Concepts in computation of preequilibrium neutron emission from heavy ion reactions

P. K. Sarkar*
HP Unit, Variable Energy Cyclotron Centre, 1/AF, Bidhannagar, Calcutta 700064, India

Maitreyee Nandy†
Saha Institute of Nuclear Physics, 1/AF, Bidhannagar, Calcutta 700064, India
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Integral forms of the Boltzmann-like transport equations are derived to treat neutron emissions from heavy ion collisions. The derivations are based on the probability of contribution (score) from nucleon occupied phase space bins to the double differential emission spectra. Two-body interactions among the nucleons in ''hot spot'' and ''cold spot'' regions of the composite system resulting in creation, annihilation, or energy redistribution of the colliding particles are considered. The resulting equations are solved numerically in energy-angle bins to study thermal and kinematic equilibrium of the target + projectile (^{165}Ho+^{20}Ne) excited system at different projectile energies (10 MeV/A to 30 MeV/A).

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I. INTRODUCTION

Calculation of the energy-angle distribution of ejectiles from heavy ion collisions (HIC's) remains a subject of current interest. A number of phenomenological and theoretical proposals have been made [1–9] for this purpose in the last few decades. The issue, however, is yet to be resolved satisfactorily. Several attempts have been made to reveal the underlying physical concepts in HIC's. We try to understand the computational concepts behind such theoretical estimations. Among the different theoretical approaches describing the dynamics of HIC's, we will confine our attention to those based on some Boltzmann-like transport equations [5–8]. These formalisms describe the reaction dynamics through the evolution of a one-body distribution function representing the mean occupation in an element of phase space volume. They have been successful in describing global features such as the collective flow, inclusive cross sections, single particle spectra, etc., of intermediate energy HIC reactions. The transport equations solved so far for this purpose are integro-differential in form.

In recent work [10] we solved the integral form of the transport equation simulteneously in the ''cold spot'' and the ''hot spot'' parts of the composite system to predict the neutron emission spectra with good accuracy. In [10] we provided only intuitive but no formal derivation of the equations. In the present work we attempt to derive the equations from a computational viewpoint based on concepts of probability. To do so we introduce the concept of ''computer score,'' which is the expected contribution of a particle to a computational array set up to collect and sort particles ejected from the HIC in specified energy-angle bins. Initially, equations are derived for the evolution with the number of collisions, ignoring spatial transition. Consequently, time dependence, spatial transport, and mean field effects are taken into account. Thereafter, we address the issue of equilibration; that is, after how many binary collisions the computations may be swiched over to statistical evaporative decay from the preequilibrium (PEQ) emissions. For that purpose we study the thermal and kinematic equilibrium of the composite system evolving with the number of binary interaction stages. The composite system considered for numerical solutions of the integral equations formulated in energy-angle bins is ^{165}Ho+^{20}Ne. The calculations are repeated for different projectile (^{20}Ne) energies in the range 10 MeV/A to 30 MeV/A.

II. DERIVATIONS

After the fusion of two nuclei we consider a composite system subdivided into two distinct temperature zones—a hot spot and a cold spot. In the hot spot the momentum of the particles is described by a finite temperature Fermi distribution while in the cold spot a zero temperature Fermi distribution is considered. For computational purposes the particles are binned according to their energy and angle (E,Ω) inside the nucleus. The corresponding energy-angle bin outside the nucleus is denoted by (ϵ,ω) where

$$\epsilon = E - E_F - S_\nu .$$

Here E_F is the Fermi energy and S_ν is the separation energy of ν type particles with respect to the composite (target + projectile) nucleus. The direction ω outside the nucleus is related to the direction Ω inside it through the effects of refraction at the nuclear surface. Each energy-angle bin (E,Ω) is represented by a single particle, assigned an importance w which is the average number of particles occupying that particular bin. That is, if n be the total number of particles and $P(E,\Omega)$ be the probability of a particle occupying the bin (E,Ω), then $w=nP(E,\Omega)$. When a particle of importance w escapes from the bin (E,Ω) to the outside, it gives a score $s=w$ in the corresponding bin (ϵ,ω) with probability $P_e(E)$. The probability of escape from the composite system, $P_e(E)$, is given by

*Electronic address: pks@veccal.ernet.in
†Electronic address: mnandy@hp2.saha.ernet.in

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$$P_e(E)=\frac{\lambda_C(E)}{\lambda_C(E)+\lambda_t(E)}, \tag{1}$$

where $\lambda_C(E)$ is the rate of emission of a particle with energy E and $\lambda_t(E)$ is the rate of two-body interactions of the particle with other nucleons in the system. Here, the score s is defined as the mean or expected contribution by the particle inside the composite system to the quantity of interest, which, in the present case, is the double differential emission multiplicity. A particle therefore contributes a score only when it escapes (is emitted from) the system. The score is a variable having any value between 0 and ∞ (the situation of a negative score does not arise here, otherwise s could have values ranging from $-\infty$ to ∞).

