Double-hump solitary waves in quadratically nonlinear media with loss and gain

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We report the existence of a family of bright chirped localized waves in quadratic media with loss and gain. It is shown that the fundamental field component of the symbiotic solitary wave may exhibit a double-hump shape. The conditions of the solitary wave's existence are identified. Numerical experiments disclose different scenarios of instability as well as domains of rather robust behavior of these objects upon propagation.

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It is now widely accepted that optical media with a quadratic, or $\chi^{(2)}$, nonlinearity exhibit a diversity of phenomena which can be exploited in all-optical signal processing, amplification, pulse compression, etc. (see Refs [1,2], and references therein). Recent investigations also reveal many interesting fundamental properties of $\chi^{(2)}$ media. In particular, different types of dichromatic solitary waves [mutually locked fundamental field (FF) and second harmonic (SH)] in conservative quadratic media were identified and their potential use for signal routing and steering was discussed [3,4]. The experimental observation of spatial soliton propagation in quadratic bulk media [5] and film waveguides [6], including their interactions [7], as well as the existence of narrow temporal [8] and spatiotemporal solitons (light bullets) [9], are additional stimulating factors for further studies of solitonic regimes in the $\chi^{(2)}$ environment.

Many realistic physical systems exhibit inherent losses which may lead to adiabatic soliton shaping or even soliton decay [10]. Recently it was numerically shown by Torner [11] that a linear gain experienced by the FF is redistributed between harmonics, and this process can be used for soliton amplification. To realize a (quasi)stable solitonic regime in nonconservative systems, optical amplifiers are required for loss compensation. Thus the study of solitary wave propagation under the combined action of gain and loss is a practically relevant issue. In cubic media such investigations have a long history. In this environment the evolution is governed by the complex Ginzburg-Landau equation. A survey on different solutions to this equation and to its modified versions can be found in Ref. [12]. Very recently taking loss and gain effects into account it was shown that robust, but eventually unstable, dichromatic shocklike [13] and single-hump chirped bright solitary [14] waves may exist in a quadratic medium.

In this paper, we aim to identify double-hump chirped solitary waves in nonconservative quadratic media, and to prove their robustness numerically. We emphasize that the Ginzburg-Landau equation admits shocklike and bright solitary solutions, but no double-hump ones. As far as conservative systems are concerned, double-hump solitary waves have been shown to exist in homogeneous [15] as well as corrugated [16] $\chi^{(2)}$ media. They are unstable with various decay scenarios [15] in the former case, whereas they are stable in Bragg waveguides [16].

The system of equations describing pulse or beam propagation in a quadratically nonlinear medium with loss and gain, which in Ref. [14] was called $\chi^{(2)}$ Ginzburg-Landau equations, has the form

$$iA_x + D_1 A_{ss} + 2A^*B + i\gamma_1 A = 0, \tag{1a}$$

$$iB_x + kB + D_2 B_{ss} + A^2 + i\gamma_2 B = 0, \tag{1b}$$

where x is the propagation distance, s is the transverse coordinate in the spatial or retarded time in the temporal case, A and B are normalized envelopes of the first and second harmonics, k is the phase mismatch, $\gamma_{1,2}$ are linear gain or loss coefficients, and $D_{1,2} = D'_{1,2} + iD''_{1,2}$ are complex valued coefficients, where $D'_{1,2}$ accounts for dispersion and diffraction and $D''_{1,2}$ for bandwidth-limited amplification or filtering in the temporal case. In the spatial case $D''_{1,2}$ occurs in the equations for the mean fields if the optical axis fluctuates around a zero mean (the fluctuating spatial walk off) [13].

System (1) has double-hump (in the fundamental field) chirped bright solitary wave solutions

$$A = a\,\sinh(\lambda s)[\cosh(\lambda s)]^{-2+i\varepsilon} e^{i(Qx+\varphi_1)},$$

$$B = b[\cosh(\lambda s)]^{-2+2i\varepsilon} e^{i(2Qx+\varphi_2)}, \tag{2}$$

where the amplitudes and the relative phase are given by

$$a^4 = 4b^2\lambda^4|D_2|^2(4\varepsilon^4 + 13\varepsilon^2 + 9),$$

$$b^2 = \lambda^4|D_1|^2(\varepsilon^4 + 13\varepsilon^2 + 36)/4, \tag{3}$$

$$\tan(\varphi_2 - 2\varphi_1) = \frac{6-\varepsilon^2 - 5d_1\varepsilon}{d_1(6-\varepsilon^2) + 5\varepsilon}, \tag{4}$$

where the chirp parameter ε is a solution of

$$2(d_1+d_2)(\varepsilon^4 - 20\varepsilon^2 + 9) + 15(d_1 d_2 - 1)(\varepsilon^3 - 3\varepsilon) = 0. \tag{5}$$

