Laser pulse modulation instabilities in plasma channels

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In this paper the modulational instability associated with propagation of intense laser pulses in a partially stripped, preformed plasma channel is analyzed. In general, modulation instabilities are caused by the interplay between (anomalous) group velocity dispersion and self-phase modulation. The analysis is based on a systematic approach that includes finite-perturbation-length effects, nonlinearities, group velocity dispersion, and transverse effects. To properly include the radial variation of both the laser field and plasma channel, the source-dependent expansion method for analyzing the wave equation is employed. Matched equilibria for a laser beam propagating in a plasma channel are obtained and analyzed. Modulation of a uniform (matched) laser beam equilibrium in a plasma channel leads to a coupled pair of differential equations for the perturbed spot size and laser field amplitude. A general dispersion relation is derived and solved. Surface plots of the spatial growth rate as a function of laser beam power and the modulation wave number are presented.

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I. INTRODUCTION

Ultrahigh intensity lasers [1–4] are being developed for a wide range of applications. Intense laser beam propagation in plasma channels [5–28] has applications in such areas as ultrabroadband radiation generation [29–32], optical harmonic generation [33–38], x-ray generation [39,40], inertial confinement fusion [41–45], and laser driven acceleration [46–74]. These and other applications provide a motivation for studying the physics of intense laser fields interacting with matter [75–88]. The propagation characteristics of an intense laser pulse in a medium can be markedly different from those in a vacuum [89–93]. For example, intense laser beams propagating in plasmas are subject to numerous instabilities [48,75,83,86,94,95]. In a vacuum the refractive index is unity and the scale length for diffraction of a laser beam with waist (minimum) spot size r_{SO} is the Rayleigh length $Z_R = \pi r_{SO}^2/\lambda$, where λ is the wavelength. In a dielectric medium there are additional contributions to the refractive index that lead to significant departures from free space propagation [29,48,76,77]. The additional contributions include effects due to the relativistic motion of free electrons in plasma and the nonlinear motion of atomic electrons in partially stripped plasma. Either one of these effects can lead to self-focusing that counteracts the diffractive tendency of the laser beam. Further, the diffraction of the laser beam can also be overcome by the presence of a preformed density channel in the plasma.

Most analyses of laser propagation in dielectrics and plasmas have been based on the paraxial form of the wave equation; that is equivalent to assuming that the longitudinal variations along the laser pulse are long compared to a wavelength. However, advancing technology has led to the development of extremely short laser pulses, some of which are no longer than a few optical cycles. In this limit short pulse effects and nonparaxial propagation can become important [19–21,25]. In this paper the nonparaxial, axisymmetric propagation of ultrashort, high intensity laser pulses in partially stripped plasmas is examined. The analysis allows for short pulse effects, nonlinear (relativistic and anharmonic atomic electron) effects, and includes a preformed parabolic plasma channel.

Intense laser beams propagating in plasmas are subject to many instabilities, such as Raman and modulational instabilities [48,78,83]. In Raman instabilities plasma waves play a fundamental role in scattering the primary laser beam into other frequencies or directions. Modulational instabilities, which are the subject of this paper, do not require the excitation of plasma waves and can result in distortions of the laser beam envelope or self-focusing. The physical basis for modulational instability is group velocity dispersion in the presence of self-phase modulation (also referred to as photon acceleration [83]). The modulational instability of laser beams in plasmas and dielectrics has been the subject of several publications [48,76,77,83,94,95]. Much of the early work considered one-dimensional models in which the laser beam is represented as a plane wave. Extension of these models to represent the finite transverse size of a laser beam has been effected by employing plane waves with a nonzero transverse wave number [76,77,95].

In this paper an equilibrium (matched) solution for a laser beam in a plasma channel is obtained. A dispersion relation for axisymmetric modulational perturbations about the equilibrium is obtained and analyzed. The analysis of modulational instability of laser beams in plasmas presented here marks a significant advance in various directions. First, the analysis allows for transverse variations of the laser field and the plasma channel. Thus, the finite transverse extent of the laser beam is self-consistently included in the formulation and there is no need to introduce an arbitrary transverse wave number in the analysis. Second, since the analysis includes transverse variations as well as nonlinearities, relativistic focusing and the notion of a critical power for self-focusing automatically emerges in the calculations. Hence, the interplay between modulational instability and self-focusing is clarified by the analysis.

The organization of this paper is as follows. In Sec. II a reduced wave equation for the slowly varying envelope of the laser electric field is derived that includes nonlinear (relativistic and ponderomotive) effects, nonparaxial (i.e., short pulse) effects, and allows for the presence of a preformed plasma channel. The derivation employs a systematic approach that, in principle, includes the dispersive effects of free (plasma) electrons as well as of bound (atomic) electrons to all orders. In Sec. III a reduced wave equation is derived and the linear refractive index and group velocity dispersion parameter is defined. The reduced wave equation is transformed to the group velocity coordinates for further analysis. The final form of the wave equation involves the transverse coordinates, the propagation distance, and the coordinate relative to the pulse centroid. The solutions of such an equation can be parametrized in terms of amplitude, phase, radius of curvature of the phase fronts, and spot size of the laser beam. This parametrization is effected in Sec. IV, making use of the source dependent expansion approach and employing Laguerre-Gaussian eigenfunctions. The laser beam and channel are assumed to be axisymmetric. In Sec. V an equilibrium solution of an envelope equation for the spot size is obtained and related to the preformed plasma channel parameters. A perturbed form of the envelope equation is also obtained and analyzed. A general stability analysis of the equilibrium solution is presented in Sec. VI. It is shown that perturbations on a uniform laser beam envelope are susceptible to a modulational instability. The analysis presented includes the effects of finite perturbation lengths. It is shown that these effects modify the modulational instability qualitatively by limiting the parameter space in which there is an instability. Surface plots displaying the growth rate of the instability are given and concluding remarks are presented in Sec. VII.

II. DERIVATION OF REDUCED WAVE EQUATION

In this section we derive an equation describing the evolution of the envelope of an electromagnetic field propagating in a preformed plasma channel. The approach adopted in this paper permits inclusion of dispersion, finite laser pulse length, and nonlinear effects. The propagation medium is described by a linear and nonlinear polarization field due to bound electrons as well as a linear and nonlinear plasma current due to free electrons. The electric field in the medium is governed by the wave equation [76,77]

$$(\nabla^2 - c^{-2}\partial^2/\partial t^2)\mathbf{E} = 4\pi c^{-2}(\partial^2\mathbf{P}/\partial t^2 + \partial \mathbf{J}_p/\partial t), \tag{1}$$

where $\mathbf{E}(\mathbf{r},t)$ is the electric field, $\nabla^2 = \nabla_\perp^2 + \partial^2/\partial z^2$, z is the axial propagation direction, $\mathbf{P}(\mathbf{r},t)$ is the total polarization field, and $\mathbf{J}_p(\mathbf{r},t)$ is the plasma current density. In obtaining Eq. (1) we have neglected a small source term proportional to the gradient of the high frequency component of the plasma density. The polarization field consists of a linear and nonlinear contribution, i.e., $\mathbf{P} = \mathbf{P}_L + \mathbf{P}_{\mathrm{NL}}$ where $\mathbf{P}_L$ is first order in the electric field $\mathbf{E}$ and $\mathbf{P}_{\mathrm{NL}}$ is nonlinear in $\mathbf{E}$.