The relaxation process or the energy sharing among the nucleons in the composite system takes place through a series of binary or two-body interactions. The importance, energy, and direction of the particle keep changing in each interaction but each energy-angle bin is always represented by a single particle. In any two-body interaction one of the following can happen.

(i) A particle-hole pair can be created, which occurs with probability P^+. So, finally we have two particles and the importance of the representative particle changes to $w'=2w$ (with probability P^+).

(ii) The particle can be annihilated and the corresponding probability is P^-. Finally, we have zero particles in this case and the importance of the particle becomes $w'=0$ (with probability P^-).

(iii) Neither creation nor annihilation takes place, instead only the energy and the direction of the particle change in the scattering. This occurs with probability P^0. The importance of the particle remains unchanged, i.e., $w'=w$ (with probability P^0).

It may be noted that $P^+ + P^- + P^0 = 1$. Therefore, for a particle going into a collision with importance w, the average value of its importance after the interaction is $w'_{av} = w(2P^+ + P^0) = w(P^+ + P^- + P^0 + P^+ - P^-) = w(1 + P^+ - P^-)$.

A. Score accumulation probability

Since the computation is aimed at determining the energy-angle distribution of emitted particles, we formulate the score accumulation probabilities for each energy-angle bin. The concept of such score accumulation probabilities has been utilized earlier to analyze Monte Carlo transport problems [11,12]. We define $\psi_N(E,\Omega,w,s)ds$ as the probability that a particle now existing with energy E, direction Ω, and importance w will give a score of ds around s in the corresponding outside bin (ϵ_N,ω_N) exactly after N collisions when its energy-angle inside will become (E_N,Ω_N).

We then have for particles escaping without any collision inside (i.e., $N=0$)

$$\psi_0(E_0,\Omega_0,w_0,s)ds = P_e(E_0)\,\delta(s-w_0)ds. \tag{2}$$

Here, $P_e(E_0)$ gives the probability that the particle with energy E_0 escapes the system and w_0 denotes the importance of a particle occupying the bin (E_0,Ω_0). The δ function ensures that the score is unique and is equal to w_0.

A particle suffering only one collision within the system can contribute to the score in the following manner. It remains within the system with probability $[1-P_e(E_0)]$ (nonescape probability) and collides inside the system and then emerges from the collision with energy dE_1 around E_1 and direction $d\Omega_1$ around Ω_1 with probability $K(E_0,\Omega_0 \rightarrow E_1,\Omega_1)dE_1 d\Omega_1$. Here $K(E_0,\Omega_0\rightarrow E_1,\Omega_1)dE_1 d\Omega_1$ is the scattering kernel (see Sec. II C) and conserves momentum as well as the Pauli exclusion principle during each nucleon-nucleon collision. The importance of the particle changes to $w_1=w_0(1+P_1^+-P_1^-)$. Finally, the particle escapes with probability $P_e(E_1)$ and gives a score w_1 in the outside bin (ϵ_1,ω_1). We therefore have

$$\begin{aligned}\psi_1(E_0,\Omega_0,w_0,s)ds &= [1-P_e(E_0)]\int dE_1 \int d\Omega_1 \\ &\quad \times K(E_0,\Omega_0\rightarrow E_1,\Omega_1)P_e(E_1)\,\delta(s-w_1)ds.\end{aligned} \tag{3}$$

Similarly, for a particle making exactly N collisions, first it makes $N-1$ collisions and remains in the system without escaping, and then undergoes the Nth collision, escapes from the system, and gives a score equal to its final weight $w_N = w_0\Pi_{i=1}^N f_i$ where $f_i=(1+P_i^+-P_i^-)$. We can now write the equation as

$$\begin{aligned}\psi_N(E_0,\Omega_0,w_0,s)ds &= [1-P_e(E_0)]\prod_{i=1}^{N-1}\left(\int dE_i \int d\Omega_i K_{i-1,i}T_i\right) \\ &\quad \times \int dE_N \int d\Omega_N K(E_{N-1},\Omega_{N-1}\rightarrow E_N,\Omega_N)P_e(E_N) \\ &\quad \times \delta(s-w_N)ds,\end{aligned} \tag{4}$$

where $T_i=1-P_e(E_i)$ and $K_{i-1,i}=K(E_{i-1},\Omega_{i-1}\rightarrow E_i,\Omega_i)$.

Now using Eq. (2) one can rewrite Eq. (3) as

$$\begin{aligned}\psi_1(E_0,\Omega_0,w_0,s)ds &= [1-P_e(E_0)]\int dE_1 \int d\Omega_1 \\ &\quad \times K(E_0,\Omega_0\rightarrow E_1,\Omega_1)\psi_0(E_1,\Omega_1,w_1,s)ds.\end{aligned} \tag{5}$$

Similarly, Eq. (4) can be rewritten as

$$\begin{aligned}\psi_N(E_0,\Omega_0,w_0,s)ds &= [1-P_e(E_0)]\int dE_1 \int d\Omega_1 \\ &\quad \times K(E_0,\Omega_0\rightarrow E_1,\Omega_1)\psi_{N-1}(E_1,\Omega_1,w_1,s)ds.\end{aligned} \tag{6}$$

Now we define the score accumulation probability as

$$\psi(E_0,\Omega_0,w_0,s)ds = \sum_{N=0}^{\infty}\psi_N(E_0,\Omega_0,w_0,s)ds. \tag{7}$$

Here, $\psi(E_0,\Omega_0,w_0,s)ds$ is the probability that a particle emerging from a collision or introduced into the system with energy E_0, direction Ω_0, and importance w_0 will give a total score of ds around s in all its future events.