The width parameter and soliton wave vector are

$$\lambda^2 = \gamma_1 / D''_1(\varepsilon^2 + 2\varepsilon d_1 - 1) > 0, \tag{6}$$

$$Q = \lambda^2 D''_1[d_1(\varepsilon^2 - 1) - 2\varepsilon], \tag{7}$$

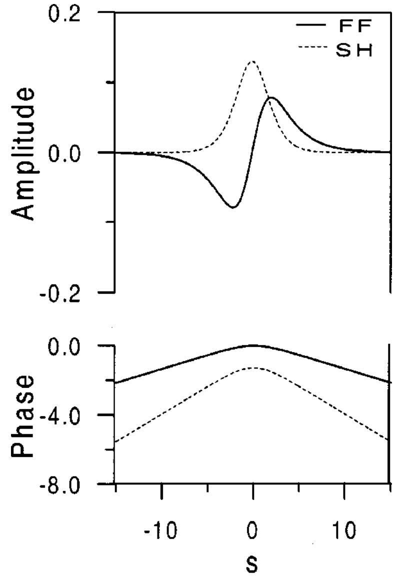

FIG. 1. Amplitude and phase profiles of the solution (2); system parameters: $\gamma_1=0.07$, $\gamma_2=-0.05$, $d_1=d_2=-2$, $D_1^*=-0.5$, $D_2^*=-0.25$, and $k=-0.06$, resulting in the soliton parameters $\varepsilon=1.03$, $\lambda=0.18$, $a=0.16$, $b=0.13$, $\varphi_1=0$, and $\tan(\varphi_2)=-3.25$.

respectively, where we have introduced $d_{1,2}=D'_{1,2}/D''_{1,2}$. This exact solution exist if the constraints

$$2\gamma_1 D_2''(1-\varepsilon d_2)=\gamma_2 D_1''(\varepsilon^2+2\varepsilon d_1-1), \tag{8}$$

$$k=2\lambda^2[(1-\varepsilon^2)D_1'+2\varepsilon D_1''+D_2'+\varepsilon D_2''] \tag{9}$$

are satisfied. These constraints ensure the balances between gain and loss and between up and down conversion which are necessary for stationary solutions to exist. An analysis of the constraints reveals that solutions exist in different domains of parameter space.

Both the shapes and the phases of the double hump solutions (2) are shown in Fig. 1 for a particular set of system parameters. We note a nonzero relative phase shift in the soliton center [cf. Eq. (4)]. The stability of the solitary wave solution was checked numerically. For this purpose we have performed extensive numerical experiments by using the exact soliton solution (2) for various consistent combinations of the relevant system parameters as initial conditions for the system (1). Strictly speaking, all solutions turned out to be unstable exhibiting different instability scenarios. But our studies also revealed domains where the solutions proved quite robust. A typical evolution of the FF and SH components in the domain of robustness is displayed in Fig. 2. The anticipated onset of background instability has been observed for large propagation distances, i.e., more than ten diffraction or dispersion lengths. This instability manifests itself by creating new peaks on the soliton tails which are spread away from the main part of the soliton, eventually leading to a stochastic field pattern. Another kind of evolution is shown in Fig. 3, where an initially single hump SH field splits into two parts, this process being initiated by the double hump FF. These two double-hump fields create a bound state that propagates up to a distance where the background instability comes into play.

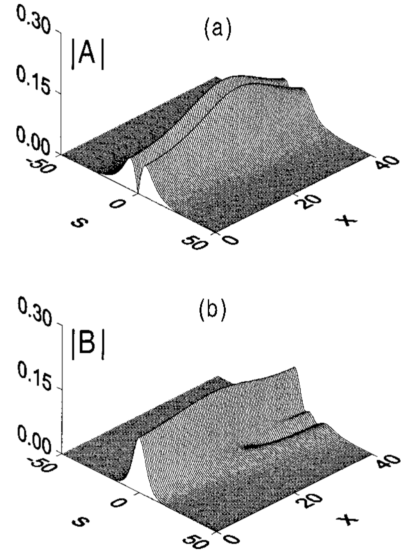

FIG. 2. Robust evolution of the FF and SH of solution (2); parameters as in Fig. 1.

System (1) admits some interesting particular solutions. Here we mention three of them permitting an explicit representation of the soliton parameters in terms of system parameters. The first one is a chirp-free solution, $\varepsilon=0$, existing for $d_1=-d_2$, including the dispersionless limit $d_1=d_2=0$. For $d_1=-d_2$, e.g., opposite signs of dispersion, the expressions for the FF and SH amplitudes and width parameter simplify to

$$b^2=\left|\frac{\gamma_1}{\gamma_2}\right|a^2=9\gamma_1^2(1+d_1^2), \quad \lambda^2=-\frac{\gamma_1}{D_1''}=\frac{\gamma_2}{2D_2''}>0. \tag{10}$$

We note that the existence of chirp-free solutions is a remarkable fact for a non-conservative system, e.g., they do not appear in the (cubic) Ginzburg-Landau equation. The evolution of this solution is shown in Fig. 4. It is evident that the soliton preserves its shape, but that its amplitude and width oscillate upon propagation until the soliton breaks up, (not shown in Fig. 4).

