeed effective. We found that in addition to the well-known relation between the scattering angle and escape energy [1], the radial momentum and energy gain of the electron is strongly correlated. In particular, as long as its final radial momentum is finite, an electron will achieve net energy gain. The maximum electron energy gain depends on the laser strength. Even for electrons initially at rest, the acceleration can be remarkable at relativistic laser intensities ($I\lambda^2 \geqslant 10^{18}$ W μm^2/cm^2, where I and λ are laser intensity and wavelength) and the affected electrons can easily leave the interaction region. For laser intensities above 10^{19} Wμm^2/cm^2, energy gains of the order of MeV can be realized.

The Lagrange equations for electron motion in a laser field can be written as [8]

$$d_t(\mathbf{p}-\mathbf{a}) = -c\nabla(\mathbf{a}\cdot\mathbf{u}), \tag{2}$$

$$d_t\gamma = \mathbf{u}\cdot\partial_t\mathbf{a}, \tag{3}$$

where $\mathbf{u}$ is the velocity of electrons normalized by c, $\mathbf{a}$ is the vector potential normalized by mc^2/e, $\mathbf{p}=\gamma\mathbf{u}$ is the normalized momentum, $\gamma=(1-u^2)^{-1/2}$ is the relativistic factor or normalized energy, and the ∇ in Eq. (2) acts on $\mathbf{a}$ only. Note that Eqs. (2) and (3) are exact.

As solution of the one-dimensional wave equation, the vector potential of a planar laser pulse can be expressed as $\mathbf{a}=\mathbf{a}(\eta)$, where $\eta=z-ct$, $a_z=0$, and we have used the Coulomb gauge. Since the vector potential is independent of x and y and depends on z and t only through η, Equations (2) and (3) lead to three integration constants, namely, $p_{x,y}-a_{x,y}=C_{1,2}$ and $\gamma-p_z=C_3$, from which we get the relation

$$\gamma=[1+C_3^2+(C_1+a_x)^2+(C_2+a_y)^2]/2C_3, \tag{4}$$

where the definition of γ has been used. For an electron entering the laser field with γ_i and leaving with γ_f, Eq. (4) yields $\gamma_i=\gamma_f$. That is, there is no net energy gain for the electron in a planar laser pulse.

However, around the peak of the laser pulse the electron has considerable energy, namely, up to $\gamma=\sqrt{1+p_\perp^2+p_z^2}=1+a^2/2$ for an initially stationary electron. Here $p_\perp=a$ is from the transverse quiver motion, driven roughly at the laser frequency ω by the electric force of laser, and $p_z=a^2/2$ is from the longitudinal motion driven by the magnetic or ponderomotive force. In circularly polarized light, the latter contains only a slowly varying drift toward the regions of low field intensity, while in linearly polarized light it contains a 2ω oscillation as well. We should emphasize that at relativistic intensities the electron drift is *slowly varying* but it is by no means slow, as is often stated in the literature. In fact, for $a>2$ the drift velocity is even larger than the quiver velocity. This crucial fact makes ponderomotive acceleration effective and promising.

For a focused laser pulse, the vector potential near the focus can be modeled as

$$\mathbf{a}=a_0\exp(-\eta^2/L^2-\rho^2/R^2)\hat{\mathbf{a}} \tag{5}$$

where $\hat{\mathbf{a}}=\cos(k\eta)\hat{\mathbf{x}}+\epsilon\sin(k\eta)\hat{\mathbf{y}}$, $\rho=\sqrt{x^2+y^2}$, L and R are the pulse width and minimum spot size, k is the wave number, a_0 the peak amplitude, $\epsilon=0$ for linear and $\epsilon=1$ for circular polarization. We have assumed that the Rayleigh length $z_R=kR^2/2$ is much larger than the pulse width L.

Thus, the vector potential depends on z and t only through $\eta=z-ct$, and its amplitude is independent of the azimuthal coordinate ϕ. That is, we have $\mathbf{a}=\mathbf{a}(\eta,\rho)$ and Eqs. (2) and (3) can be rewritten as

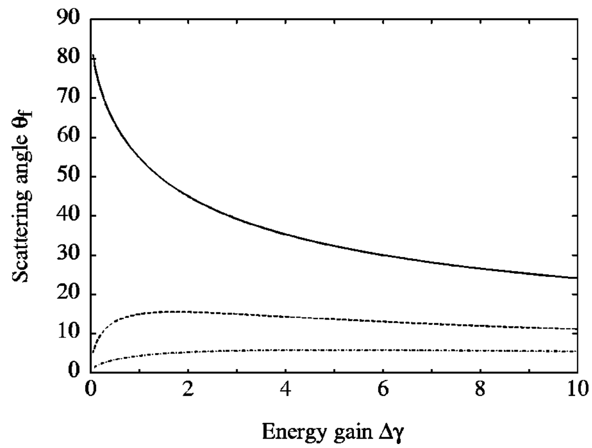

FIG. 1. Relation between the scattering angle θ_f (in degrees) and energy gain $\Delta\gamma$, where $\gamma_i=1$ (solid line), 2 (dashed line), and 5 (dotted line).