A. Linear source terms

The relationship between the Fourier transforms of $\mathbf{P}_L$ and $\mathbf{E}$ is given by

$$\hat{\mathbf{P}}_L(\mathbf{r},\omega) = \hat{\chi}_L(\mathbf{r},\omega)\hat{\mathbf{E}}(\mathbf{r},\omega), \tag{2}$$

where $\hat{\mathbf{P}}_L$ and $\hat{\mathbf{E}}$ are the Fourier transforms of $\mathbf{P}_L$ and $\mathbf{E}$ and $\chi_L(r,\omega)$ is the frequency dependent linear scalar susceptibility which may also be a function of r in order to provide optical guiding. The convention for the Fourier transform pairs is

$$\hat{\mathbf{P}}_L(\mathbf{r},\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathbf{P}_L(\mathbf{r},t)e^{i\omega t}dt, \tag{3a}$$

$$\mathbf{P}_L(\mathbf{r},t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\hat{\mathbf{P}}_L(\mathbf{r},t)e^{-i\omega t}d\omega. \tag{3b}$$

The relationship between $\mathbf{P}_L$ and $\mathbf{E}$ is given by

$$\mathbf{P}_L(r,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{t}\chi_L(t-t')\mathbf{E}(\mathbf{r},t')dt', \tag{4}$$

which in terms of Fourier transforms results in Eq. (2). The linear part of the plasma current density is given by

$$\frac{\partial \mathbf{J}_{p,L}}{\partial t} = \frac{\omega_P^2}{4\pi}\mathbf{E}(\mathbf{r},t), \tag{5}$$

where $\omega_p(r) = [4\pi q^2 n_p(r)/m]^{1/2}$ is the plasma frequency and n_p is the plasma density which may be spatially tapered, i.e., have a minimum on axis, to optically guide the beam. The electric field is represented in the following form:

$$\mathbf{E}(\mathbf{r},t) = E(\mathbf{r},t)(1/2)e^{i(k_0 z - \omega_0 t)}\hat{e}_x + \text{c.c.}, \tag{6}$$

where $E(\mathbf{r},t)$ denotes the amplitude and is a slowly varying function of z and t, k_0 and ω_0 are, respectively, the wave number and frequency of the carrier field, and $\hat{e}_x$ is a unit vector in the x direction denoting the polarization. To obtain an envelope equation describing the evolution of $E(\mathbf{r},t)$ it is convenient to first neglect the nonlinear contributions from both the polarization field as well as from the plasma current density. Taking a Fourier transform of Eq. (1) without the nonlinear source terms gives

$$(\nabla^2 + \omega^2/c^2)\hat{\mathbf{E}}(\mathbf{r},\omega) = -4\pi(\omega^2/c^2)\hat{\mathbf{P}}_L(\mathbf{r},\omega) + (\omega_P^2/c^2)\hat{\mathbf{E}}(\mathbf{r},\omega) \tag{7}$$

Since we will be considering Gaussian beams we introduce the spot size r_c, which will be defined later. Equation (7) can now be written as

$$\left(\nabla^2 + \frac{4}{r_c^2} + \frac{\omega^2}{c^2}n_L^2(r,\omega)\right)\hat{\mathbf{E}}(\mathbf{r},\omega) = 0, \tag{8}$$

where

$$n_L(r,\omega) = [n_b^2(r,\omega) - \omega_p^2(r)/\omega^2 - 4c^2/(\omega r_c)^2]^{1/2}, \tag{9}$$

is the linear part of the total refractive index, and

$$n_b(r,\omega) = [1 + 4\pi\hat{\chi}_L(r,\omega)]^{1/2}, \tag{10}$$

is the refractive index due to both vacuum and bound electrons. The effective transverse wave number for a Gaussian beam having spot size r_c is $k_\perp = 2/r_c$. To obtain Eq. (8) the term $4/r_c^2$ has been added and subtracted to the wave operator. In the analysis that follows this will emerge as an important step in identifying the slowly varying envelope of the optical field.

1. Plasma channel

To form an optical channel the linear index of refraction must be a maximum on axis and decrease radially. To optically guide a beam having a Gaussian transverse profile, the square of the refractive index must decrease as r^2, i.e.,

$$n_L^2(r,\omega) = n_L^2(\omega)(1 - r^2/R_{ch}^2), \tag{11}$$

where $n_L(\omega) = n_L(r=0,\omega)$ and R_{ch} is the channel radius. Substituting the Fourier transform of $\mathbf{E}(\mathbf{r},t)$, together with the representation for $\mathbf{E}(\mathbf{r},t)$ in Eq. (6), into the wave equation, yields

$$\left(\nabla^2 + \frac{4}{r_c^2} + \frac{\omega^2}{c^2} n_L^2(r,\omega)\right)\hat{E}(\mathbf{r},\omega-\omega_0)\exp(ik_0 z) = 0. \tag{12}$$

Equation (12) can be rewritten as

$$\left[\nabla_\perp^2 + \frac{\partial^2}{\partial z^2} + 2k_0\left(i\frac{\partial}{\partial z} + K(\omega) + \frac{2}{k_0 r_c^2}\right) - \frac{\omega_0^2}{c^2} n_L^2(\omega_0)\frac{r^2}{R_{ch}^2}\right]\hat{E}(\mathbf{r},\omega-\omega_0) = 0, \tag{13}$$

where $K(\omega) = [\beta^2(\omega) - k_0^2]/2k_0 \cong \beta(\omega) - k_0$ and

$$\beta(\omega) = \frac{\omega}{c} n_L(\omega) \tag{14}$$

is the mode propagation constant (wave number). In Eq. (13) the part of $n_L(r,\omega)$ which depends on r^2 has been evaluated at ω_0, this approximation is valid for a wide channel, i.e., $r_c^2/R_{ch}^2 \ll 1$.

2. Mode propagation constant

Since $\hat{E}(\mathbf{r},\omega-\omega_0)$ is the Fourier transform of the slowly varying amplitude $E(\mathbf{r},t)$ the frequency dependent function $\beta(\omega)$ may be expanded about ω_0 [29],

$$\beta(\omega) = \frac{\omega}{c} n_L(\omega) \approx \beta_0 + (\omega-\omega_0)\beta_1 + \frac{1}{2}(\omega-\omega_0)^2\beta_2 + \cdots \tag{15}$$

where

$$\beta_n = [d^n\beta(\omega)/d\omega^n]_{\omega=\omega_0}. \tag{16}$$

In Eq. (15), β_2 is related to group velocity dispersion (GVD). Substituting Eq. (15) into Eq. (13) and taking the inverse Fourier transform yields

$$\left[\nabla_\perp^2 + \frac{\partial^2}{\partial z^2} + 2k_0\left(i\frac{\partial}{\partial z} + \beta_0 - k_0 + \frac{2}{k_0 r_c^2} + i\beta_1\frac{\partial}{\partial t} - \frac{\beta_2}{2}\frac{\partial^2}{\partial t^2} - \frac{i}{6}\beta_3\frac{\partial^3}{\partial t^3}\right) - \beta_0^2\frac{r^2}{R_{ch}^2}\right]E(\mathbf{r},t) = 0. \tag{17}$$

In obtaining Eq. (17) we have used the relationship,

$$\int_{-\infty}^{\infty}(\omega-\omega_0)^n\hat{E}(\mathbf{r},\omega-\omega_0)e^{-i(\omega-\omega_0)t}d\omega = \sqrt{2\pi}(i)^n\frac{\partial^n E(\mathbf{r},t)}{\partial t^n}. \tag{18}$$

B. Nonlinear source terms

The effects of the nonlinear polarization field and plasma current are assumed small and can be included in the envelope equation approximately. The channel as well as the optical beam is assumed broad. This implies, among other things, that the linear refractive index varies little from the axis, $r=0$, to the optical spot size, $r=r_c$, i.e., $r_c^2/R_{ch}^2 \ll 1$.

1. Nonlinear polarization field

The nonlinear polarization field due to bound electrons is given by

$$\mathbf{P}_{\rm NL}(\mathbf{r},t) = \chi_{\rm NL}\langle\mathbf{E}\cdot\mathbf{E}\rangle\mathbf{E}(\mathbf{r},t), \tag{19}$$

where $\chi_{\rm NL}$ is the scalar third-order susceptibility of the neutral gas and the brackets $\langle\,\rangle$ denote a time average. In this approximation the third harmonic component of the nonlinear polarization field is neglected and the nonlinear response is assumed to be instantaneous. Equation (19) can be expressed in terms of the nonlinear refractive index n_2,

$$\mathbf{P}_{\rm NL}(\mathbf{r},t) = \frac{1}{4\pi}[2n_{L0}n_2 I(\mathbf{r},t)]\mathbf{E}(\mathbf{r},t), \tag{20}$$

where $n_{L0} = n_L(0,\omega_0)$ is the linear index evaluated at $r=0$ and $\omega=\omega_0$, $n_2 = (8\pi^2/n_{L0}^2 c)\chi_{\rm NL}$ is the nonlinear refractive index, $I(\mathbf{r},t) = (c/4\pi)n_{L0}\langle\mathbf{E}\cdot\mathbf{E}\rangle$ is the intensity, and $|n_2 I| \ll n_{L0} - 1$ has been assumed. The refractive index is the sum of the linear and nonlinear contributions and in the absence of relativistic effects is

$$n(r,\omega) = n_L(r,\omega) + n_2 I. \tag{21}$$

2. Nonlinear plasma current

The nonlinear contribution to the plasma current density originates from plasma waves and relativistic effects. The plasma waves, i.e., wakefields, can be generated by the ponderomotive force, i.e., radiation pressure, associated with the electromagnetic field envelope. The relativistic contribution to the plasma current density is due to relativistic changes in the mass of the oscillating electrons. The nonlinear part of the plasma current density is given by

$$\frac{\partial\mathbf{J}_{p,\rm NL}}{\partial t} = \frac{\omega_p^2}{4\pi}\left(\frac{\delta n_p}{n_p} - \frac{\delta m}{m}\right)\mathbf{E}(\mathbf{r},t), \tag{22}$$

where δn_p is the perturbed plasma density due to the generated plasma wave and δm is the change of the electron's mass due to relativistic effects. This paper is devoted to the analysis of self-phase modulation, due to relativistic and nonlinear atomic electron effects. Analysis of wakefield excitation is postponed to a forthcoming paper; hereafter δn_p is neglected.

