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Tricritical points in the Sherrington-Kirkpatrick model in the presence of discrete random fields

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The infinite-range-interaction Ising spin glass is considered in the presence of an external random magnetic field following a trimodal (three-peak) distribution. Such a distribution corresponds to a bimodal added to a probability p_0 for a field dilution, in such a way that at each site the field h_i obeys $P(h_i) = p_+ \delta(h_i - h_0) + p_0 \delta(h_i) + p_- \delta(h_i + h_0)$. The model is studied through the replica method and phase diagrams are obtained within the replica-symmetry approximation. It is shown that the border of the ferromagnetic phase may present, for conveniently chosen values of p_0 and h_0 , first-order phase transitions, as well as tricritical points at finite temperatures. Analogous to what happens for the Ising ferromagnet under a trimodal random field, it is verified that the first-order phase transitions are directly related to the dilution in the fields: the extensions of these transitions are reduced for increasing values of p_0 . Whenever the δ function at the origin becomes comparable to those at $h_i = \pm h_0$, first-order phase transitions disappear; in fact, the threshold value p_0^* , above which all phase transitions are continuous, is calculated analytically as $p_0^* = 2(e^{3/2} + 2)^{-1} \approx 0.30856$. The ferromagnetic boundary at zero temperature also exhibits an interesting behavior: for $0 < p_0 < p_0^*$, a single tricritical point occurs, whereas if $p_0 > p_0^*$ the critical frontier is completely continuous; however, for $p_0 = p_0^*$, a fourth-order critical point appears. Stability analysis of the replica-symmetric solution is performed and the regions of validity of such a solution are identified; in particular, the Almeida-Thouless line in the plane field versus temperature is shown to depend on the weight p_0 .

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I. INTRODUCTION

Among disordered magnets [1], spin glasses [2–4], and ferromagnets in the presence of random fields [5–8] may be singled out as two of the most puzzling and controversial systems in condensed matter physics. The random-field Ising model (RFIM), introduced by Imry and Ma [9], has attracted much interest since the identification of its physical realizations. Probably the most important physical conception of the RFIM is a diluted Ising antiferromagnet in the presence of a uniform magnetic field [10,11]. Many diluted antiferromagnets have now been investigated, in such a way that systems like $\text{Fe}_x\text{Zn}_{1-x}\text{F}_2$ and $\text{Fe}_x\text{Mg}_{1-x}\text{Cl}_2$ are nowadays considered as standard experimental realizations of the RFIM [12,13].

From the theoretical point of view, many important ingredients remain unknown. At the mean-field level, it is well known that different probability distributions for the random fields may lead to distinct phase diagrams, e.g., a Gaussian probability distribution yields a continuous ferromagnetic-paramagnetic boundary [14], whereas for a bimodal distribution, this boundary exhibits a continuous piece at high temperatures ending up at a tricritical point, which is followed by a first-order phase transition at low temperatures [15]. Such a contrast in the mean-field phase diagrams of the

RFIM with bimodal and Gaussian probability distributions has been proven rigorously [16]. Indeed, Aharony [15] argued that whenever an analytic symmetric distribution for the fields presents a minimum at zero field, one should expect a tricritical point and a first-order transition for sufficiently low temperatures. Further studies of the RFIM at the mean-field level have considered a trimodal (three-peak) distribution [17,18]

$$P(h_i) = p_+ \delta(h_i - h_0) + p_0 \delta(h_i) + p_- \delta(h_i + h_0), \quad (1.1)$$

in its symmetrical form, i.e., $p_+ = p_- = \frac{1}{2}(1 - p_0)$. Such a distribution, which may be interpreted as a bimodal added to a dilution in the fields with probability p_0 [17], is expected to mimic real systems better than its bimodal counterpart. It has been shown that the field dilution plays an important role in the occurrence of the tricritical point: distinct analyses lead to slightly different estimates for the threshold value above which the tricritical point disappears (whereas the analysis of Mattis [17] shows that the tricritical point vanishes for $p_0 > 0.25$, according to Kaufman *et al.* [18] such a behavior should occur for $p_0 > 0.24$). Whether the features in the mean-field phase diagrams of the RFIM should prevail in short-range-interaction models represents a point that has attracted a lot of interest [19–22]. For the three-dimensional

RFIM, recent Monte Carlo simulations detect a jump in the magnetization but no latent heat, for both bimodal [19] and Gaussian [20] distributions, whereas high-temperature series expansions [21] and a zero-temperature scaling analysis [22] find a continuous transition for both distributions. However, in four dimensions the same zero-temperature analysis [22] leads to a first-order transition in the bimodal case and a continuous one for a Gaussian distribution, in agreement with the mean-field predictions. Apart from that, the low-temperature phase of the RFIM, in finite dimensions, may present a nontrivial structure, with a complicated free-energy landscape, as suggested by perturbative analyses [23,24].