$$d_t(\gamma u_\rho - a_\rho) = -c\mathbf{u}\cdot\partial_\rho\mathbf{a}, \tag{6}$$

$$d_t(\gamma u_\phi - a_\phi)=0, \tag{7}$$

$$d_t(\gamma u_z)=d_t\gamma=-c\mathbf{u}\cdot\partial_\eta\mathbf{a}, \tag{8}$$

leading to two integration constants

$$\gamma u_\phi - a_\phi = C_1 \quad \text{and} \quad \gamma(1-u_z)=C_2, \tag{9}$$

corresponding to the cyclic properties of $\mathbf{a}$ mentioned above.

When the electron eventually leaves the laser field, from Eq. (9) and the definition of γ we obtain

$$\tan^2\theta_f=(2C_2\gamma_f-C_2^2-1)/(\gamma_f-C_2)^2, \tag{10}$$

$$\gamma_f^2u_{\rho f}^2-2C_2\gamma_f=-(1+C_1^2+C_2^2), \tag{11}$$

where γ_f is the normalized escape energy, and θ_f is the scattering angle measured from the direction of laser propagation. Equation (10) is the well-known relation between the scattering angle and the escape energy, and has been verified in numerical simulations and experiments [2–4,8,9].

The maximum scattering angle is

$$\theta_{f\,\max}=\tan^{-1}(C_2/\sqrt{1-C_2^2}), \tag{12}$$

which occurs for $\gamma_f=C_2^{-1}$. Note that Eqs. (10) and (12) are valid as long as the laser field depends on t and z only through η. Results from simulations with exact laser fields [i.e., without invoking $\mathbf{a}=\mathbf{a}(\eta,\rho)$] showed that slight deviation from this dependence may occur [4].

For an electron initially moving ahead of the laser pulse with $\mathbf{u}_i=\sqrt{1-\gamma_i^{-2}}\hat{\mathbf{z}}$, we have $C_1=0$ and $C_2=\gamma_i-\sqrt{\gamma_i^2-1}$. Figure 1 shows the relation between the scattering angle θ_f and the energy gain γ_f-1, where $\gamma_i=1$ (solid line), 2 (dashed line), and 5 (dotted line). For electrons initially at rest ($\gamma_i=1$), the maximum scattering angle can be up to 90°. Moreover, the scattering angle decreases steadily with increasing escape energy. For electrons with higher initial energy, the scattering will be confined to smaller angles.

Using the same initial conditions, Eq. (11) leads to another important result, namely,

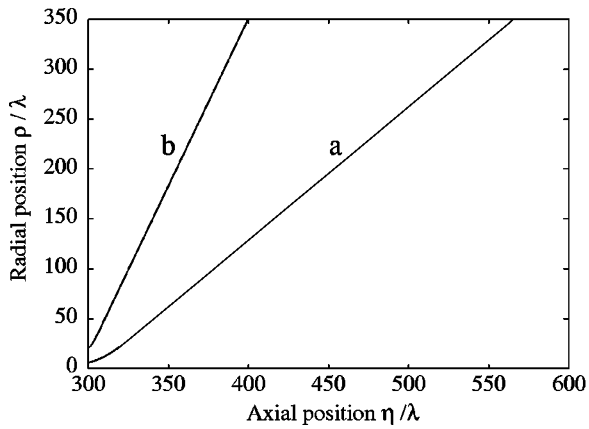

FIG. 2. Drift of electrons starting from radial positions $\rho_i = 0.3R$ (curve a) and R (curve b), for $\gamma_i = 1$, $\epsilon = 1$, $a_0 = 2$, $L = 30\lambda$, and $R = 20\lambda$.

$$\Delta\gamma \equiv \gamma_f - \gamma_i = \gamma_f^2 u_{\rho f}^2 / 2C_2, \tag{13}$$

for the energy gain. It shows that the electron will achieve net energy gain as long as its final radial momentum $\gamma_f u_{\rho f}$ is nonzero. This conclusion demonstrates the importance of the radial drift motion: without it there can be no net acceleration, as in the case for a planar wave field.

By setting $\mathbf{u} = \mathbf{v} + \mathbf{a}/\gamma$ and using the above mentioned initial conditions, we can rewrite Eqs. (6)–(8) as

$$d_t(\gamma v_\rho) = -(c/2\gamma)\partial_\rho a^2 - c v_\rho \partial_\rho a_\rho, \tag{14}$$

$$d_t(\gamma v_z) = d_t \gamma = -(c/2\gamma)\partial_\eta a^2 - c v_\rho \partial_\eta a_\rho, \tag{15}$$

where the electron quiver motion driven by **a** has been precluded. Other fast-varying terms in Eqs. (14) and (15) also have little contribution and can thus be ignored.