Summing both sides of Eq. (6) from $N=1$ to ∞ we get

$$\sum_{N=1}^{\infty}\psi_N(E_0,\Omega_0,w_0,s)ds = [1-P_e(E_0)]\int dE_1\int d\Omega_1 K(E_0,\Omega_0\rightarrow E_1,\Omega_1) \times\sum_{N=1}^{\infty}\psi_{N-1}(E_1,\Omega_1,w_1,s)ds. \quad (8)$$

Adding $\psi_0(E_0,\Omega_0,w_0,s)ds$ on both sides of Eq. (8) and using Eq. (7) we get,

$$\psi(E_0,\Omega_0,w_0,s)ds = P_e(E_0)\,\delta(s-w_0)ds+[1-P_e(E_0)]\int dE_1\int d\Omega_1 \times K(E_0,\Omega_0\rightarrow E_1,\Omega_1)\psi(E_1,\Omega_1,w_1,s)ds. \quad (9)$$

B. Expected emission multiplicity

The mean or expected score contributed by a particle with (E_0,Ω_0,w_0) can be obtained as

$$M(E_0,\Omega_0,w_0)=\int_0^{\infty}\psi(E_0,\Omega_0,w_0,s)s\,ds. \quad (10)$$

Multiplying Eq. (9) by s and integrating, we get

$$M(E_0,\Omega_0,w_0) = P_e(E_0)w_0+[1-P_e(E_0)]\int dE_1\int d\Omega_1 \times K(E_0,\Omega_0\rightarrow E_1,\Omega_1)M(E_1,\Omega_1,w_1). \quad (11)$$

Now we can write $M(E,\Omega,mw)=mM(E,\Omega,w)$, which implies that the mean score contributed by a particle of importance mw is m times the mean score contributed by a particle of importance w [11,12]. We have $w_1=(1+P_1^+-P_1^-)w_0$. Therefore, we can write Eq. (11) as

$$M(E_0,\Omega_0,w_0)=P_e(E_0)w_0+[1-P_e(E_0)] \times\int dE_1\int d\Omega_1(1+P_1^+-P_1^-) \times K(E_0,\Omega_0\rightarrow E_1,\Omega_1)M(E_1,\Omega_1,w_0). \quad (12)$$

Equation (12) is an integral equation of the mean score contributed by a particle with energy E_0, direction Ω_0, and importance w_0. This integral equation can be solved for any (E_0,Ω_0) if w_0 is known. Now, if n_0 excited particles are in the system with energy-angle distribution $P_0(E_0,\Omega_0)$ then w_0 at (E_0,Ω_0) can be obtained as $w_0=n_0P_0(E_0,\Omega_0)$. Calculation of the initial number of particles n_0 and their distribution $P_0(E_0,\Omega_0)$ is described in Sec. II D. Equation (12) has been used in [10] as well as in the present work to obtain numerical results. However, we go further in our theoretical analysis to include time dependence, spatial transport, and mean field effects to make the formulation of the integral equation complete.

1. Inclusion of time dependence

It may be noted here that time evolution of the system is not explicitly considered in this formulation. Instead we have considered the evolution of the system with the number of collisions. Time dependence, however, can easily be incorporated in the formulations. To do that we have to consider time as an explicit variable and have to replace the escape and collision probabilities with escape and collision rates. Equation (12) will then appear as

$$M(E_0,\Omega_0,w_0,t_0)=\lambda_C(E_0)\Delta t w_0+\lambda_t(E_0)\Delta t\int dE_1\int d\Omega_1 \times(1+P_1^+-P_1^-)\times K(E_0,\Omega_0\rightarrow E_1,\Omega_1) \times M(E_1,\Omega_1,w_0,t_0+\Delta t). \quad (13)$$

Here, $M(E_0,\Omega_0,w_0,t_0)$ is the mean score contributed by a particle existing with energy E_0, direction Ω_0, and importance w_0 during a time interval Δt after t_0.

Evolution with a number of interactions is not an observable, i.e., M_N for any N cannot be observed as it can at any instant of time. However, M_N summed over all N is physically observable and gives the double differential emission multiplicity. There is at least one advantage of using the number of collisions in place of time—it results in less computational effort. Since no scattering happens between two collisions, we can evaluate the integrals in Eq. (12) only at integer values of N. Time evolution, on the other hand, requires a large amount of calculation as in this case the scattering and emission have to be evaluated for a large number of Δt time bins.