The Ising spin-glass (ISG) problem has become, nowadays, one of the most controversial issues in the physics of disordered magnets. Its mean-field theory, based on the solution of the infinite-range-interaction model, the so-called Sherrington-Kirkpatrick (SK) model [25], presents a quite nontrivial behavior. The correct low-temperature solution, as proposed by Parisi [26], consists of a continuous order-parameter function (i.e., an infinite number of order parameters) associated with many low-energy states, a procedure which is usually denominated the replica-symmetry breaking (RSB). Furthermore, a transition in the presence of an external magnetic field, known as the Almeida-Thouless (AT) line [27], is found in the solution of the SK model: such a line separates a low-temperature region, characterized by RSB, from a high-temperature one, where a simple one-parameter solution, denominated as replica-symmetric (RS) solution, is stable. The validity of the results of the SK model for the description of real (short-range-interaction) systems represents a very polemic question [2]. The rival theory is the droplet model [28], based on domain-wall renormalization-group arguments for spin glasses [29,30]. According to the droplet model, the low-temperature phase of any *finite-dimensional* short-range spin glass should be described in terms of a single thermodynamic state (together, of course, with its corresponding time-reversed counterpart), i.e., essentially a RS-type of solution. Obviously, the droplet model becomes questionable for increasing dimensionalities, where one expects the existence of a finite upper critical dimension—believed to be six for the ISG [31]—above which the mean-field picture should prevail. Recent analyses of short-range ISG's on diamond hierarchical lattices (on which the Migdal-Kadanoff renormalization group is exact) have found evidence of the droplet picture [32]; however, the applicability of such lattices for the description of ISG's on Bravais lattices is doubtful [33,34]. Numerical simulations are very hard to carry out for short-range ISG's on a cubic lattice, due to large thermalization times [33]; as a consequence, no conclusive results in three-dimensional systems are available. However, in four dimensions the critical temperature is much higher, making thermalization easier; in this case, many works claim to have observed some mean-field features [35].

From the theoretical point of view these two problems (RFIM and ISG), have been, in most cases, studied separately, with a few exceptions [36–40]. However, many diluted antiferromagnets, like $\text{Fe}_x\text{Zn}_{1-x}\text{F}_2$ [41] and $\text{Fe}_x\text{Mg}_{1-x}\text{Cl}_2$ [42,43], are able to exhibit, within certain concentration ranges, random-field, spin-glass, or both behaviors. For $\text{Fe}_x\text{Zn}_{1-x}\text{F}_2$, one gets a RFIM for $x \geq 0.40$ and an

ISG for $x \leq 0.24$, whereas for intermediate concentrations ($0.24 \leq x \leq 0.40$) one may observe both behaviors depending on the magnitude of the applied external magnetic field [RFIM (ISG) for small (large) magnetic fields], with a crossover between them; this latter effect was observed in $\text{Fe}_{0.31}\text{Zn}_{0.69}\text{F}_2$ [41]. Certainly, such properties are expected to be properly explained only if one considers a model that takes into account both spin-glass and random-field ingredients. Indeed, the crossover observed in $\text{Fe}_{0.31}\text{Zn}_{0.69}\text{F}_2$ was also found in a study of the SK model under a Gaussian random field [38]. On the other hand, a study of the SK model in the presence of a bimodal random field produced interesting results, with first-order phase transitions and tricritical points [39]; such results may be relevant for explaining the first-order phase transitions observed in $\text{Fe}_x\text{Mg}_{1-x}\text{Cl}_2$ [13].

In the present work we study the SK model in the presence of a random field following a trimodal probability distribution [see Eq. (1.1)]. In addition to that, one may interpolate between the bimodal distribution and a behavior that is qualitatively analogous to the Gaussian one, since by monitoring the δ function at the origin, one is able to control the presence of tricritical points. In the next section we define the model and, through the use of the replica method, we find its free-energy density, equations of state, and equations for the validity of the RS solution. In Sec. III we exhibit and discuss the phase diagrams of the model. Finally, in Sec. IV we present our conclusions.

II. THE MODEL AND REPLICA FORMALISM

The mean-field theory of the ISG is usually formulated as a set of N spins, each of them interacting with all others [a total of $1/2N(N-1)$ interactions], known as the SK model [25]. The SK model in the presence of an external random magnetic field may be defined in terms of the Hamiltonian [38,39],

$$\mathcal{H} = - \sum_{(ij)} J_{ij} S_i S_j - \sum_i h_i S_i, \quad (2.1)$$

where $S_i = \pm 1$, with $i = 1, 2, \dots, N$, and the interactions are infinite-range-like, i.e., the sum $\sum_{(i,j)}$ applies to all distinct pairs of spins. The coupling constants $\{J_{ij}\}$ and the random fields $\{h_i\}$ are quenched variables, following independent probability distributions,

$$P(J_{ij}) = \left(\frac{N}{2\pi J^2} \right)^{1/2} \exp \left[- \frac{N}{2J^2} \left(J_{ij} - \frac{J_0}{N} \right)^2 \right], \quad (2.2)$$

with $P(h_i)$ given by Eq. (1.1) ($p_+ + p_0 + p_- = 1$). Let us, for the moment, keep the trimodal probability distribution in its general form of Eq. (1.1); later on, we will see that the ferromagnetic boundary does not exist for $p_+ \neq p_-$, and so, in such a case, we will be restricted to the symmetrical form $p_+ = p_- = \frac{1}{2}(1 - p_0)$. It should be mentioned that the above randomnesses ($\{J_{ij}\}$ and $\{h_i\}$) are usually correlated in real systems; herein for the sake of simplicity we shall consider two independent probability distributions. Therefore, for a given realization of bonds and site fields, ($\{J_{ij}\}, \{h_i\}$), one

has a corresponding free energy, $F(\{J_{ij}\}, \{h_i\})$, such that the average over the disorder, $[\]_{J,h}$, may be performed as independent integrals,

$$[F(\{J_{ij}\}, \{h_i\})]_{J,h} = \int \prod_{(ij)} [dJ_{ij} P(J_{ij})] \prod_i [dh_i P(h_i)] \times F(\{J_{ij}\}, \{h_i\}). \quad (2.3)$$