To second order in the field amplitude the fractional change in the electron's mass is $\delta m/m = \mathbf{a}\cdot\mathbf{a}/2$, where $\mathbf{a}$ is the unitless normalized vector potential associated with the electromagnetic field, i.e., the vector potential multiplied by q/mc^2. In terms of $\mathbf{a}$, the electron momentum in the oscillating electromagnetic field is $\mathbf{a}mc$. The magnitude of $\mathbf{a}$ is often referred to as the laser strength parameter. For mildly relativistic electron oscillations $|\mathbf{a}|\ll 1$ and for a linearly polarized laser beam, $|\mathbf{a}| = 8.6\times 10^{-10}\lambda[\mu\mathrm{m}]I^{1/2}[\mathrm{W/cm^2}]$. Using the electric field representation in Eq. (6), we find that

$$\mathbf{a}(\mathbf{r},t) \cong -iq\frac{E(\mathbf{r},t)}{mc\omega_0}\frac{e^{i(k_0 z-\omega_0 t)}}{2}\mathbf{e}_x + \text{c.c.} \tag{23a}$$

and, neglecting harmonics,

$$\mathbf{a}\cdot\mathbf{a} \cong \frac{1}{2}\left(\frac{q}{mc\omega_0}\right)^2 |E(\mathbf{r},t)|^2, \tag{23b}$$

where we have assumed $|\partial \ln(E)/\partial t| \ll \omega_0$.

Substituting Eqs. (20) and (22) into Eq. (17), with the nonlinear source terms reinstated, and using Eq. (23a), the equation for the complex amplitude becomes

$$\left[\nabla_\perp^2 + \frac{\partial^2}{\partial z^2} + 2k_0\left(i\frac{\partial}{\partial z} + \beta_0 - k_0 + \frac{2}{k_0 r_c^2} + i\beta_1\frac{\partial}{\partial t} - \frac{\beta_2}{2}\frac{\partial^2}{\partial t^2} - \frac{i}{6}\beta_3\frac{\partial^3}{\partial t^3} + \cdots\right) - \beta_0^2\frac{r^2}{R_{ch}^2}\right] b(\mathbf{r},t) \cong -\left(\frac{\omega_{p0}}{2c}\right)^2(1+R)|b|^2 b(\mathbf{r},t), \tag{24}$$

where

$$b(\mathbf{r},t) = \frac{|q|}{mc}\frac{E(\mathbf{r},t)}{\omega_0}, \tag{25a}$$

$$R = \frac{4\pi q^2 c}{\lambda_0^2 r_e^2}\frac{\omega_0^2}{\omega_{p0}^2} n_{L0}^2 n_2, \tag{25b}$$

$r_e = q^2/mc^2$ is the classical electron radius, $\lambda_0 = 2\pi c/\omega_0$ is the vacuum wavelength, and $\omega_{p0} = \omega_p(0)$ is the on-axis plasma frequency in terms of the on-axis density $n_{p0} = n_p(0)$. Note that $b(\mathbf{r},t)$ is proportional to the electric field amplitude and is only approximately equal to the magnitude of the normalized vector potential. The quantity R is the ratio of the critical powers for relativistic focusing in a plasma and nonlinear focusing in a gas [76,77], i.e. $R = P_p/P_a$ where

$$P_p = 2c(q/r_e)^2 n_{L0}(\omega_0/\omega_{p0})^2, \tag{26a}$$

$$P_a = \lambda_0^2/(2\pi n_{L0} n_2). \tag{26b}$$

The linear refractive index appears in the definition of P_p in Eq. (26a) since the laser intensity is proportional to $n_{L0}E^2$. In general, when the laser power exceeds either of these critical powers, focusing occurs [21,76,77]. The total nonlinear focusing power consists of contributions from both P_p and P_a and is given by

$$P_{\text{crit}} = P_p P_a/(P_a + P_p). \tag{27}$$

The quantity R in Eq. (25b) is also equal to the ratio P_p/P_a, which in practical units [76,77] is given by

$$R = P_p/P_a = \frac{1.22\times 10^{40} n_{L0}^2 n_2[\mathrm{cm^2/W}]}{\lambda_0^4[\mu\mathrm{m}] n_p[\mathrm{cm^{-3}}]}. \tag{28}$$

III. NONLINEAR REDUCED WAVE EQUATION

The nonlinear reduced wave equation becomes, neglecting β_3 and higher order dispersion terms,

$$\left[\nabla_\perp^2 + \frac{\partial^2}{\partial z^2} + 2k_0\left(i\frac{\partial}{\partial z} + \beta_0 - k_0 + \frac{2}{k_0 r_c^2} + i\beta_1\frac{\partial}{\partial t} - \frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}\right) - \beta_0^2\frac{r^2}{R_{ch}^2} + \kappa_{\mathrm{NL}}^2|b(\mathbf{r},t)|^2\right] b(\mathbf{r},t) = 0, \tag{29}$$

where

$$\kappa_{\mathrm{NL}}^2 = \frac{\omega_{p0}^2}{4c^2}(1+R), \tag{30}$$

in which the first term represents relativistic effects and the second term bound electron effects.

It is convenient at this point to change variables from (z,t) to (η, ξ) where $\eta = z$, $\xi = (z - v_g t)$, and v_g is the group velocity. In terms of these new variables $\partial/\partial z = \partial/\partial\eta + \partial/\partial\xi$ and $\partial/\partial t = -v_g\partial/\partial\xi$. Setting $v_g = 1/\beta_1$ and $k_0 = \beta_0 = \omega_0 n_L(\omega_0)/c$, Eq. (29) reduces to

$$\left[\nabla_\perp^2 + 2k_0\left(i\frac{\partial}{\partial\eta} - \frac{1}{2}\beta_2 v_g^2\frac{\partial^2}{\partial\xi^2} + \frac{2}{k_0 r_c^2}\right) + 2\frac{\partial^2}{\partial\eta\partial\xi} - k_0^2\frac{r^2}{R_{ch}^2} + \kappa_{\mathrm{NL}}^2|b(r,\xi,\eta)|^2\right] b(r,\xi,\eta) = 0, \tag{31}$$

where we have neglected $1/k_0$ compared to $\beta_2 v_g^2$ and $\partial^2/\partial\eta^2$ compared to $2k_0\partial/\partial\eta$; this approximation is valid for the case of interest here. Note that the condition $k_0 = \beta_0$ defines the dispersion relation $k_0 = \omega_0 n_L(\omega_0)/c$.