2. Inclusion of spatial transport

So far, we have not considered spatial transition of the particles as this is not required when one defines the emission probability as given by Eq. (1). To consider spatial transition we have to define a transport kernel as $T(E,\Omega,r\rightarrow r')dr'$, the probability that a particle existing at r with energy and direction (E,Ω) will make a collision at dr' around r' before making any other collision. We also have to define a volume V of the composite system such that if r' is not within V then we consider emission of the particle. It may be noted that we have explicitly assumed that the energy and direction of a particle do not change between two collisions. This assumption is not valid when we consider mean field effects later.

We have

$$\int_{r'\in V}T(E,\Omega,r\rightarrow r')dr'+\int_{r'\notin V}T(E,\Omega,r\rightarrow r')dr'=1, \quad (14)$$

where the first term in the left side of Eq. (14) is the probability that the particle remains within the composite system and the second term gives the probability of escape. The second term now replaces $P_e(E)$ as given in Eq. (1).

If l is the distance between r and r' then the probability that a particle travels the distance without having a collision is given by

$$p(l)=\exp[-\Sigma(E)l], \tag{15}$$

where $\Sigma(E)$ is the total macroscopic nucleon-nucleon interaction cross section at energy E and is the reciprocal of the nucleon mean free path inside nuclear matter. The macroscopic cross section has the dimension of inverse of length and can be calculated as the product of the nucleon-nucleon scattering cross section and the number of nucleons per unit volume.

Using this transport kernel Eq. (12) can be rewritten as

$$\begin{aligned}M(r,E_0,\Omega_0,w_0)=&\int_{r'\notin V}T(E_0,\Omega_0,r\to r')dr'w_0\\&+\int_{r'\in V}T(E_0,\Omega_0,r\to r')dr'\\&\times\int dE_1\int d\Omega_1\times(1+P_1^+-P_1^-)\\&\times K(E_0,\Omega_0\to E_1,\Omega_1)M(r',E_1,\Omega_1,w_0).\end{aligned} \tag{16}$$

Solution of this equation requires spatial coordinates to be included along with energy and angle, thus resulting in energy-angle-space bins.

A more generalized version of Eq. (15) can be written as

$$p(l)=\exp\left(-\int_0^l\Sigma(E,l')dl'\right), \tag{17}$$

where the macroscopic cross section keeps changing along the path length of the particle. This can happen when there is a variation in the number density of particles with coordinate position or there is an external force acting on the particle to change its energy and direction between collisions (e.g., the mean field effect).

3. Inclusion of mean field effects

If we wish to account for the mean field effect we have to modify the transport kernel in Eq. (16) to include change in energy and direction during spatial transition. We obtain

$$\begin{aligned}M(r,E_0,\Omega_0,w_0)=&\int_{r'\notin V}T^+(E_0,\Omega_0,r\to E_1',\Omega_1',r')dr'w_0\\&+\int_{r'\in V}T^*(E_0,\Omega_0,r\to E_1',\Omega_1',r')dr'\\&\times\int dE_1\int d\Omega_1(1+P_1^+-P_1^-)\\&\times K(E_1',\Omega_1'\to E_1,\Omega_1)M(r',E_1,\Omega_1,w_0).\end{aligned} \tag{18}$$

Now if we consider that the mean field effect is confined within the nuclear volume,

$$\begin{aligned}&\int_{r'\notin V}T^+(E_0,\Omega_0,r\to E_1',\Omega_1',r')dr'\\&\quad=T^*(E_0,\Omega_0,r\to E_1',\Omega_1',r'')\\&\qquad\times\int_{r'\notin V}T(E_1',\Omega_1',r''\to r')dr'.\end{aligned} \tag{19}$$

where r'' is a point on the boundary of the volume V.

In case the mean field effects extend beyond the nuclear volume then

$$\begin{aligned}&\int_{r'\notin V}T^+(E_0,\Omega_0,r\to E_1',\Omega_1',r')dr'\\&\quad=\int_{r'\notin V}T^*(E_0,\Omega_0,r\to E_1',\Omega_1',r')dr'.\end{aligned} \tag{20}$$

Here, $T^*(E_0,\Omega_0,r\to E_1',\Omega_1',r')dr'$ is defined as the probability that a particle at r with energy and direction (E_0,Ω_0) will have a collision within dr' around r' before having any other collision while during its transition from r to r' its energy and direction change from (E_0,Ω_0) to (E_1',Ω_1').

Now, if l is the distance between r and r' and $U[\rho(r'')]$ is the external force acting on the particle at any point r'', then

$$E_1'=E_0+\int_r^{r'}U[\rho(r'')]dr''=E_0+\overline{U}l, \tag{21}$$

where $\rho(r)$ is the nucleon density at r and $\overline{U}$ is the average external force the particle experiences during its transition from r to r'. $\overline{U}$ depends on the nucleon density surrounding the flight path of the particle as well as on the importance of the particle.

C. Description of the scattering kernel

As mentioned earlier the composite system is subdivided into two systems—a hot spot and a cold spot. Partial equilibrium in each subsystem is assumed to describe them by two different sets of thermodynamic parameters. The effect of nuclear excitation is included in the hot spot, which contains all the excited particles and holes and is responsible for emission of particles. Nucleon motion inside the hot spot is described by a finite temperature Fermi distribution, while that in the cold spot is described by a zero temperature Fermi distribution.