The usual procedure consists in applying the replica method [3,4], in such a way as to get the free energy per spin as

$$-\beta f = \lim_{N \rightarrow \infty} \frac{1}{N} [\ln Z(\{J_{ij}\}, \{h_i\})]_{J,h} = \lim_{N \rightarrow \infty} \lim_{n \rightarrow 0} \frac{1}{Nn} \times ([Z^n]_{J,h} - 1), \quad (2.4)$$

where Z^n is the partition function of n copies of the system defined in Eq. (2.1) and $\beta = 1/T$ (we work in units $k_B = 1$). Standard calculations lead to

$$\beta f = -\frac{(\beta J)^2}{4} + \lim_{n \rightarrow 0} \frac{1}{n} \min g(m^\alpha, q^{\alpha\beta}), \quad (2.5)$$

where

$$g(m^\alpha, q^{\alpha\beta}) = \frac{\beta J_0}{2} \sum_\alpha (m^\alpha)^2 + \frac{(\beta J)^2}{2} \sum_{(\alpha\beta)} (q^{\alpha\beta})^2 - p_+ \ln \text{Tr}_\alpha \exp(\mathcal{H}_{eff}^+) - p_0 \ln \text{Tr}_\alpha \exp(\mathcal{H}_{eff}^0) - p_- \ln \text{Tr}_\alpha \exp(\mathcal{H}_{eff}^-), \quad (2.6a)$$

$$\mathcal{H}_{eff}^\pm = \beta J_0 \sum_\alpha m^\alpha S^\alpha + (\beta J)^2 \sum_{(\alpha\beta)} q^{\alpha\beta} S^\alpha S^\beta \pm \beta h_0 \sum_\alpha S^\alpha, \quad (2.6b)$$

$$\mathcal{H}_{eff}^0 = \beta J_0 \sum_\alpha m^\alpha S^\alpha + (\beta J)^2 \sum_{(\alpha\beta)} q^{\alpha\beta} S^\alpha S^\beta. \quad (2.6c)$$

In the equations above, the sum indexes α and β ($\alpha, \beta = 1, 2, \dots, n$) are replica labels and $\sum_{(\alpha\beta)}$ denote sums over distinct pairs of replicas.

The extrema of the functional $g(m^\alpha, q^{\alpha\beta})$ give us the equilibrium equations for the magnetization and spin-glass order parameters, respectively,

$$m^\alpha = p_+ \langle S^\alpha \rangle_+ + p_0 \langle S^\alpha \rangle_0 + p_- \langle S^\alpha \rangle_-, \quad (2.7a)$$

$$q^{\alpha\beta} = p_+ \langle S^\alpha S^\beta \rangle_+ + p_0 \langle S^\alpha S^\beta \rangle_0 + p_- \langle S^\alpha S^\beta \rangle_- \quad (\alpha \neq \beta), \quad (2.7b)$$

where $\langle \rangle_\pm$ and $\langle \rangle_0$ refer to thermal averages with respect to the ‘‘effective Hamiltonians’’ \mathcal{H}_{eff}^\pm and \mathcal{H}_{eff}^0 in Eqs. (2.6b) and (2.6c), respectively.

If one assumes the replica-symmetry (RS) ansatz [25],

$$m^\alpha = m, \quad \forall \alpha; \quad q^{\alpha\beta} = q, \quad \forall (\alpha\beta), \quad (2.8)$$

the free energy per spin [Eq. (2.5)] and the equilibrium conditions [Eqs. (2.7)] become

$$\beta f = -\frac{(\beta J)^2}{4} (1-q)^2 + \frac{\beta J_0}{2} m^2 - p_+ \int \mathcal{D}z \ln(2 \cosh \xi^+) - p_0 \int \mathcal{D}z \ln(2 \cosh \xi^0) - p_- \int \mathcal{D}z \ln(2 \cosh \xi^-), \quad (2.9)$$

$$m = p_+ \int \mathcal{D}z \tanh \xi^+ + p_0 \int \mathcal{D}z \tanh \xi^0 + p_- \int \mathcal{D}z \tanh \xi^-, \quad (2.10)$$

$$q = p_+ \int \mathcal{D}z \tanh^2 \xi^+ + p_0 \int \mathcal{D}z \tanh^2 \xi^0 + p_- \int \mathcal{D}z \tanh^2 \xi^-, \quad (2.11)$$

where

$$\int \mathcal{D}z \dots = \int_{-\infty}^{\infty} \left(\frac{1}{2\pi} \right)^{1/2} dz \exp(-z^2/2) \dots, \quad (2.12)$$

and

$$\xi^\pm = \beta J_0 m + \beta J q^{1/2} z \pm \beta h_0, \quad (2.13a)$$

$$\xi^0 = \beta J_0 m + \beta J q^{1/2} z. \quad (2.13b)$$

Although the spin-glass order parameter [Eq. (2.11)] is always induced by a nonzero random field ($p_0 < 1$), it may still contribute to a nontrivial behavior; this is provided by the instability of the RS solution. Such an instability occurs at the AT line [27],

$$\left(\frac{T}{J} \right)^2 = p_+ \int \mathcal{D}z \text{sech}^4 \xi^+ + p_0 \int \mathcal{D}z \text{sech}^4 \xi^0 + p_- \int \mathcal{D}z \text{sech}^4 \xi^-, \quad (2.14)$$

which may be obtained through the simultaneous solution of Eqs. (2.14), (2.10), and (2.11).

In the next section we shall consider the phase diagrams of the model and the regions of instability of the RS solution, worked out from Eqs. (2.9)–(2.14).