A. Linear refractive index

The total refractive index $n_L(\omega)$ is given by

$$n_L(\omega) = [n_b^2(\omega) - \Omega^2/\omega^2]^{1/2}, \tag{32}$$

where $n_b(\omega)$ is the refractive index associated with the bound electrons, $\Omega^2 = \omega_{p0}^2 + 4c^2/r_c^2$ and Ω^2/ω^2 represents the square of the refractive index associated with the free electrons (plasma) and finite spot size effects. Assuming $\Omega^2/\omega^2 \ll 1$, the total index can be written as

$$n_L(\omega) \cong n_b(\omega) - \frac{1}{2n_{b0}} \frac{\Omega^2}{\omega^2}, \tag{33}$$

where $n_{b0} = n_b(\omega_0)$. With this approximate form for the index the dispersion parameters β_1 and β_2 are

$$\beta_1 = \frac{1}{c}\left(n_b(\omega) + \omega \frac{\partial n_b}{\partial \omega} + \frac{1}{2n_{b0}} \frac{\Omega^2}{\omega^2} \right)_{\omega=\omega_0}, \tag{34a}$$

$$\beta_2 \cong \frac{1}{c}\left(2\frac{\partial n_b}{\partial \omega} + \omega \frac{\partial^2 n_b}{\partial \omega^2} - \frac{1}{n_{b0}} \frac{\Omega^2}{\omega^3} \right)_{\omega=\omega_0}. \tag{34b}$$

The phase and group velocities on axis $(r=0)$ are, respectively, $v_{ph} = \omega_0/k_0 = c/n_L(\omega)$, and $v_g = 1/\beta_1$,

$$v_{ph} \cong \frac{c}{n_{b0}}\left(1 + \frac{1}{2n_b^2} \frac{\Omega^2}{\omega^2} \right)_{\omega=\omega_0}, \tag{35a}$$

$$v_g \cong \frac{c}{n_{b0}}\left(1 - \frac{\omega}{n_b} \frac{\partial n_b}{\partial \omega} - \frac{1}{2n_b^2} \frac{\Omega^2}{\omega^2} \right)_{\omega=\omega_0}. \tag{35b}$$

B. Group velocity dispersion (GVD)

The GVD parameter β_2 has units of see^2/cm and is given by

$$\beta_2 = \frac{\partial \beta_1}{\partial \omega_0} = \frac{\partial}{\partial \omega_0}(v_g^{-1}) = -\frac{1}{v_g^2} \frac{\partial v_g}{\partial \omega_0}. \tag{36}$$

The GVD parameter can also be written as the sum of contributions from bound electrons, free, i.e., plasma, electrons, and finite spot size effects,

$$\beta_2 = \beta_{2b} + \beta_{2p} + \beta_{2\perp}, \tag{37}$$

where

$$\beta_{2b} = \frac{1}{c}\left(2\frac{\partial n_b}{\partial \omega} + \omega \frac{\partial^2 n_b}{\partial \omega^2} \right)_{\omega=\omega_0}, \tag{38a}$$

is due to bound electrons and can be either less than or greater than zero,

$$\beta_{2p} = -\frac{1}{n_{b0} c} \frac{\omega_p^2(0)}{\omega_0^3}, \tag{38b}$$

is due to plasma electrons and is always negative, and

$$\beta_{2\perp} = -\frac{1}{n_{b0} c} \frac{4c^2}{\omega_0^3 r_c^2}, \tag{38c}$$

is due to the finite spot of the optical beam and is also always negative. The group velocity near the frequency ω_0 is

$$v_g(\omega) \cong v_g(\omega_0) + \left(\frac{\partial v_g}{\partial \omega} \right)_{\omega=\omega_0} \delta\omega,$$
$$= v_g(\omega_0) - v_g^2(\omega_0)\beta_2 \delta\omega, \tag{39}$$

where $\delta\omega = \omega - \omega_0$. Note that for $n_{b0}=1$, i.e., no bound electrons, the GVD parameter is negative and given by

$$\beta_2 \cong -\frac{1}{\omega_0 c}\left(\frac{\omega_p^2(0)}{\omega_0^2} + \frac{4c^2}{\omega_0^2 r_c^2} \right). \tag{40}$$

IV. ANALYSIS OF WAVE EQUATION

Equation (31) is the model equation and forms the basis for the subsequent analysis in this paper. It is a partial differential equation for the complex-valued, slowly varying, normalized electric field amplitude $b(\xi,\eta)$. Equation (31) describes the propagation of a finite-pulse-length laser beam in a partially stripped plasma channel. To analyze this equation it is useful to first separate the radial variation from the axial propagation characteristics. This can be conveniently accomplished by employing the source dependent expansion method.

Source-dependent expansion solution

The following analysis is based on the source-dependent expansion (SDE) method developed in Ref. [96]. Assuming axisymmetry, in the SDE method the representation for the amplitude of the Gaussian beam is

$$b(r,\xi,\eta) = a(\xi,\eta) e^{i\psi(\xi,\eta)} e^{-[1-i\alpha(\xi,\eta)]r^2/r_s^2(\xi,\eta)}, \tag{41}$$

where a, ψ, α, and r_s are real functions of ξ and η. Here α is inversely proportional to the wavefront radius of curvature and r_s is the spot size. In the SDE method the wave equation is written in the form

$$(\nabla_\perp^2 + 2ik_0 \partial/\partial\eta) b(r,\xi,\eta) = S(r,\xi,\eta), \tag{42}$$

where $\nabla_\perp^2 = r^{-1}\partial(r\partial/\partial r)/\partial r$, r is the radial coordinate, $S(r,\xi,\eta) = S_g(r,\xi,\eta) + S_{\not\parallel}(r,\xi,\eta)$, and

$$S_g(r,\xi,\eta) = \left(k_0^2 \frac{r^2}{R_{ch}^2} - \frac{4}{r_c^2} - \kappa_{\rm NL}^2 |b(r,\xi,\eta)|^2 \right) b(r,\xi,\eta), \tag{43a}$$

$$S_{\not\parallel}(r,\xi,\eta) = (k_0\beta_2 v_g^2 \partial^2/\partial\xi^2 - 2\partial^2/\partial\eta\partial\xi) b(r,\xi,\eta). \tag{43b}$$

The source term S_g contains the contributions from channel guiding, finite spot size and nonlinearities while the source term $S_{\not\parallel}$ contains finite pulse length effects including group velocity dispersion. In the SDE method the equations governing a, ψ, α, and r_s are given by

$$\frac{\partial \ln(a r_s)}{\partial \eta} = (F)_I, \tag{44a}$$

$$\frac{\partial \psi}{\partial \eta} + \frac{(1+\alpha^2)}{k_0 r_s^2} - \frac{\alpha}{r_s} \frac{\partial r_s}{\partial \eta} + \frac{1}{2} \frac{\partial \alpha}{\partial \eta} = -(F)_R, \tag{44b}$$

$$\frac{\partial r_s}{\partial \eta} - \frac{2\alpha}{k_0 r_s} = -r_s(G)_I, \tag{44c}$$

$$\frac{\partial \alpha}{\partial \eta} - \frac{2(1+\alpha^2)}{k_0 r_s^2} = 2(G)_R - 2\alpha(G)_I, \tag{44d}$$

where $\varepsilon(\xi,\eta) = a(\xi,\eta)\exp[i\psi(\xi,\eta)]$ and the subscripts R, I denote real and imaginary parts, respectively, and

$$F(\xi,\eta) = \frac{1}{2k_0\varepsilon(\xi,\eta)} \int_0^\infty d\chi S(\chi,\xi,\eta) e^{-(1+i\alpha)\chi/2}, \tag{45a}$$

$$G(\xi,\eta) = \frac{1}{2k_0\varepsilon(\xi,\eta)} \int_0^\infty d\chi S(\chi,\xi,\eta)(1-\chi) e^{-(1+i\alpha)\chi/2}, \tag{45b}$$

with $\chi = 2r^2/r_s^2$. Combining Eqs. (44a), (44b), and (44c) we find that

$$\frac{\partial \psi}{\partial \eta} = -\frac{2}{k_0 r_s^2} - (F+G)_R. \tag{46}$$

To proceed with the analysis we substitute the source function $S(r,\xi,\eta) = S_g(r,\xi,\eta) + S_\ell(r,\xi,\eta)$, given by Eqs. (43) into Eq. (45) and evaluate the integrals. The functions F and G are found to be

$$F(\xi,\eta) = \frac{1}{2k_0}\left(\frac{k_0^2 r_s^2(\xi,\eta)}{2R_{ch}^2} - \frac{4}{r_c^2} - \kappa_{\mathrm{NL}}^2 \frac{a^2(\xi,\eta)}{2} + A(\xi,\eta) + \frac{1}{2}B(\xi,\eta) + \frac{1}{2}C(\xi,\eta) \right), \tag{47a}$$

$$G(\xi,\eta) = -\frac{1}{2k_0}\left(\frac{k_0^2 r_s^2(\xi,\eta)}{2R_{ch}^2} + \kappa_{\mathrm{NL}}^2 \frac{a^2(\xi,\eta)}{4} + \frac{1}{2}B(\xi,\eta) + C(\xi,\eta) \right). \tag{47b}$$