Other particular solutions occur if the (i) SH or (ii) FF absorption or gain equal zero ($\gamma_2=0$ or $\gamma_1=0$, respectively), and the net gain of either wave vanishes, i.e., the peak gain equals the losses. In case (i) the solution parameters are

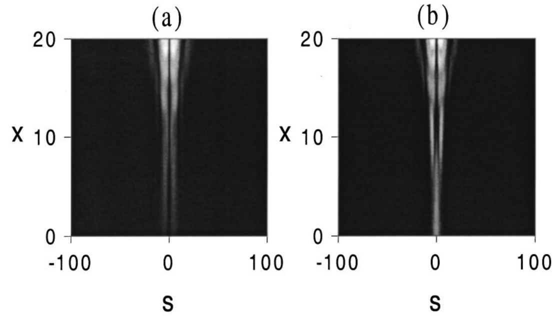

FIG. 3. Evolution of the FF and SH of solution (2); parameters: $\gamma_1=-0.39$, $\gamma_2=0.1$, $d_1=d_2=-2$, $D_1^*=-0.5$, $D_2^*=-0.25$, and $k=-0.27$ leading to $\varepsilon=-2.90$, $\lambda=0.2$, $a=0.56$, $b=0.34$, $\varphi_1=0$, and $\tan(\varphi_2)=3.25$.

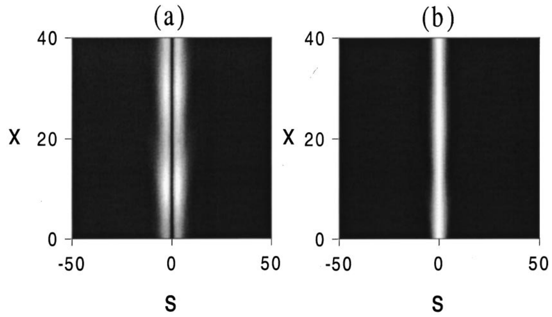

FIG. 4. Evolution of a chirp-free soliton for the parameters $\gamma_1 = -0.05$, $\gamma_2 = 0.05$, $d_1 = -d_2 = -2$, $D_1^* = -0.5$, $D_2^* = -0.25$, $k = 0.1$, $\lambda = 0.316$, $a = 0.33$, $b = 0.33$, $\varphi_1 = 0$, and $\tan(\varphi_2) = -0.5$.

$$\varepsilon = d_2^{-1}, \quad \lambda^2 = \frac{\gamma_1 d_2^2(27d_2^2 - 2)}{D_1''(3d_2^2+2)(3d_2^2-1)} > 0,$$

and the relation $d_1 = d_2[(18d_2^2 - 13)]/[(27d_2^2 - 2)]$ must be satisfied. The normalized width $w = \lambda^{-1}\sqrt{|\gamma_1/D_1''|}$ as a function of the parameter d_2 is shown in Fig. 5 for $(\gamma_1 D_1'') > 0$. As can be inferred from Fig. 5, the solution exists for $0 < |d_2| < \sqrt{2/27}$ and $|d_2| > 1/\sqrt{3}$. In case (ii) there are two solutions with the respective chirp $\varepsilon_\pm = -d_1 \pm \sqrt{1+d_1^2}$ and width parameters $\lambda_\pm^2 = \gamma_2/[2D_2''(1-\varepsilon_\pm d_2)] > 0$. Equation (5) provides the relation between the parameters d_1 and d_2.

In conclusion, we find a family of exact chirped double-hump solitary-wave solutions to the system of equations describing the wave propagation in quadratic nonlinear media with loss and gain. The system parameter constraints for the solutions to exist are the consequences of the mutual balancing of gain and loss as well as up and down conversion processes. Numerics reveal various instability scenarios, but also domains of fairly robust behavior of these waves.

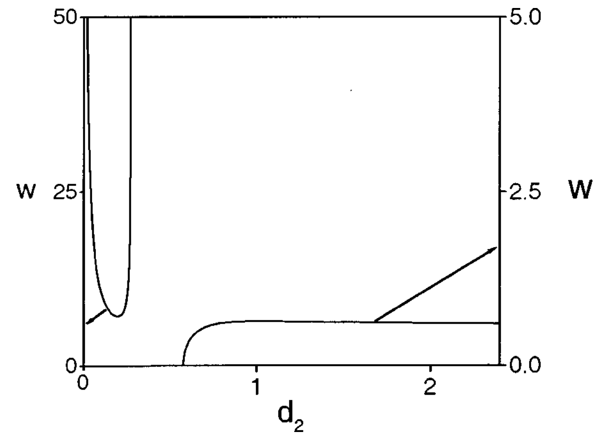

FIG. 5. Normalized soliton width w as a function of the dispersion parameter d_2 for $\gamma_2 = 0$.

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