III. RESULTS AND DISCUSSION

Let us first consider the case $J_0 = 0$; one may easily see that the only nontrivial behavior in this case is given by the AT instability in the plane magnetic field versus temperature, which may now be obtained from the solution of Eqs. (2.11) and (2.14). The integrals involving ξ^- may be easily transformed through the change of variables $z \rightarrow -z$, in such a way that the AT line may be obtained by solving the set of equations,

$$\left(\frac{T}{J} \right)^2 = (1-p_0) \int \mathcal{D}z \text{sech}^4(\beta J q^{1/2} z + \beta h_0) + p_0 \int \mathcal{D}z \text{sech}^4(\beta J q^{1/2} z), \quad (3.1a)$$

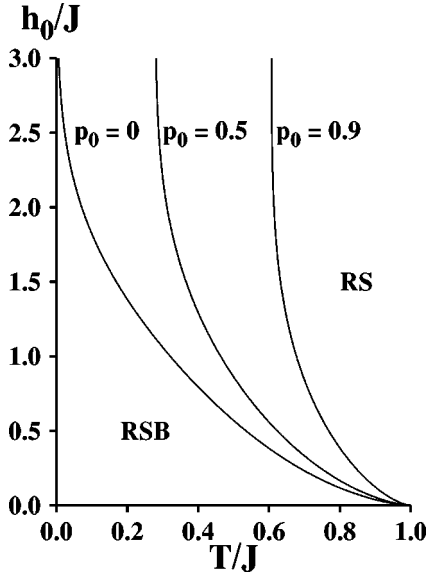


FIG. 1. The AT lines, for the SK model in the presence of a trimodal random field, in the plane h_0 versus T (in units of J), for typical values of p_0 .

$$q = (1-p_0) \int \mathcal{D}z \tanh^2(\beta J q^{1/2} z + \beta h_0) + p_0 \int \mathcal{D}z \tanh^2(\beta J q^{1/2} z). \quad (3.1b)$$

It should be pointed out that the equations above are valid for arbitrary values of the weights in the probability distribution of Eq. (1.1), with $p_+ + p_- = 1 - p_0$; although the AT line changes with field dilution, it is not altered under a field inversion. The AT lines in the plane magnetic field versus temperature are exhibited in Fig. 1, for typical values of p_0 . Clearly, the AT line for the bimodal distribution ($p_0=0$) [39] is identical to that of the SK model in the presence of a uniform magnetic field [27], due to the property of invariance under field inversion. For $0 < p_0 < 1$, one may calculate analytically the behavior of the AT line in the low-field regime ($T \cong J$),

$$1 - \frac{T}{J} \cong \left[\frac{3(1-p_0)}{4} \right]^{1/3} \left(\frac{h_0}{J} \right)^{2/3}, \quad (3.2)$$

which leads to a slightly modified amplitude, but the same low-field exponent as the standard AT line [27]. If one considers $p_0 \sim 0$, the low-temperature behavior of the AT line may be easily calculated,

$$\frac{T}{J} \cong \frac{4}{3} \frac{1}{\sqrt{2\pi}} \left[(1-p_0) \exp\left(-\frac{h_0^2}{2J^2}\right) + p_0 \right], \quad (3.3)$$

which exhibits the usual exponential decay [27], but with a shift toward higher temperatures for increasing values of p_0 . In all other situations, the AT lines were calculated by solving Eqs. (3.1) numerically. One notices that for high values of p_0 , the integrals multiplying p_0 in Eqs. (3.1) contribute significantly, in such a way that the AT lines become slightly independent of h_0 , for h_0 large enough, as shown in Fig. 1.

From now on, we will be restricted to $J_0 > 0$; in this case, as far as RS is concerned, if $p_+ \neq p_-$ Eqs. (2.10) and (2.11) yield nonzero magnetization and spin-glass order parameters, leading to trivial behavior. Therefore, for the rest of this paper we will concentrate on a symmetrical trimodal distribution, i.e., $p_+ = p_- = \frac{1}{2}(1-p_0)$. In this case, the random field still induces a nonzero value for the parameter q , forcing spin-glass order (unlike the spontaneous order of the SK model in the zero external field [25]). Therefore, the only possible phase transition within the RS approximation is the one associated with the magnetization, as in the case of the bimodal distribution [39]. Hence, two phases are possible, namely, the ferromagnetic ($m \neq 0$, $q \neq 0$) and the independent ($m=0$, $q \neq 0$) ones. Although in the RFIM this latter phase is usually said to be paramagnetic, in the present problem, within the RS approximation, we shall keep the nomenclature independent, for reasons that will become clear soon.

The critical frontier separating these two phases may be found by solving the equilibrium equations (2.10) and (2.11); in the case of first-order phase transitions, we shall make use of the free energy per spin [Eq. (2.9)] as well. Expanding Eq. (2.10) in powers of m one gets

$$m = A_1(q)m + A_3(q)m^3 + A_5(q)m^5 + O(m^7), \quad (3.4)$$

where the coefficients depend on q [which in its turn depends on m through Eq. (2.11)]. Expanding Eq. (2.11) in powers of m ,

$$q = q_0 + \frac{(\beta J_0)^2 \Gamma}{1 - (\beta J)^2 \Gamma} m^2 + O(m^4), \quad (3.5)$$

with

$$\Gamma = (1-p_0)(1-4\rho_1^+ + 3\rho_2^+) + p_0(1-4\rho_1^0 + 3\rho_2^0), \quad (3.6)$$

$$\rho_k^+ = \int \mathcal{D}z \tanh^{2k}(\beta J q_0^{1/2} z + \beta h_0), \quad (3.7a)$$

$$\rho_k^0 = \int \mathcal{D}z \tanh^{2k}(\beta J q_0^{1/2} z), \quad (3.7b)$$

where q_0 is independent of m , corresponding to the solution of Eq. (2.11) with $m=0$. Substituting the above results into Eq. (3.4), one gets the m -independent coefficients of the power expansion,

$$A'_1 = \beta J_0 [1 - (1-p_0)\rho_1^+ - p_0\rho_1^0], \quad (3.8a)$$

$$A'_3 = -\frac{(\beta J_0)^3}{3} \left[\frac{1 + 2(\beta J)^2 \Gamma}{1 - (\beta J)^2 \Gamma} \right] \Gamma, \quad (3.8b)$$

$$A'_5 = -\gamma \frac{(\beta J_0)^5}{30} \left[\frac{1 + 8(\beta J)^2 \Gamma + 36(\beta J)^4 \Gamma^2 + 15(\beta J)^6 \Gamma^3}{1 - (\beta J)^2 \Gamma} \right], \quad (3.8c)$$

where

$$\gamma = (1-p_0)(-4 + 34\rho_1^+ - 60\rho_2^+ + 30\rho_3^+) + p_0(-4 + 34\rho_1^0 - 60\rho_2^0 + 30\rho_3^0). \quad (3.9)$$