In obtaining Eqs. (47) the source function $S_\ell(r,\xi,\eta)$ was expressed in the following form, displaying the r dependencies explicitly,

$$S_\ell(r,\xi,\eta) = [A(\xi,\eta) + B(\xi,\eta)(r/r_s)^2 + C(\xi,\eta) \times (r/r_s)^4] b(r,\xi,\eta), \tag{48}$$

where

$$A(\xi,\eta) = k_0\beta_2 v_g^2 \left(\frac{\partial^2 \ln(\varepsilon)}{\partial \xi^2} + \left(\frac{\partial \ln(\varepsilon)}{\partial \xi} \right)^2 \right) - 2\left(\frac{\partial^2 \ln(\varepsilon)}{\partial \eta \partial \xi} + \frac{\partial \ln(\varepsilon)}{\partial \eta} \frac{\partial \ln(\varepsilon)}{\partial \xi} \right), \tag{49a}$$

$$B(\xi,\eta) = r_s^2(\xi,\eta) k_0 \beta_2 v_g^2 \left(\frac{\partial^2 q}{\partial \xi^2} + 2\frac{\partial q}{\partial \xi} \frac{\partial \ln(\varepsilon)}{\partial \xi} \right) - 2r_s^2(\xi,\eta) \times \left(\frac{\partial^2 q}{\partial \eta \partial \xi} + \frac{\partial q}{\partial \xi} \frac{\partial \ln(\varepsilon)}{\partial \eta} + \frac{\partial q}{\partial \eta} \frac{\partial \ln(\varepsilon)}{\partial \xi} \right), \tag{49b}$$

$$C(\xi,\eta) = r_s^4(\xi,\eta) k_0 \beta_2 v_g^2 \left(\frac{\partial q}{\partial \xi} \right)^2 - 2r_s^4(\xi,\eta) \frac{\partial q}{\partial \xi} \frac{\partial q}{\partial \eta}, \tag{49c}$$

and $q(\xi,\eta) = -[1 - i\alpha(\xi,\eta)]/r_s^2(\xi,\eta)$.

V. LASER ENVELOPE EQUATION IN PLASMA CHANNEL

Combining Eqs. (44c) and (44d) we obtain an equation for the laser spot size

$$\frac{\partial^2 r_s}{\partial \eta^2} - \frac{4}{k_0^2 r_s^3} - \frac{4}{k_0 r_s}(G)_R + \left(\frac{2\alpha}{k_0 r_s} + \frac{\partial r_s}{\partial \eta} \right)(G)_I + r_s \frac{\partial}{\partial \eta}(G)_I = 0. \tag{50}$$

This is a differential equation for the spot size r_s, involving $\partial/\partial\eta$ and $\partial/\partial\xi$ derivatives and other laser beam parameters. It includes the effects of a plasma channel, nonlinearities, and dispersion. Equation (50) can be reduced to an envelope equation for the spot size alone in various limits. These will be discussed in a forthcoming paper. Here, we shall confine the presentation to a simple but important limit.

In the absence of finite pulse length effects, i.e., neglecting S_ℓ, we find that

$$F(\xi,\eta) = \frac{1}{2k_0}\left(\frac{k_0^2 r_s^2(\xi,\eta)}{2R_{ch}^2} - \frac{4}{r_c^2} - \kappa_{\mathrm{NL}}^2 \frac{a^2(\xi,\eta)}{2} \right), \tag{51a}$$

$$G(\xi,\eta) = -\frac{1}{2k_0}\left(\frac{k_0^2 r_s^2(\xi,\eta)}{2R_{ch}^2} + \kappa_{\mathrm{NL}}^2 \frac{a^2(\xi,\eta)}{4} \right). \tag{51b}$$

Substituting Eq. (51b) into Eq. (50) we obtain an envelope equation, in the long pulse limit [21]

$$\frac{\partial^2 r_s}{\partial \eta^2} - \frac{4}{k_0^2 r_s^3}\left[1 - \hat{P}_0(\xi) - \left(\frac{r_s}{r_c} \right)^4 \right] = 0, \tag{52}$$

where $\hat{P}_0 = P_0/P_{\mathrm{crit}}$ is the normalized power, P_0 is the laser power, P_{crit} is the critical laser power for nonlinear focusing, i.e.,

$$\hat{P}_0(\xi) = \kappa_{\mathrm{NL}}^2 a^2(\xi,\eta) r_s^2(\xi,\eta)/8, \tag{53}$$

and r_c is defined as

$$r_c = 2^{1/2}\left(\frac{R_{ch}}{k_0} \right)^{1/2}. \tag{54}$$

In the absence of a plasma channel ($R_{ch} \to \infty$) Eq. (52) predicts catastrophic focusing for $P_0 > P_{\mathrm{crit}}$. This is an artifact of the third order nonlinearity in the wave equation, Eq. (24). Mode conversion, higher order nonlinearities and ionization effects will prevent the beam from focusing down indefinitely.

The Rayleigh length associated with the spot size r_c is

$$Z_{R0} = k_0 r_c^2/2 = n_L(\omega_0)\pi r_c^2/\lambda_0. \tag{55}$$

A. Plasma channel

If the optical channel is formed by the plasma electrons and not the bound atomic electrons the depth of the plasma density necessary to optically guide a beam with matched spot size r_c is [97]

$$\Delta n \cong (\pi r_e r_c^2)^{-1}, \tag{56}$$

where we have assumed a plasma density variation of the form

$$n_p(r) = n_{p0} + \Delta n r^2/r_c^2, \tag{57}$$

and $r_e = e^2/mc^2$ is the classical electron radius. In the long pulse limit

$$\frac{\partial (a(\xi,\eta) r_s(\xi,\eta))}{\partial \eta} = 0, \tag{58a}$$

$$\frac{\partial \psi}{\partial \eta} = -\frac{2}{k_0 r_s^2}\left(1 - \frac{3}{2}\hat{P}_0(\xi)\right) + \frac{2}{k_0 r_c^2}. \tag{58b}$$

From Eq. (58a) we find that in the absence of finite length effects the laser power profile does not change with propagation distance η.

B. Matched pulse

For a matched pulse in a channel each of the quantities a, r_s, and α are independent of the propagation distance η. However, the phase ψ can be a function of η. For a matched beam Eqs. (44) yield

$$r_{s0}(\xi) = r_c(1 - \hat{P}_0(\xi))^{1/4}, \tag{59a}$$

$$\psi_0(\xi,\eta) = \mu_0(\xi)\,\eta = \left(1 - \frac{1-(3/2)\hat{P}_0(\xi)}{(1-\hat{P}_0(\xi))^{1/2}}\right)\frac{\eta}{Z_{R0}}, \tag{59b}$$

$$\alpha_0 = 0. \tag{59c}$$

C. Perturbed envelope equation

The envelope equation for a nearly matched pulse is found by setting

$$r_s(\xi,\eta) = r_{s0}(\xi) + r_1(\xi,\eta), \tag{60}$$

where $|r_1/r_{s0}| \ll 1$, and r_{s0} is given by Eq. (59a). Substituting Eq. (60) into the envelope equation in Eq. (52) gives

$$\frac{\partial^2 r_1}{\partial \eta^2} + \frac{4}{Z_{R0}^2} r_1 = 0, \tag{61}$$

where the period of the envelope oscillation for a nearly matched pulse is πZ_{R0}.

VI. ANALYSIS OF MODULATION INSTABILITY

In this section, the modulation instability of an initially uniform laser beam in a plasma channel is considered. First, a simple one-dimensional (1D) model is employed in order to illustrate the effects of finite-perturbation-length terms. The modulation instability is then examined using the SDE formalism, which accounts for the Gaussian profile of the beam. For an infinitely wide beam, we show how the SDE reduces to the one-dimensional limit.

