Static $A+B\rightarrow 0$ annihilation process in arbitrary dimension

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(Received 7 April 1999)

The pair annihilation $A+B\rightarrow 0$ of static particles distributed at random in a d-dimensional space is studied in the large time regime for a tunneling law. The consequences of a superposition approximation used to close the hierarchy of equations describing the process are analytically explored. It is shown that the density and pair correlations of surviving particles have scaling expressions which display the ordering and clustering effects.

PACS number(s): 05.40.−a, 05.70.Ln

I. INTRODUCTION

In the static annihilation model ([1] and references therein), particles are initially distributed at random, remain immobile and are removed by a fusion reaction $A+A\rightarrow 0$, (type I) or $A+B\rightarrow 0$ (type II), with isotropic reaction rate $w(r)$, for any pair of particles separated by distance r. The rate is usually chosen as a tunneling law $w(r)=w_0\exp(-r/r_0)$ [2] or a power-law $w(r)=w_0 r^{-\delta}$ whith $\delta>1$ [3]. In the following, we study the type II model in a d-dimensional space, for a tunneling law and in case of equal concentration of A and B particles.

An interesting aspect of these reactions at large time is their tendency to self-organization (see [4] and references therein), which is well described in terms of the reaction radius [5] $R(t)$, defined by $tw(R(t))\simeq 1$. In practice, at time t, all interactive pairs whose separation r is less than $R(t)$ have disappeared, and on the range $R(t)\lessapprox r\lessapprox 2R(t)$ the fluctuations of their number dominate the reaction. To be more definite, one considers the density of surviving particles and the two-particle correlation functions. There is only one density $\rho(t)$ [$\rho(t)=\rho_A$ for type I and $\rho(t)=\rho_A=\rho_B$ for type II], one correlation $X(r,t)=X_{AA}$ for type I and two correlations for type II, $X_1(r,t)=X_{AB}$ and $X_2(r,t)=X_{AA.}=X_{BB}$. It then appears that the density vanishes as $\rho(t)=C/R^{\alpha}(t)$, where the exponent α is given by $\alpha=d$ (type I) or $\alpha=d/2$ (type II) and that the correlations, which are initially normalized to unity, $X_i(r,t=0)=1$, tend towards nontrivial functions.

These results have been firstly derived from heuristic considerations [6] or on the basis of numerical evidence [4,7]. In particular, in the work of Refs. [4,7], the hierarchical system [8] which describes the reaction as an infinite set of evolution equations for the correlation functions has been investigated within a Kirkwood approximation [9]. This approximation is to express the three-particle correlations in terms of the two-particle ones, in such a way that the hierarchical set reduces to a closed system for the density and the two-particle correlation functions, which have been numerically obtained. A comparison made for various dimensions of their values with the physical ones, given by a Monte Carlo simulation of the process, indicates that the Kirkwood approximation works extremely well. In one dimension, the Kirkwood approximation can be implemented into the annihilation reaction through a particular model which can be solved. In that way analytic results have been obtained for the exponent α and the prefactor C of the density for type I [10] and type II [11] reactions. This model also shows that the asymptotic expression of the two-particle correlations is given by a universal function of the scaled variable $x=r/R(t)$.

On the other hand, we have recently obtained all these results, for the type I reaction [12], from a rigorous resolution at large time of the hierarchical equations within the Kirkwood approximation. The derivation is thus more tranparent than with the model used for the one-dimensional case and valid in any dimension. We therefore consider in this work the type II reaction and, with our method of Ref. [12], rigorously explore some consequences of the Kirkwood closure relation. We are just doing here analytically the analysis numerically performed in Refs. [4,7].

Our starting point are the evolution equations for the density and for the two-particle correlation functions.When the three-particle correlation function appearing in these coupled equations is expressed in term of the previous functions through a Kirkwood superposition approximation, they reduce to a closed system. Our main result is to show that the density and the correlations have, at large time, scaling expressions which involve two functions $h_i(x)$, where x is defined as $x=r/R(t)$. In fact, one finds for the density exponent $\alpha=d/2$, for the density factor $C=h_1\ (x=\infty)=h_2\ (x=\infty)$ and for the correlations $X_i[r=xR(t),t]=h_i(x)/h_i(\infty)$. The functions $h_i(x)$ are solution of an integral system of coupled equations explicitly soluble in one dimension, and we recover some of our previous results , the other cases being tractable by a numerical iteration. Our plan is the following.