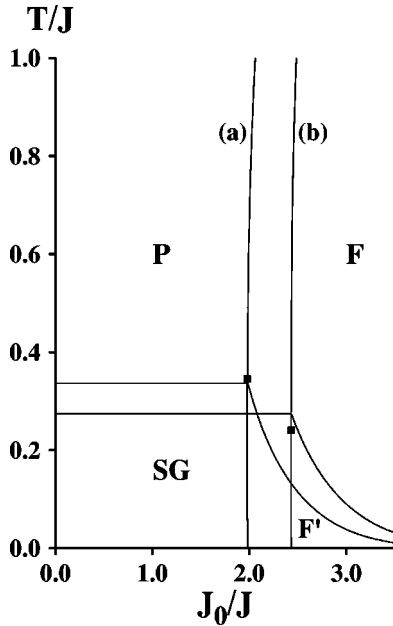


FIG. 2. Phase diagram T versus J_0 (in units of J) of the SK model in the presence of a trimodal random field with $p_0=0.3$, compared with that of the bimodal case ($p_0=0$), for conveniently chosen values of h_0 . (a) $h_0/J=0.9573$ ($p_0=0$); (b) $h_0/J=1.53526$ ($p_0=0.3$). The ferromagnetic boundaries are continuous, except for the points where $A'_3=0$ [cf. Eq. (3.8b)], represented by black squares. These choices signal lower bounds for h_0 , above which first-order phase transitions occur. The phase nomenclature is specified in the text, with the low-temperature phases SG and F' delimited by AT lines.

The critical frontier may be determined using standard procedures, as described below.

(i) For continuous phase transitions, $A'_1=1$ and $A'_3<0$.

(ii) A first-order phase transition occurs whenever $A'_1=1$ and $A'_3>0$; the proper critical frontier should be found, in this case, through a Maxwell construction, i.e., by equating the free energies of the two phases.

(iii) When both types of phase transition are present, the continuous and first-order critical frontiers meet at a tricritical point [45], which defines the limit of validity of the series expansions; beyond the tricritical point the magnetization is discontinuous. The location of such a point is determined by setting $A'_1=A'_3=0$, with the condition $A'_5<0$ satisfied.

In Figs. 2–4 we show three qualitatively distinct ferromagnetic boundaries of the present problem, for a typical value of p_0 ($p_0=0.3$), compared with those of the bimodal probability distribution ($p_0=0$). In Fig. 2 there is a single point along the ferromagnetic boundary at which $A'_3=0$; such a point may not be considered as tricritical, since there is no first-order phase transition. However, for any value of h_0 greater than those of Fig. 2 [$h_0/J=0.9573$ ($p_0=0$) and $h_0/J=1.53526$ ($p_0=0.3$)], one gets first-order phase transitions, and at least one tricritical point. In Fig. 3 we show situations where two tricritical points appear along the ferromagnetic boundary; we have verified that, for a fixed value of p_0 , such a behavior occurs within a narrow interval of h_0 . In Fig. 4 a single tricritical point emerges, separating a continuous boundary (high temperatures) from a first-order critical frontier (low temperatures). From such phase diagrams,

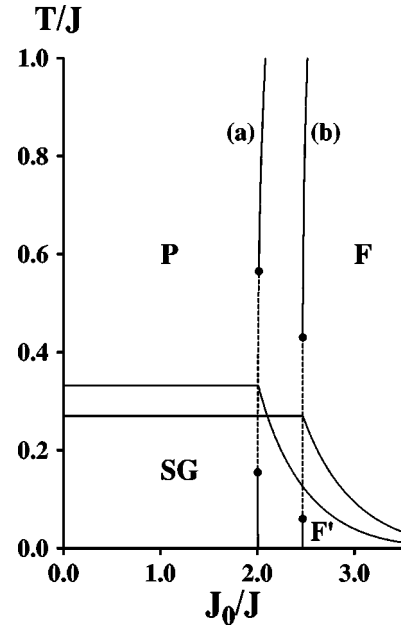


FIG. 3. Phase diagram T versus J_0 (in units of J) of the SK model in the presence of a trimodal random field with $p_0=0.3$, compared with that of the bimodal case ($p_0=0$), for conveniently chosen values of h_0 , in such a way as to obtain two tricritical points (black circles) along the ferromagnetic boundary. (a) $h_0/J=0.97$ ($p_0=0$); (b) $h_0/J=1.558$ ($p_0=0.3$). The dashed lines stand for first-order phase transitions. The phase nomenclature is the same as in Fig. 2.

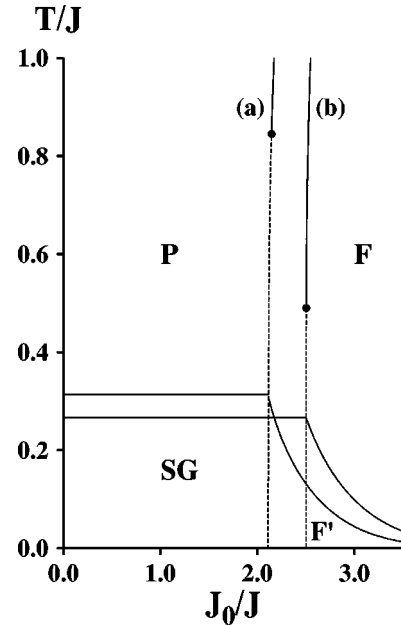


FIG. 4. Phase diagram T versus J_0 (in units of J) of the SK model in the presence of a trimodal random field with $p_0=0.3$, compared with that of the bimodal case ($p_0=0$), for conveniently chosen values of h_0 , in such a way as to obtain a single tricritical point (black circle) along the ferromagnetic boundary. (a) $h_0/J=1.02$ ($p_0=0$); (b) $h_0/J=1.58$ ($p_0=0.3$). The phase nomenclature and line representations are as in Figs. 2 and 3.

one notices that the main effect of the field dilution is to push the tricritical points toward lower temperatures, i.e., the temperature range over which the first-order transitions occur decreases.