A. One-dimensional modulation instability

The longitudinal modulation instability for a long laser pulse can be analyzed by taking the 1D (plane wave) limit of Eq. (31),

$$\left[2k_0\left(i\frac{\partial}{\partial \eta} - \frac{1}{2}\beta_2 v_g^2 \frac{\partial^2}{\partial \xi^2}\right) + 2\frac{\partial^2}{\partial \eta \partial \xi} + \kappa_{\rm NL}^2 |b(r,\xi,\eta)|^2\right] b(r,\xi,\eta) = 0. \tag{62}$$

The equilibrium for a long beam, i.e., $\partial/\partial\xi = 0$, is

$$b_0(\eta) = a_0 \exp\left(2i\hat{P}_0 \frac{\eta}{Z_{R0}}\right), \tag{63}$$

where a_0 is independent of ξ and

$$\kappa_{\rm NL}^2 |b_0|^2 = \frac{4k_0}{Z_{R0}}\hat{P}_0, \tag{64}$$

[cf. Eq. (53)]. The perturbed equilibrium is given by

$$b(\xi,\eta) = b_0(\eta) + a_1(\xi,\eta)\exp\left(2i\hat{P}_0\frac{\eta}{Z_{R0}}\right), \tag{65}$$

where $|a_1| \ll |b_0|$. Substituting Eq. (65) into Eq. (62) gives

$$i\frac{\partial a_1}{\partial \eta} - \frac{\beta_2}{2} v_g^2 \frac{\partial^2 a_1}{\partial \xi^2} + i\frac{2}{k_0 Z_{R0}}\hat{P}_0 \frac{\partial a_1}{\partial \xi} + \frac{1}{k_0}\frac{\partial^2 a_1}{\partial \xi \partial \eta} + \frac{2}{Z_{R0}}\hat{P}_0(a_1 + a_1^*) = 0. \tag{66}$$

Since the equilibrium is independent of ξ, the perturbed amplitude can be written as

$$a_1(\xi,\eta) = a_+(\eta)\exp(ik\xi) + a_-(\eta)\exp(-ik\xi). \tag{67}$$

Substituting Eq. (67) into Eq. (66) yields

$$\left[\left(1 - \frac{k^2}{k_0^2}\right)\frac{\partial^2}{\partial \eta^2} \pm i\left(\frac{8}{Z_{R0}}\hat{P}_0 + \beta_2 v_g^2 k^2\right)\frac{k}{k_0}\frac{\partial}{\partial \eta} + \frac{\beta_2^2}{4}v_g^4 k^4 + \frac{2}{Z_{R0}}\hat{P}_0 \beta_2 v_g^2 k^2 - \frac{4}{Z_{R0}^2}\hat{P}_0^2 \frac{k^2}{k_0^2}\right] a_\pm(\eta) = 0. \tag{68}$$

Taking $a_\pm$ to vary with η as $\exp(\pm iK\eta)$, the following dispersion relation is obtained:

$$(1-\hat{k}^2)\hat{K}^2 + 8(\hat{P}_0 + \hat{\beta}_2\hat{k}^2)\hat{k}\hat{K} - 16(\hat{\beta}_2^2\hat{k}^2 + \hat{P}_0\hat{\beta}_2 - \hat{P}_0^2/4)\hat{k}^2 = 0, \tag{69}$$

where the unitless quantities are $\hat{k}=k/k_0$, $\hat{K}=Z_{R0}K$, and $\hat{\beta}_2=(1/8)v_g^2k_0^2Z_{R0}\beta_2$. Substituting the expression for β_2, i.e., Eq. (37), into the definition of $\hat{\beta}_2$ gives

$$\hat{\beta}_2 \simeq \frac{v_g^2}{8}k_0^2Z_{R0}\beta_{2b} - \frac{1}{4}\left(1+\frac{\omega_p^2(0)r_c^2}{4c^2}\right), \tag{70}$$

where the last term in the expression for $\hat{\beta}_2$ is due to plasma electrons and finite spot size effects and $v_g \approx c/n_{b0}$ has been assumed.

The dispersion relation in Eq. (69) can be readily solved to show that the modulation instability is excited provided $\hat{\beta}_2$ is sufficiently negative, i.e.,

$$\hat{\beta}_2 + 3\hat{P}_0/4 < 0, \tag{71}$$

with the range of unstable wave numbers given by

$$\hat{k}^2 \leqslant -\frac{\hat{P}_0(3\hat{P}_0/4+\hat{\beta}_2)}{(\hat{P}_0/2+\hat{\beta}_2)^2}. \tag{72}$$

The maximum growth rate occurs at

$$\hat{k} = \pm\left(\frac{2(\hat{P}_0/2+\hat{\beta}_2)^2}{\hat{P}_0|3\hat{P}_0/4+\hat{\beta}_2|}-1\right)^{-1/2}, \tag{73}$$

and has the value

$$\Gamma_{\max} = (K_I)_{\max} = 2\hat{P}_0|3\hat{P}_0/4+\hat{\beta}_2|[(\hat{P}_0/2+\hat{\beta}_2)^2 - \hat{P}_0|3\hat{P}_0/4+\hat{\beta}_2|]^{-1/2}. \tag{74}$$

Equation (71) defines a cutoff power for the one-dimensional modulation instability, given by $(P_{\text{cutoff}})_{1D}/P_{\text{crit}} = -(4/3)\hat{\beta}_2$. If the mixed derivative term representing finite-perturbation-length effects, i.e., $\partial^2/\partial\xi\partial\eta$ in Eq. (62) is neglected the dispersion relation in Eq. (69) reduces to [29]

$$\hat{K}^2 = 16(\hat{\beta}_2^2\hat{k}^4 + \hat{P}_0\hat{\beta}_2\hat{k}^2). \tag{75}$$

A modulation instability exists if $\hat{\beta}_2<0$ and $\hat{k}^2 \leqslant \hat{P}_0/|\hat{\beta}_2|$, with spatial growth rate, i.e., $\Gamma=\hat{K}_I$,

$$\Gamma = 4|\hat{\beta}_2\hat{k}|(\hat{P}_0/|\hat{\beta}_2| - \hat{k}^2)^{1/2}. \tag{76}$$

The growth rate is symmetric with respect to $\hat{k}$ and has a maximum value of $\Gamma_{\max}=2\hat{P}_0$ which occurs at $\hat{k}=\pm[\hat{P}_0/(2|\hat{\beta}_2|)]^{1/2}$.

The parameter space stability boundary described by Eq. (72) is shown as the solid curve in Fig. 1. The appearance of a cutoff power defined by Eq. (71), above which the pulse is stable, is due to the inclusion of the mixed derivative term in Eq. (62), i.e., a finite-perturbation-length effect. Without this term, the stability boundary is given by the dashed curve in Fig. 1 which is described by $\hat{k}^2 \leqslant \hat{P}_0/|\hat{\beta}_2|$.

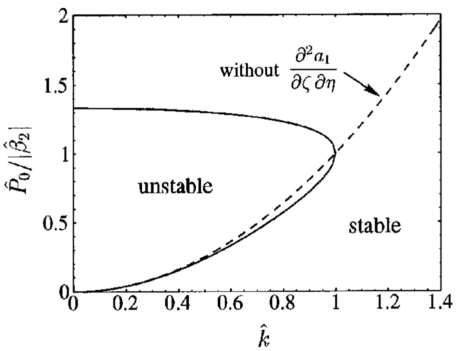

FIG. 1. Stability boundaries of the one-dimensional model in the parameter space $(k,\hat{P}_0/\hat{\beta}_2)$ calculated with (solid curve) and without (dashed curve) the mixed derivative term in Eq. (62).

B. Modulation instability of a Gaussian beam

Equations (44a)–(44d) together with Eqs. (47a), (47b) describe a laser pulse propagating in a plasma channel. The propagation model contains relativistic and finite-perturbation-length effects, GVD, as well as atomic electron nonlinearities. We now analyze the modulational instability associated with the laser beam amplitude and spot size. The laser beam equilibrium is assumed to be uniform $(\partial/\partial\xi=0)$ with arbitrary power $(\hat{P}_0<1)$. Although the equilibrium is independent of ξ the perturbed quantities are functions of both ξ and η (and are assumed to be nonlocal, i.e., extend over all ξ and η). The equilibrium (matched beam) solution to Eqs. (44) is given by Eq. (59).