In the composite system, two-body scattering may take place either between two nucleons in the hot spot or between one nucleon in the hot spot and another in the cold spot. Accordingly, the scattering kernel that includes the Pauli blocking effects is divided into two parts,

$$\begin{aligned}K(E,\Omega\to E',\Omega')=&\xi K_\beta^H(E,\Omega\to E',\Omega')\\&+(1-\xi)K^C(E,\Omega\to E',\Omega'),\end{aligned} \tag{22}$$

where ξ is the probability of an interaction taking place inside the hot spot and is equal to the ratio of the number of nucleons in the hot spot to the total number of nucleons in the composite system.

The scattering kernel in the hot spot, K_β^H, which depends on the temperature ($=1/\beta$) of the hot spot, is further subdivided into three parts corresponding to creation (with probability P_β^+), annihilation (with probability P_β^-), and redistribution (with probability P_β^0). A complete description of these kernels can be found in [13].

On the other hand, K^C, the scattering kernel between one nucleon in the hot spot and another in the cold spot, can be described by the standard Kikuchi-Kawai [14] scattering. This scattering results only in the creation of a particle-hole pair since all levels below the Fermi energy in the cold spot are filled up.

D. Number and distribution of initial excited particles

The number of excited particles just at the time of fusion of two nuclei is calculated by considering the overlap of three momentum spheres (target, projectile, and composite system) [15,16]. The radius of each of the three spheres equals the Fermi momentum. The momentum sphere of the composite system is centered at the center of mass of the system. The centers of the momentum spheres of the target and the projectile are separated from that of the composite system by their respective c.m. momenta per nucleon. From the overlap of the three spheres the initial number of excited particles is determined as the number of nucleons in the target or the projectile spheres that remain outside the momentum sphere of the composite system. The momentum volume remaining outside the Fermi sphere of the composite nucleus represents the momentum volume of the excited particles. Then the initial number of excited particles from the projectile or target is given by

$$n_{0i}=\overline{n}_i\int d\mathbf{p} \tag{23}$$

with the integration carried out over appropriate limits of the momentum vector $\mathbf{p}$. From the Fermi gas model the density of the momentum states ($\overline{n}_i$) is given by

$$\overline{n}_i=\frac{V_i}{2\pi^3h^3}=\frac{2r_0^3A_i}{3\pi^2h^3}, \tag{24}$$

where i stands for projectile or target and V_i is the relevant volume. Thus n_{0T} is the number of initial excited particles contributed by the target and n_{0P} is that from the projectile.

Evaluation of $P_0(E,\Omega)dE\,d\Omega$ requires the energy-angle distribution of the initial excited particles. This is obtained by coupling the incident velocity of the projectile with the Fermi velocities of the constituent nucleons for the initial excited particles contributed by the projectile. When the target-projectile fusion takes place, an excited system is formed and the constituent nucleons move about with a finite temperature ($T_0=1/\beta_0$) Fermi distribution. After coupling the projectile velocity to this distribution we get [10],

$$P_{0P}(E_0,\Omega_0)dE\,d\Omega_0=\frac{3}{8\pi E_F^{3/2}}\frac{\sqrt{E_0}}{\sqrt{E_0+E_{inc}-2\sqrt{E_0E_{inc}}\cos\eta}}\frac{dE_0d\Omega_0}{1+\exp[\beta_0(E_0+E_{inc}-2\sqrt{E_0E_{inc}}\cos\eta-\mu_0)]}, \tag{25}$$

where μ_0 is the corresponding chemical potential.

Since the target is at rest in the laboratory system, P_{0T} is given by a zero temperature Fermi distribution with isotropic angular distribution. It may be noted at this point that all energies and angles in the present calculations are defined in the laboratory frame.

E. Calculation of nuclear temperature

Along with evaluation of the initial β_0 (reciprocal of the temperature), β_N at each stage of interaction, N, needs to be calculated, as part of the scattering kernel K depends on this parameter. Estimation of β_N is done from the entropy of the system, S_N, as

$$\beta_N=\frac{dS_N}{dE_C}=\frac{d}{dE_C}[\ln\rho_N(E_C)], \tag{26}$$

where $\rho_N(E_C)$ is the average density of states at excitation E_C after N binary interactions. $\rho_N(E_C)$ is obtained as the statistically weighted average of the partial level densities $\rho_n(E_C)$ of all possible n exciton (particle + hole) states that may be formed after N two-body interactions. Now, we define the average number of excitons after N collisions, $\overline{n}_N$, such that

$$\rho_N(E_C)=\rho_{\overline{n}_N}^-(E_C). \tag{27}$$

For $N=0$, $\overline{n}_N=n_0$. For $N>0$ we define $\overline{n}_N$ by the following recursion relation:

$$\overline{n}_N=\overline{n}_{N-1}(1+P_N^+-P_N^-). \tag{28}$$

We use the expression for $\rho_n(E_C)$ given by Ericson [17],

$$\rho_n(E_C)=\frac{g^nE_C^{n-1}}{p!h!(n-1)!}, \tag{29}$$

where p and h are the number of excited particles and holes in the n-exciton state and g is the single particle level density. Using Eqs. (26), (27), and (29), β_N can be obtained as

$$\beta_N=\frac{\overline{n}_N-1}{E_C}. \tag{30}$$

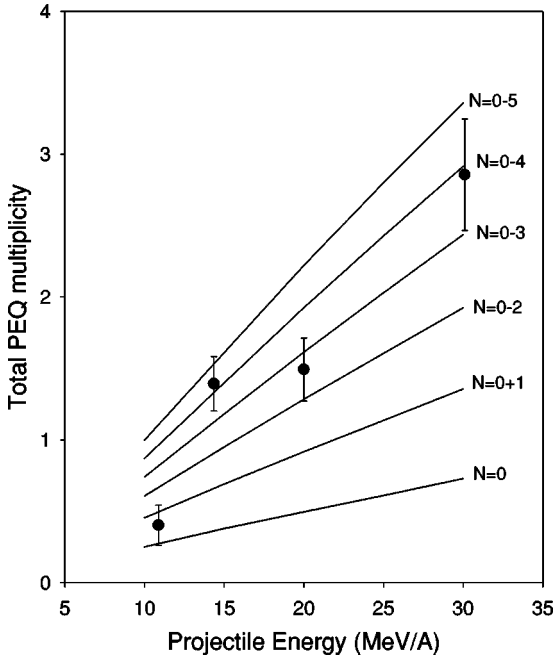

FIG. 1. Plot of PEQ neutron multiplicity against projectile energy (in MeV/nucleon) for the reaction ^{20}Ne+^{165}Ho. Solid points with error bars are experimental data [22,23]; the present calculations for contributions up to different interaction stages (indicated beside each curve) are shown as solid lines.

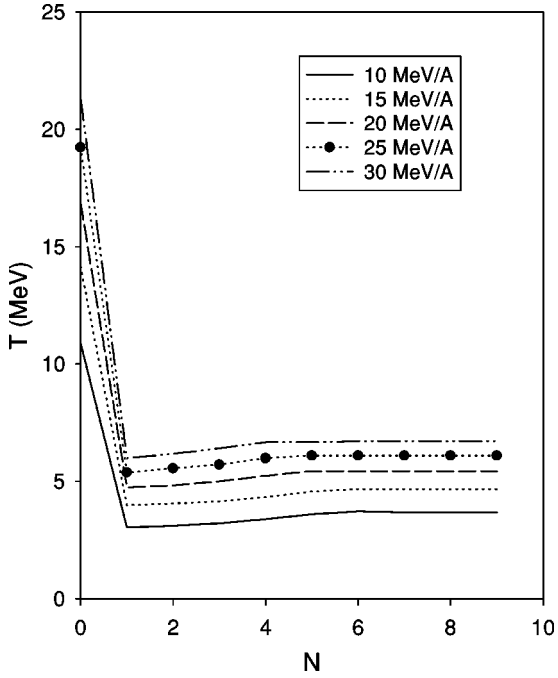

FIG. 2. Plot of the hot spot temperature T against interaction stage N at different projectile energies for the reaction ^{20}Ne $+^{165}$Ho.

The chemical potential μ_N is determined using the method outlined by Kittel [18].

III. EQUILIBRATION OF THE COMPOSITE SYSTEM

We now try to find out at what stage of the interaction process the composite system reaches thermal and kinematic equilibrium. We first attempt to evaluate this from the emission of the particles and accordingly compare experimentally observed total PEQ neutron multiplicities with those obtained from our calculations using Eq. (12). We have used in our calculations the emission rate $\lambda_C(E)$ as given by [19]

$$\lambda_C(E)=\frac{(2S+1)mE\sigma_{inv}(E)}{\pi^2\bar{h}^3 g}, \tag{31}$$

where S and m are the spin and mass of the ejectile, g is the single particle level density, and $\bar{h}=h/2\pi$. $\sigma_{inv}(E)$ is the cross section for the reverse reaction, which is calculated by the method of Chatterjee *et al.* [20]. In estimating the two-body interaction rate $\lambda_t(E)$, we have used the empirical relation of Blann [21],

$$\lambda_t(E)=[(1.4\times 10^{21})(E+B)-(6.0\times 10^{18})(E+B)^2]/k, \tag{32}$$

where B is the binding energy of the ejectile and k is an adjustable parameter for introducing the Pauli blocking effect. We have chosen $k=1.0$ in our calculations.

Figure 1 gives a plot of total PEQ neutron multiplicity against energy per nucleon of the projectile for the ^{165}Ho $+^{20}$Ne reaction. The solid circles with error bars are experimental observations [22,23] and the solid lines give calculated results with contributions up to different stages of interaction N (as noted beside each curve). It can be observed that with increasing projectile energy more interaction stages need to be considered to account for the total PEQ multiplicity. Apparently, this conclusion is not counterintuitive as with increasing projectile energy the temperature of the initial hot spot becomes higher and it is likely that more interactions may be needed to dissipate this larger amount of energy among other nucleons to attain thermal equilibrium.