As mentioned before, although the spin-glass order parameter is always induced by the random field, it may still exhibit interesting behavior, associated with the instability of the RS solution. The AT instabilities, given by the solution of Eqs. (2.10), (2.11), and (2.14) with $p_+ = p_- = \frac{1}{2}(1-p_0)$, yield two distinct lines in the phase diagrams of Figs. 2–4, depending on whether one is in the independent phase ($m=0$) or in the ferromagnetic ($m \neq 0$) one. In the former case, the AT line is a straight line (independent of J_0), whereas in the latter, it presents the usual decrease with temperature for increasing values of J_0 , in such a way that for low temperatures one gets exponential decays,

$$\frac{T}{J} \cong \frac{4}{3} \frac{1}{\sqrt{2\pi}} \left\{ \frac{1}{2} (1-p_0) \exp \left[-\frac{(J_0+h_0)^2}{2J^2} \right] + p_0 \exp \left[-\frac{J_0^2}{2J^2} \right] + \frac{1}{2} (1-p_0) \exp \left[-\frac{(J_0-h_0)^2}{2J^2} \right] \right\}. \quad (3.10)$$

Herein we shall adopt the usual criteria for identification of the regions where RS is stable and those throughout which a RSB procedure is necessary [3,4]. The two regions with zero magnetization will be associated with the paramagnetic (high temperatures) and spin-glass (low temperatures) phases, whereas those with nonzero magnetization will be associated with the ferromagnetic (high temperatures) and mixed-ferromagnetic (low temperatures) phases. The several phases exhibited in our phase diagrams are identified as follows.

Paramagnetic (P)	($m=0$; q : RS);
Spin-Glass (SG)	($m=0$; q : RSB);
Ferromagnetic (F)	($m \neq 0$; q : RS);
Mixed Ferromagnetic (F')	($m \neq 0$; q : RSB).

It should be mentioned that the present low-temperature results are questionable inside the phases F' and SG, due to the instability of the RS solution; in particular, the point for $p_0=0.3$ where $A'_3=0$ in Fig. 2, as well as the low-temperature tricritical points of Fig. 3, may completely disappear under a RSB procedure. However the high-temperature tricritical points, like those of Figs. 3 and 4, are inside the region of stability of the RS solution and will persist under more general treatments; we believe that such points are reminiscent of the tricritical point of the bimodal RFIM.

The two AT lines mentioned above usually meet at a continuous ferromagnetic boundary; however, these lines do not match each other across first-order phase transitions [39,40,46]: there is a small (but finite) gap between them in Figs. 3 and 4.

Let us now investigate the ferromagnetic boundary at zero temperature; for $T=0$ the spin-glass order parameter is trivial ($q=1$), in such a way that one gets for the free energy and magnetization,

$$f = -\frac{J_0}{2} m^2 - \frac{h_0}{2} (1-p_0) \left[\operatorname{erf} \left(\frac{J_0 m + h_0}{J\sqrt{2}} \right) - \operatorname{erf} \left(\frac{J_0 m - h_0}{J\sqrt{2}} \right) \right] - \frac{J}{\sqrt{2\pi}} (1-p_0) \left\{ \exp \left[-\frac{(J_0 m + h_0)^2}{2J^2} \right] + \exp \left[-\frac{(J_0 m - h_0)^2}{2J^2} \right] \right\} - \frac{2J}{\sqrt{2\pi}} p_0 \left\{ \exp \left[-\frac{(J_0 m)^2}{2J^2} \right] \right\}, \quad (3.11a)$$

$$m = \frac{1}{2} (1-p_0) \left[\operatorname{erf} \left(\frac{J_0 m + h_0}{J\sqrt{2}} \right) + \operatorname{erf} \left(\frac{J_0 m - h_0}{J\sqrt{2}} \right) \right] + p_0 \operatorname{erf} \left(\frac{J_0 m}{J\sqrt{2}} \right). \quad (3.11b)$$

Using a similar procedure as the one for finite temperatures, one may expand Eq. (3.11b),

$$m = a_1 m + a_3 m^3 + a_5 m^5 + O(m^7), \quad (3.12)$$

where

$$a_1 = \sqrt{\frac{2}{\pi}} \frac{J_0}{J} \left[(1-p_0) \exp \left(-\frac{h_0^2}{2J^2} \right) + p_0 \right], \quad (3.13a)$$

$$a_3 = \frac{1}{6} \sqrt{\frac{2}{\pi}} \left(\frac{J_0}{J} \right)^3 \left[(1-p_0) \left(\frac{h_0^2}{J^2} - 1 \right) \exp \left(-\frac{h_0^2}{2J^2} \right) - p_0 \right], \quad (3.13b)$$

$$a_5 = \frac{1}{120} \sqrt{\frac{2}{\pi}} \left(\frac{J_0}{J} \right)^5 \left[(1-p_0) \left(\frac{h_0^4}{J^4} - 6 \frac{h_0^2}{J^2} + 3 \right) \times \exp \left(-\frac{h_0^2}{2J^2} \right) - 3p_0 \right]. \quad (3.13c)$$