To analyze the stability of the laser pulse, the pulse amplitude, $a(\xi,\eta)$, the phase, $\psi(\xi,\eta)$, the spot size, $r_s(\xi,\eta)$, and the curvature parameter, $\alpha(\xi,\eta)$, are perturbed about the uniform equilibrium values given by Eqs. (59), that is, $a(\xi,\eta)=a_0+a_1(\xi,\eta)$, $\psi(\xi,\eta)=\psi_0(\eta)+\psi_1(\xi,\eta)$, $r_s(\xi,\eta)=r_{s0}+r_1(\xi,\eta)$ and $\alpha(\xi,\eta)=\alpha_0+\alpha_1(\xi,\eta)$. The equilibrium quantities have subscripts 0 and are independent of ξ and the small, perturbed quantities have subscripts 1. Similarly, the functions F and G are perturbed, $F(\xi,\eta)=F_0+F_1(\xi,\eta)$, and $G(\xi,\eta)=G_0+G_1(\xi,\eta)$. The equations describing the perturbed laser beam quantities r_1, α_1, a_1, and ψ_1, from Eqs. (44) are

$$\frac{1}{r_{s0}}\frac{\partial r_1}{\partial\eta} + \frac{1}{a_0}\frac{\partial a_1}{\partial\eta} = (F_1)_I, \tag{77a}$$

$$\frac{\partial\psi_1}{\partial\eta} - \frac{2}{k_0r_{s0}^3}r_1 + \frac{1}{2}\frac{\partial\alpha_1}{\partial\eta} = -(F_1)_R, \tag{77b}$$

$$\frac{\partial r_1}{\partial\eta} - \frac{2}{k_0r_{s0}}\alpha_1 = -r_{s0}(G_1)_I, \tag{77c}$$

$$\frac{\partial\alpha_1}{\partial\eta} + \frac{4}{k_0r_{s0}^3}r_1 = 2(G_1)_R. \tag{77d}$$

The forms for F_0, G_0, F_1, and G_1 are given in the Appendix.

C. Dispersion relation

The full dispersion relation for the modulation instability is obtained by taking the perturbed quantities to vary like $\exp(iK\eta+ik\xi)$ in the full set of perturbed equations, Eqs. (77). An intermediate step in the derivation of the dispersion relation is the coupled equations for the Fourier transforms of $r_1(\xi,\eta)$ and $a_1(\xi,\eta)$, these are

$$\left(D(\hat{k},\hat{K})-\frac{4(1-\hat{P}_0/4)}{(1-\hat{P}_0)^{1/2}}E(\hat{k},\hat{K})-\frac{4(1-\hat{P}_0/2)}{(1-\hat{P}_0)}\right)\frac{\tilde{r}_1(\hat{k},\hat{K})/r_c}{(1-\hat{P}_0)^{1/4}}$$

$$=\frac{\hat{P}_0}{(1-\hat{P}_0)}[2+(1-\hat{P}_0)^{1/2}E(\hat{k},\hat{K})]\frac{\tilde{a}_1(\hat{k},\hat{K})}{a_0}, \tag{78a}$$

$$\left(D(\hat{k},\hat{K})+\frac{2\hat{P}_0}{(1-\hat{P}_0)^{1/2}}E(\hat{k},\hat{K})\right)\frac{\tilde{a}_1(\hat{k},\hat{K})}{a_0}$$

$$=-\left(D(\hat{k},\hat{K})+\frac{\hat{P}_0}{(1-\hat{P}_0)^{1/2}}E(\hat{k},\hat{K})\right)\frac{\tilde{r}_1(\hat{k},\hat{K})/r_c}{(1-\hat{P}_0)^{1/4}}, \tag{78b}$$

where

$$D(\hat{k},\hat{K})=(1-\hat{k}^2)\hat{K}^2+2(4\hat{\beta}_2\hat{k}^2+\hat{\mu}_0)\hat{k}\hat{K}-16\hat{\beta}_2^2\hat{k}^4+\hat{\mu}_0^2\hat{k}^2, \tag{79a}$$

$$E(\hat{k},\hat{K})=(\hat{K}-4\hat{\beta}_2\hat{k})\hat{k}, \tag{79b}$$

and $\hat{\mu}_0=Z_{R0}\mu_0=1-(1-3\hat{P}_0/2)/(1-\hat{P}_0)^{1/2}$, $\hat{k}=k/k_0$, $\hat{K}=Z_{R0}K$, $Z_{R0}=k_0r_c^2/2$, and $\hat{\beta}_2=(1/8)v_g^2k_0^2Z_{R0}\beta_2$. In Eqs. (78) $\tilde{r}_1$ and $\tilde{a}_1$ are the Fourier amplitudes of r_1 and a_1. The full dispersion relation is obtained by combining Eqs. (78) to give

$$\begin{aligned}&[(1-\hat{P}_0)D(\hat{k},\hat{K})-(4-\hat{P}_0)(1-\hat{P}_0)^{1/2}E(\hat{k},\hat{K})-4+2\hat{P}_0]\\&\quad\times[(1-\hat{P}_0)^{1/2}D(\hat{k},\hat{K})+2\hat{P}_0E(\hat{k},\hat{K})]\\&\quad+\hat{P}_0[2+(1-\hat{P}_0)^{1/2}E(\hat{k},\hat{K})][(1-\hat{P}_0)^{1/2}D(\hat{k},\hat{K})\\&\quad+\hat{P}_0E(\hat{k},\hat{K})]=0.\end{aligned} \tag{80}$$

The quantity $(1-\hat{P}_0)^{1/2}$ appears in various terms throughout the dispersion relation in Eq. (80). This is related to the fact that $\hat{P}_0=1$ has a physical significance associated with focusing, as indicated in the envelope equation, Eq. (52). The 1D model cannot take into account self-focusing associated with the equilibrium of the laser beam.

1. Low power limit ($\hat{P}_0=0$)

For $\hat{P}_0=0$ the full dispersion relation in Eq. (80) reduces to

$$[D(\hat{k},\hat{K})-4E(\hat{k},\hat{K})-4]D(\hat{k},\hat{K})=0. \tag{81}$$

The dispersion relation in Eq. (81) has stable roots, i.e., the imaginary part of $\hat{K}$ is zero for $\hat{k}$ real.

2. 1D limit ($r_c\to\infty$)

The dispersion relation in the one-dimensional limit is obtained by letting the spot size of the laser beam approach infinity ($r_c\to\infty$) in Eqs. (78). In this limit two dispersion relations are obtained. It is straightforward to show that for the first one, $2+(1-\hat{P}_0)^{1/2}E(\hat{k},\hat{K})=0$, the single root is stable. The other dispersion relation is

$$(1-\hat{P}_0)^{1/2}D(\hat{k},\hat{K})+2\hat{P}_0E(\hat{k},\hat{K})=0. \tag{82}$$

In the low power limit, i.e., to order $\hat{P}_0^2$, this simplifies to

$$\begin{aligned}&(1-\hat{k}^2)\hat{K}^2+8(\hat{P}_0/2+7\hat{P}_0^2/32+\hat{\beta}_2\hat{k}^2)\hat{k}\hat{K}-16(\hat{\beta}_2^2\hat{k}^2\\&\quad+\hat{P}_0\hat{\beta}_2/2-\hat{P}_0^2/16+\hat{P}_0^2\hat{\beta}_2/4)\hat{k}^2=0,\end{aligned} \tag{83}$$

which becomes identical to Eq. (69) to order $\hat{P}_0$ with the substitution $\hat{P}_0/2\to\hat{P}_0$.

Figures 2 and 3 compare the spatial growth rates, i.e., Im K, obtained from the 1D limit ($r_c\to\infty$) of the SDE dispersion relation [Eq. (82)] and from the formalism including

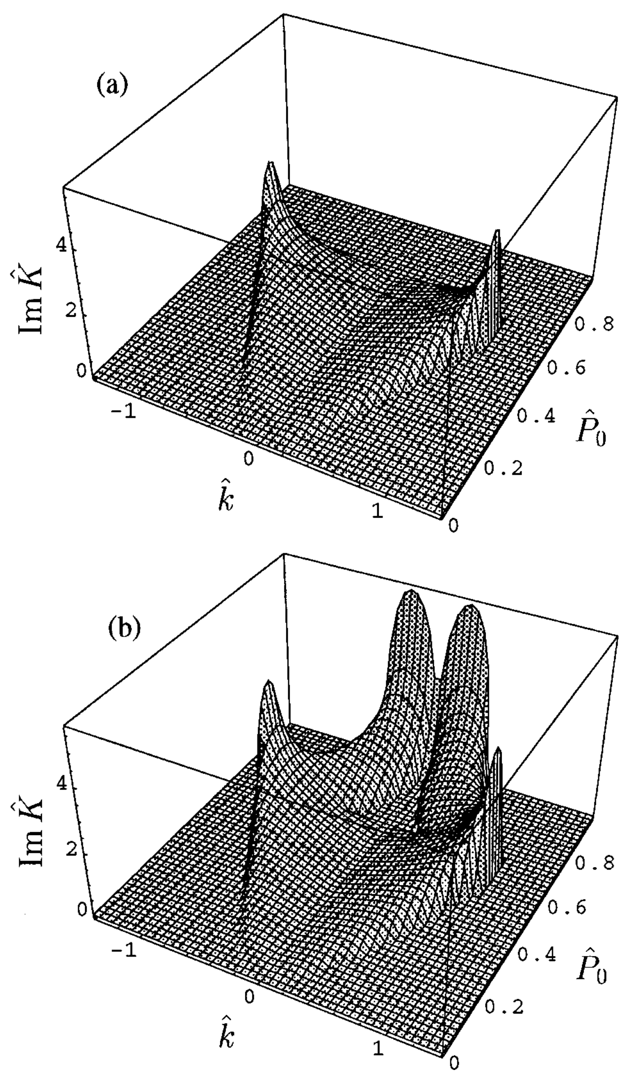

FIG. 2. Spatial growth rate (Im $\hat{K}$) versus scaled wave number $\hat{k}$ and power $\hat{P}_0$ calculated from Eq. (82) (a), and Eq. (80) (b) for $\hat{\beta}_2=-0.5$.