To investigate this point further we plot in Fig. 2 the variation in nuclear temperature T of the hot spot with different interaction stages N. This is done for different projectile energies of the same reaction as mentioned above. It is observed that the temperature of the hot spot drops sharply at the first interaction stage and stabilizes more or less thereafter. The temperatures at which such thermal equilibrium is reached is higher for higher projectile energies. There is no indication that at higher projectile energies more interactions are needed to attain thermal equilibrium. In fact, the cooling rates ($\Delta T/\Delta N$) with N also behave in a similar fashion (see Fig. 3). In this case, the initial cooling rate, though different for different projectile energies, stabilizes to almost the same value after the first interaction stage.

Next, we give in Fig. 4 a plot of the variation of the chemical potential with N. This chemical potential was calculated from the nuclear temperature [18] to describe the finite temperature Fermi distribution of the nucleon momenta in the hot spot. Here also we see that the initial values of the chemical potential differ greatly for different projectile energies but after the first interaction stage they stabilize to values that are close to each other. Thus it may be concluded that for these projectile energies (10 Mev/A to 30 MeV/A) thermal equilibrium is reached after the first interaction stage.

Now we look into the status of kinematic equilibrium of

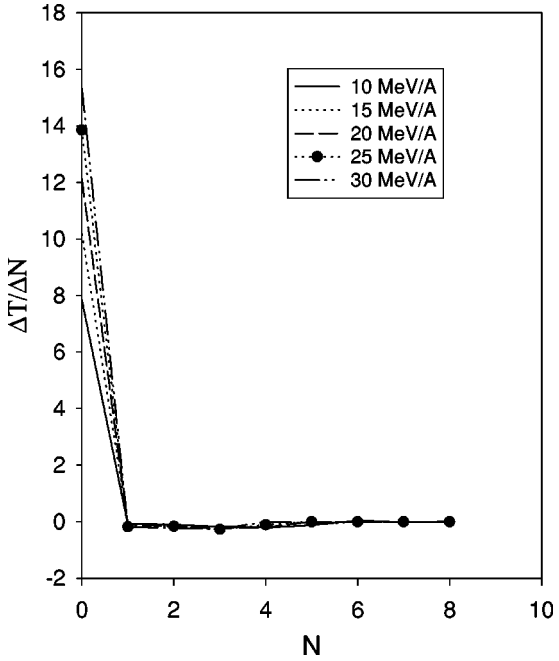

FIG. 3. Plot of the hot spot cooling rate $\Delta T/\Delta N$ against interaction stage N at different projectile energies for the reaction $^{20}\mathrm{Ne}+^{165}\mathrm{Ho}$.

the nucleons in the hot spot. To do so we first observe the number of excited particles at each stage of the interaction for different projectile energies. In Fig. 5 we give a plot of the number of excited neutrons against N. For all projectile energies the numbers of excited neutrons increase sharply and reach maxima at $N=1$. Thereafter a slow decrease is observed and a stability is apparent around $N=4$. It can be seen that the differences in the number of excited neutrons for different projectile energies are much greater at $N=1$ compared to those at $N=0$. For higher projectile energies a much larger number of particles share the excitation energies at $N=1$. This explains why the temperature of the hot spot remains almost unchanged after $N=1$.

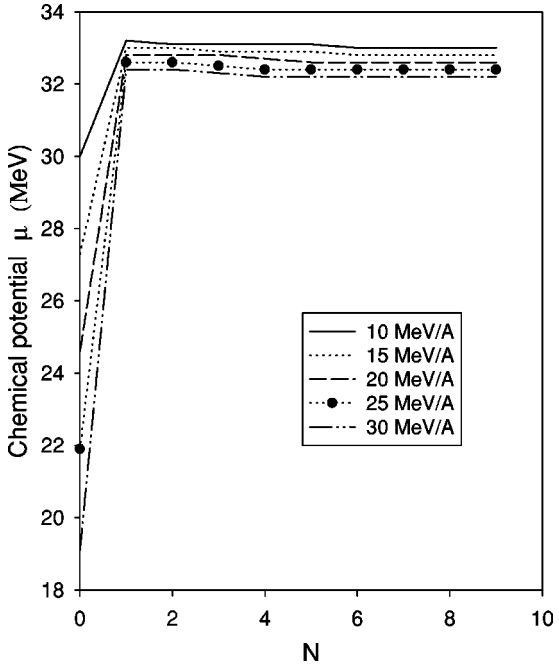

FIG. 4. Variation of the hot spot chemical potential μ with interaction stage N at different projectile energies for the reaction $^{20}\mathrm{Ne}+^{165}\mathrm{Ho}$.