The critical frontier separating the phases F' and SG is shown in Fig. 5 for typical values of p_0 . One notices that the effect of the weight p_0 is to favor the continuous line, along which $a_1=1$ with $a_3 < 0$, i.e.,

$$\frac{J_0}{J} = \sqrt{\frac{\pi}{2}} \frac{1}{p_0 + (1-p_0) \exp(-h_0^2/2J^2)}, \quad (3.14)$$

while decreasing the extension of the first-order transition line. For small values of p_0 these two lines meet at a tricritical point, obtained by solving the equations $a_1=1$, $a_3=0$, with the condition $a_5 < 0$; within the analysis for finite temperatures, this corresponds to the situation where the lower-temperature tricritical point (cf. Fig. 3) hits the zero-temperature axis. If $p_0=0$ such an effect occurs at [39]

$$\frac{h_0}{J} = 1; \quad \frac{J_0}{J} = \sqrt{\frac{\pi e}{2}} \approx 2.0664. \quad (3.15)$$

We verified that for $0 < p_0 < p_0^*$ (where p_0^* will be defined below), such a set of equations presents two solutions, although only one of them represents a tricritical point, satisfying $a_5 < 0$. By increasing p_0 inside this range, we noticed

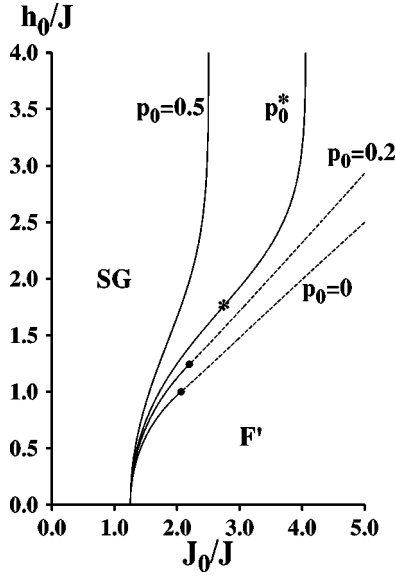


FIG. 5. The zero-temperature phase diagram h_0 versus J_0 (in units of J) of the SK model in the presence of a trimodal random field, for typical values of p_0 . If $0 < p_0 < p_0^*$ one always gets tricritical points (black circles), followed by first-order phase transitions for high values of h_0 . When $p_0 = p_0^*$, one gets a fourth-order critical point (represented by a star). Above the threshold value $p_0^* = 2(e^{3/2} + 2)^{-1} \approx 0.30856$, the critical frontier separating the phases SG and F' is continuous.

that such solutions get closer and collapse for $p_0 = p_0^*$. We calculated analytically $p_0^* = 2(e^{3/2} + 2)^{-1} \approx 0.30856$, at which a fourth-order critical point [47] (characterized by $a_1 = a_3 = a_5 = 0$, with $a_7 < 0$) occurs at

$$\frac{h_0}{J} = \sqrt{3} \approx 1.73207; \quad \frac{J_0}{J} = \frac{\sqrt{2\pi}}{6} (e^{3/2} + 2) \approx 2.70786. \quad (3.16)$$

The value p_0^* represents a threshold of p_0 above which there are no first-order transitions for any temperature $T \geq 0$. For $p_0 > p_0^*$ the second-order critical frontier of Fig. 5 approaches an asymptote for large values of h_0 ; indeed, when $p_0 \rightarrow 1$ the zero-temperature ferromagnetic boundary approaches a straight line at $J_0/J = \sqrt{\pi/2}$ [see Eq. (3.14)], characteristic of the SK model in zero field [25].

It should be mentioned that the finite-temperature vestigial points where $A'_3 = 0$, like the ones in Fig. 2, are qualitatively different from the fourth-order critical point found for $p_0 = p_0^*$ at zero temperature, even though both situations represent thresholds for the occurrence of tricritical points. In the former case, $A'_5 < 0$, whereas in the latter, $A'_5 = 0$. In Fig. 6 we exhibit the behavior of the coefficients A'_3 and A'_5 , for temperatures along the ferromagnetic frontier, for the case (b) of Fig. 2, i.e., $p_0 = 0.3$ ($h_0/J = 1.53526$), and $p_0 = p_0^*$ ($h_0/J = \sqrt{3}$). One clearly sees that the fourth-order critical point shows up only at zero temperature; its parameters, as defined in Eq. (3.16), correspond to the situation where the vestigial point of Fig. 2 collapses on the zero-temperature axis.

If $0 < p_0 < p_0^*$, it is always possible to obtain first-order phase transitions by conveniently choosing the value of h_0 .

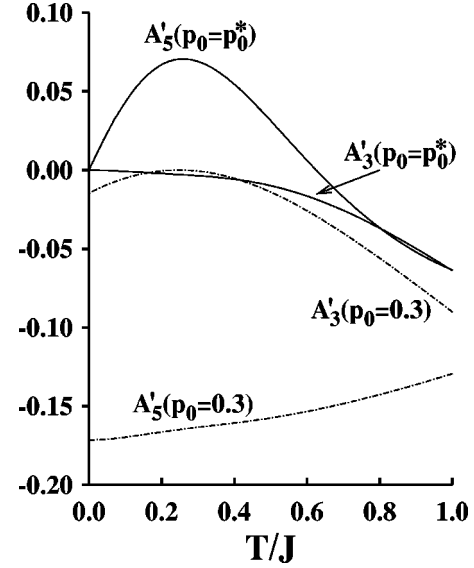


FIG. 6. The ordinate represents either the coefficient A'_3 or A'_5 [Eqs. (3.8b) and (3.8c), respectively] along the ferromagnetic boundary, for $p_0 = 0.3$ ($h_0/J = 1.53526$) (dot-dashed lines) and $p_0 = p_0^*$ ($h_0/J = \sqrt{3}$) (full lines), as a function of temperature. In the former case, $A'_3 = 0$ at $T/J \approx 0.25$ (with $A'_5 < 0$), whereas in the latter, $A'_3 = A'_5 = 0$ at $T = 0$.