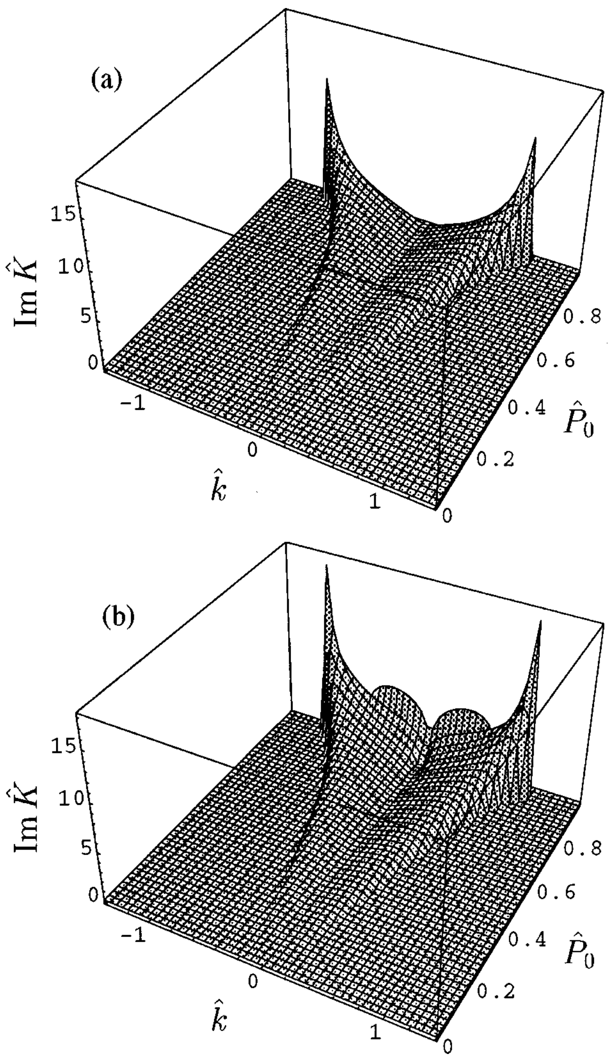

FIG. 3. Spatial growth rate (Im $\hat{K}$) versus scaled wave number $\hat{k}$ and power $\hat{P}_0$ calculated from Eq. (82) (a), and Eq. (80) (b) for $\hat{\beta}_2=-2$.

transverse variations [Eq. (80)]. Figure 2 displays Im K as a function of scaled wave number $\hat{k}$ and power $\hat{P}_0$ for $\hat{\beta}_2=-1/2$. In the 1D limit [Fig. 2(a)], there exists a cutoff power,

$$\hat{P}_{\text{cutoff}}=\frac{P_{\text{cutoff}}}{P_{\text{crit}}}=\frac{3}{8}-\hat{\beta}_2(1+2\hat{\beta}_2)-\frac{1}{8}(1+4\hat{\beta}_2)\times\sqrt{9+8\hat{\beta}_2(1+2\hat{\beta}_2)}, \tag{84}$$

above which the modulation is stable. The scaled cutoff power, $\hat{P}_{\text{cutoff}}$, tends towards unity as $\hat{\beta}_2\to-\infty$ and is zero when $\hat{\beta}_2=0$. The solutions of Eq. (80), however, show instability even when $\hat{P}_0>\hat{P}_{\text{cutoff}}$. As seen in Fig. 2(b), for $\hat{P}_0<\hat{P}_{\text{cutoff}}$, the peak growth rate increases and tends toward larger $\hat{k}$ as the power increases, while for $\hat{P}_0>\hat{P}_{\text{cutoff}}$ (in the full SDE model) the peak growth rate shifts to smaller $\hat{k}$ as the power increases. For larger values of $|\hat{\beta}_2|$ such that $\hat{P}_{\text{cutoff}}\to 1$ Fig. 3 shows that the 1D limit [Eq. (82)] and the full SDE dispersion relation [Eq. (80)] yield similar growth rates for $\hat{P}_0<1$.

VII. CONCLUSIONS

In this paper the modulational instability associated with propagation of intense, short laser pulses in a partially stripped, preformed plasma channel has been analyzed. The analysis is based on a systematic approach that includes finite-perturbation-length effects, nonlinearities, group velocity dispersion (GVD) and transverse effects. To properly include the radial variation of both the laser field and plasma channel, the source-dependent expansion method has been employed. Matched equilibria for a laser beam propagating in a plasma channel are obtained and analyzed. It is shown that modulation of a uniform (matched) laser beam equilibrium in a plasma channel leads to a coupled pair of differential equations for the perturbed spot size and laser field amplitude. A general dispersion relation is derived and solved. It is shown that in some instances finite-perturbation-length effects quench the instability above a cutoff power. Surface plots of the spatial growth rate as a function of laser beam power and the modulation wave number are presented.

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APPENDIX

The source-dependent expansion approach to the solution of the wave equation requires the evaluation of the overlap integrals F and G in Eqs. (45a) and (45b). For the equilibrium,

$$F_0=\frac{1}{2k_0}\left(\frac{2r_{s0}^2}{r_c^4}-\frac{4}{r_c^2}-\frac{4\hat{P}_0}{r_{s0}^2}\right), \tag{A1}$$

$$G_0=-\frac{1}{2k_0}\left(\frac{2r_{s0}^2}{r_c^4}+\frac{2\hat{P}_0}{r_{s0}^2}\right), \tag{A2}$$

while the perturbed integrals are given by

$$F_1(\xi,\eta)=\frac{1}{2k_0}\left(\frac{4r_{s0}}{r_c^4}r_1(\xi,\eta)-\frac{8\hat{P}_0}{r_{s0}^2}\frac{a_1(\xi,\eta)}{a_0}+A_1(\xi,\eta)+\frac{r_{s0}^2}{2}B_1(\xi,\eta)\right), \tag{A3}$$

$$G_1(\xi,\eta)=-\frac{1}{2k_0}\left(\frac{4r_{s0}}{r_c^4}r_1(\xi,\eta)+\frac{4\hat{P}_0}{r_{s0}^2}\frac{a_1(\xi,\eta)}{a_0}+\frac{r_{s0}^2}{2}B_1(\xi,\eta)\right), \tag{A4}$$

where

$$A_1(\xi,\eta)=\left(k_0\beta_2v_g^2\frac{\partial^2}{\partial\xi^2}-2\frac{\partial^2}{\partial\eta\partial\xi}-2i\frac{\partial\psi_0}{\partial\eta}\frac{\partial}{\partial\xi}\right)\left(\frac{a_1}{a_0}+i\psi_1\right), \tag{A5}$$

$$B_1(\xi,\eta)=\frac{2}{r_{s0}^2}\left(k_0\beta_2 v_g^2\frac{\partial^2}{\partial\xi^2}-2\,\frac{\partial^2}{\partial\eta\partial\xi}-2i\,\frac{\partial\psi_0}{\partial\eta}\,\frac{\partial}{\partial\xi}\right)\left(\frac{r_1}{r_{s0}}+i\,\frac{\alpha_1}{2}\right), \tag{A6}$$

and $\hat{P}_0=P_0/P_{\mathrm{crit}}$.

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