The acceleration of initially stationary electrons by a circularly polarized laser is studied by solving Eqs. (14) and (15). In this case, the field amplitude $a^2 = a_0^2 \exp(-2\eta^2/L^2 - 2\rho^2/R^2)$ and the ponderomotive force are slowly varying. Figure 2 shows the drift trajectories of two electrons starting at rest from the axial position $\eta = 300$ and the transverse positions $\rho_i = 0.3R$ and R, respectively. The other parameters are $a_0 = 2$, $\gamma_i = 1$, $\epsilon = 1$, $L = 30\lambda$, and $R = 20\lambda$. One can see that the electrons, initially stationary, are pushed away from the propagation axis by the radial component of the radiation pressure. They are also accelerated along the propagation axis. The radial drift out of the focus region terminates the laser action on the electron before it experiences the descending phase. We note that for the parameters under consideration, the quiver displacement of the electron is well below the focus spot size.

Figure 3 shows the dependence of the energy gain $\Delta\gamma$ on the initial transverse position ρ_i for laser strengths $a_0 = 2$ (○), 3 (∗), and 4 (+), respectively. The other parameters are the same as that for Fig. 2, i.e., $\gamma_i = 1$, $\epsilon = 1$, $L = 30\lambda$, and $R = 20\lambda$. One can see that for any given laser strength, there is an optimal initial radial position for which an electron gains maximum energy.

Figure 4 shows the dependence of the maximum energy gain $\Delta\gamma_{\max}$ on the laser strength a_0 for the same parameters as in Fig. 2. Note that the proposed acceleration mechanism

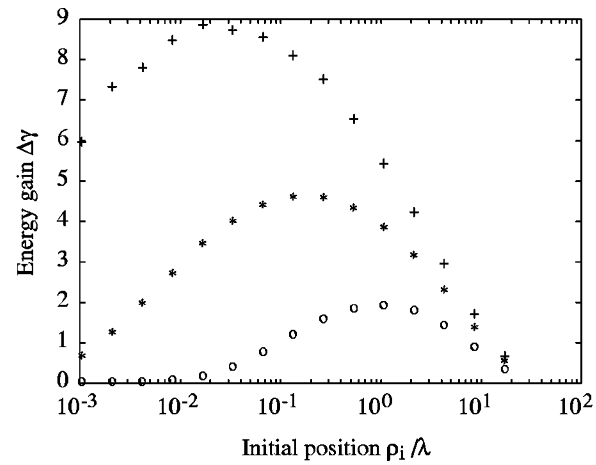

FIG. 3. Dependence of energy gain $\Delta\gamma$ on the initial radial position ρ_i, where $\gamma_i = 1$, $\epsilon = 1$, $L = 30\lambda$, $R = 20\lambda$, for laser strength $a_0 = 2$(○), 3(∗), and 4(+), respectively.

becomes noticeable only in the relativistic regime $a_0 > 1$, or $I\lambda^2 > 10^{18}$ W μm^2/cm^2. At intensities above 10^{19} W μm^2/cm^2, energy gain in the MeV level can occur even for electrons initially at rest.

We have shown that, although on a timescale greater than that of the electron quiver motion, the ponderomotive force of a tightly focused high-intensity laser can lead to net acceleration and scattering of initially slow electrons. This occurs because in the relativistic regime the electrons accelerated by the ponderomotive force can acquire sufficient radial drift speed to leave the interaction region before encountering the decelerating phase of the laser pulse. Moreover, since the direction of the ponderomotive force depends on the field gradients in the focus region, the scattering is quite isotropic. In particular, it is independent of the direction of polarization of the laser light.

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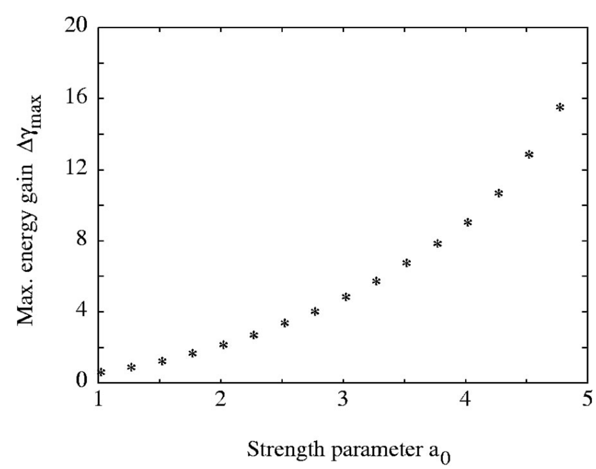

FIG. 4. Dependence of the maximum energy gain $\Delta\gamma_{\max}$ on the laser strength a_0, for $\gamma_i = 1$, $\epsilon = 1$, $L = 30\lambda$, and $R = 20\lambda$.

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