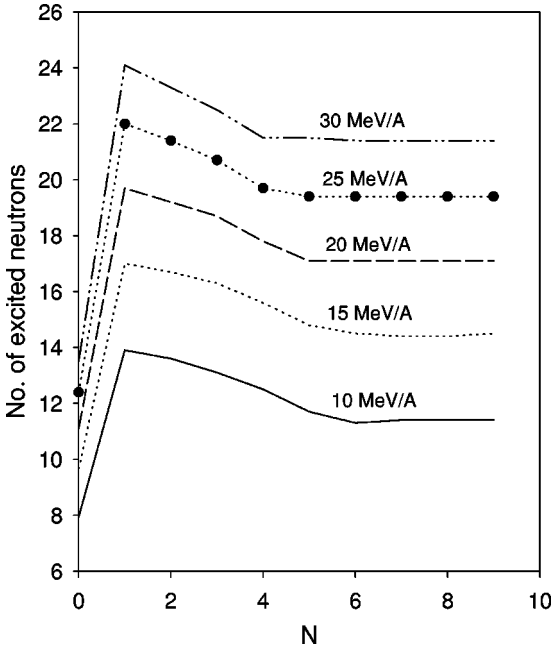

FIG. 5. Number of excited neutrons at each interaction stage N for different projectile energies of the reaction $^{20}\mathrm{Ne}+^{165}\mathrm{Ho}$.

Next, as a further probe, we examine the angular distribution of particles emitted from different interaction stages.

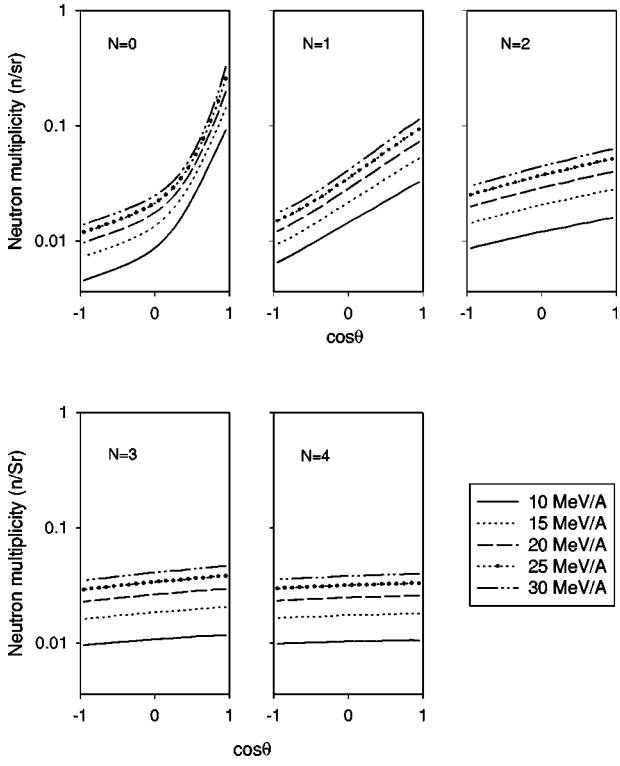

FIG. 6. Energy integrated angular distribution of neutron multiplicity for $^{20}\mathrm{Ne}+^{165}\mathrm{Ho}$ reaction at different projectile energies plotted against cosine of the emission angle θ from different interaction stages N.

PEQ emissions are predominantly peaked in the forward direction since such particles retain the memory of the incident projectile direction. With progressive number of interactions the angular distributions of the emitted particles get more and more smeared and eventually become isotropic as equilibrium sets in. In Fig. 6 we plot for $N=0-4$ the energy integrated PEQ neutron multiplicity against the cosine of the angle of emission with respect to the projectile direction. Different projectile energies are considered in each plot for the reaction $^{165}\mathrm{Ho}+^{20}\mathrm{Ne}$. For $N=0$, the sharply forward peaked direction results from the coupling of the projectile motion with the Fermi motion of its constituent nucleons for those neutrons that are contributed by the projectile. Contributions from the target, on the other hand, have isotropic distributions. As the number of interactions increases the distributions tend to become flatter and at $N=4$ appear to be almost isotropic. The slight anisotropy favoring forward emissions at $N=4$ may be due to the motion of the composite system along the projectile direction in the laboratory frame of reference. It may thus be concluded that kinematic equilibrium sets in around $N=4$ for the energy range considered in the present work.

IV. SUMMARY AND CONCLUSIONS

We have derived, based on the concepts of probability theory, the integral form of the transport equation to calculate double differential particle emission spectra from heavy ion collisions. The basic equation gives the evolution of the emission spectra with the number of interaction stages. Calculations using this formulation result in much less computaional effort. Several variants of the integral equation to consider time evolution of the composite system and to introduce spatial transport of the nucleons along with mean field effects have also been derived. The present deduction gives an insight into the computational procedures involved in HIC's from a different viewpoint and thus helps in understanding the processes involved more clearly.

Calculations performed with the present formulation to study equilibration of the target + projectile composite system result in the following conclusions for HIC's in the energy range 10 MeV/A to 30 MeV/A. (1) The number of interactions required to reach thermal and kinematic equilibrium is independent of the projectile energy. (2) Thermal equilibrium sets in much earlier than kinematic equilibrium in terms of the number of interactions.

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