In Fig. 7 we exhibit the ranges of p_0 and h_0/J throughout which first-order phase transitions and tricritical points are possible along the ferromagnetic boundary. In region (a), first-order phase transitions are conceivable at finite and zero temperature, with a single tricritical point (at finite temperatures); typical examples are shown in Fig. 4. Throughout a very narrow range [region (b)] two tricritical points appear

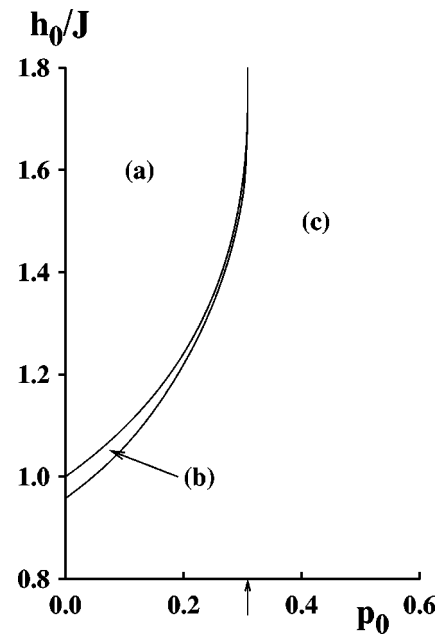


FIG. 7. Ranges of p_0 and h_0/J associated with distinct behaviors for the ferromagnetic boundary. (a) First-order phase transitions at finite and zero temperatures, with a single tricritical point at finite temperatures; (b) two tricritical points with a first-order phase transition for finite temperatures; (c) continuous phase transitions. The arrows indicates the value $p_0 = p_0^*$.

and the first-order phase transition occurs only for finite temperatures: typical examples are exhibited in Fig. 3. The region (b) is delimited by characteristic values of $(p_0, h_0/J)$: (i) the threshold for h_0/J smaller corresponds to the set of points satisfying $A_3' = 0$, but with no first-order phase transition (e.g., the vestigial points shown in Fig. 2); (ii) the delimiter for h_0/J larger corresponds to the coordinates of the tricritical points at zero temperature. The vertical line in Fig. 7 is for $p_0 = p_0^*$, defining [together with the delimiter (i) of region (b)], the range throughout which the ferromagnetic boundary is always continuous [region (c)].

IV. CONCLUSION

We have studied the Sherrington-Kirkpatrick spin glass in the presence of random fields $\{h_i\}$, following a trimodal (three-peak) probability distribution, which corresponds to a bimodal plus a probability p_0 for field dilution, i.e., $P(h_i) = p_+ \delta(h_i - h_0) + p_0 \delta(h_i) + p_- \delta(h_i + h_0)$. We have used the replica method and the phase diagrams were obtained within the replica-symmetry approximation. The boundary of the ferromagnetic phase exhibited an interesting behavior, with the presence of first-order phase transitions and tricritical points: within certain ranges for p_0 and h_0 , a single or two tricritical points were encountered. We have shown that the first-order phase transitions are directly affected by the dilution in the fields, in such a way that the extension of such lines is reduced by increasing p_0 . In fact, there is a threshold value, $p_0^* = 2(e^{3/2} + 2)^{-1} \approx 0.30856$, above which the ferromagnetic boundary is always continuous. Such effects may be reminiscent of those occurring within the mean-field theory of the Ising ferromagnet in the presence of trimodal random fields: the single tricritical point that appears in the case of a bimodal distribution [15] is removed by the presence of the δ function at the origin, whenever p_0 becomes greater than a certain value [17,18].

At zero temperature, if $0 < p_0 < p_0^*$, the ferromagnetic critical frontier exhibits a single tricritical point, with a first-order phase transition at high values of h_0 . By increasing p_0 , the first-order line gets reduced and, for $p_0 = p_0^*$, a fourth-

order critical point is observed; for $p_0 > p_0^*$, the ferromagnetic boundary is always continuous.

Although the spin-glass order parameter is induced by a random field ($p_0 < 1$), it may still contribute to a nontrivial behavior, concerning the stability of the replica-symmetric solution. We have calculated the instability of such a solution, leading to the identification of two low-temperature phases, namely, the spin-glass and mixed ferromagnetic ones. Besides that, the Almeida-Thouless line in the plane field versus temperature was shown to depend on the weight p_0 , with different amplitudes (but the same exponent) in the low-field regime, and qualitatively distinct high-field behaviors.

We have verified that whenever the ferromagnetic boundary presents both continuous and first-order transition lines meeting at a single finite-temperature tricritical point, such a point is located inside the region of stability of the replica-symmetric solution, and it will not be removed by a replica-symmetry-breaking procedure. However, when two tricritical points occur along the ferromagnetic boundary, at least one of them (the one at low temperatures) appears inside the unstable region, and its existence may be an artifact of the replica-symmetric solution.

The applicability of the present results to the description of real systems obviously depends on the survival of the mean-field characteristics in the respective short-range-interaction versions of Ising spin glasses and the Ising ferromagnet in the presence of a random field. However, the trimodal distribution employed herein is expected to mimic real systems better than the bimodal distribution itself. Although we are not aware of experimental observations that match our results, we believe that the diluted antiferromagnet $\text{Fe}_x\text{Mg}_{1-x}\text{Cl}_2$ is a good candidate, since, for conveniently chosen dilutions, it may exhibit first-order phase transitions [13], as well as a crossover from first- to second-order behavior [44].

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