In Sec. II we recall the derivation of the evolution equations for the density and for the correlations. Using the superposition approximation we give their leading expression in the large time regime. At this point we introduce the functions $\gamma_i(r,t)$ defined by $\gamma_i(r,t)=\rho(t)X_i(r,t)$, since we observe that the previous system implies simple evolution equations for $\gamma_i(r,t)$ and that these functions suffice to fix the density and the correlations through the relations $\rho(t)=\gamma_1(\infty,t)=\gamma_2(\infty,t)$ and $X_i(r,t)=\gamma_i(r,t)/\gamma_i(\infty,t)$ which

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follow from the normalization conditions $X_i(\infty,t)=1$. Moreover, as we show in Sec. III, the solutions of these equations at large time have the scaling form $\gamma_i(r,t)=h_i(x)/R^{\alpha}(t)$, provided that $h_i(x)$ and α fulfill some conditions that we investigate in Sec. IV. We summarize our findings in Sec. V.

II. PAIR CORRELATION EVOLUTION EQUATION

Let $n_A(r,t)$ and $n_B(r,t)$ be the microscopic particle concentrations of A and B particles at time t and position r, r being here a d-dimensional vector. The ensemble averaged products of these concentrations define the many-center densities $\rho_{m,n}=\langle \Pi_{i=1}^{i=m} n_A(r_i,t)\Pi_{j=1}^{j=n} n_B(r_j,t)\rangle$. In particular $\langle n_A(r,t)\rangle=\langle n_B(r,t)\rangle=\rho(t)$, $\rho_{1,1}=\rho^2(t)X_1(|r_1-r_2|,t)$, and $\rho_{2,0}=\rho_{0,2}=\rho^2(t)X_2(|r_1-r_2|,t)$ where $\rho(t)$ is the global density and $X_i(r,t)$ the pair correlation functions normalized according to $X_i(r\to\infty,t)=1$. The $A-B$ symmetry comes from the initial conditions that we consider, i.e., equal number of A and B particles. As their positions are chosen at random, $X_i(r,t=0)=1$ for any separation r. In the following we have also to consider the three-center density $\rho_{2,1}(r,r',r'')$ for a triplet A_1A_2B (or B_1B_2A) where r is the distance A_1A_2, r' and r'' being the distances A_1B and A_2B, respectively.

Taking into account the annihilation process one can write an infinite system of equations [8] coupling the many-center densities. For the density $\rho(t)$ one finds

$$-\frac{d\rho}{dt}(t)=\int w(r)\rho_{1,1}(r,t)D^d r \tag{1}$$

and considering the decays of a set of two particles (A_1B) separated by a distance r one obtains the evolution equation of $\rho_{1,1}(r,t)$. The pair itself can annihilate with a probability $w(r)\rho_{1,1}(r,t)$, or either member of the pair can annihilate with a third particle. The particle A_1 annihilates with some particle B_2, which happens with a probability $w(r')\rho_{2,1}(r'',r,r')$ where r' and r'' are the distances from B_2 to A_1 and to B, respectively. The particle B annihilates with some particle A_2, which happens with a probability $w(r')\rho_{2,1}(r'',r,r')$ where r' and r'' are the distances from A_2 to B and to A_1, respectively. One gets

$$-\frac{\partial\rho_{1,1}}{\partial t}(r,t)=w(r)\rho_{1,1}(r,t)+2\int w(r')\rho_{2,1}(r'',r,r')D^d r'. \tag{2}$$

In this equation one uses the Kirkwood approximation

$$\rho_{2,1}(r'',r,r')=\rho^3(t)X_1(r,t)X_1(r',t)X_2(r'',t) \tag{3}$$

and investigates its expression for large values of r and t. We thus decompose in Eq. (2) the r' integration region into three subdomains E_1, E_2, and $E(r)$ that we describe here in the three dimensional case, its generalization to other dimensionalities being straightforward. Let P_1 and P_2 be those planes perpendicular to the line A_1B and containing the points A_1 and B, respectively. Then $E(r)$ denotes the region between P_1 and P_2, E_1 the half-space bounded by P_1 and where $r'<r''$, and finally the remaining space is E_2, where $r'>r''$. First of all, the integration domain E_2 can be neglected due to the exponential decay of the rate $w(r')$, r' being in E_2 greater than r. On the other hand, in the half-space E_1 the distance r'' is greater than r and Eq. (3) can be written

$$\rho_{2,1}(r'',r,r')\simeq\rho^3(t)X_1(r,t)X_1(r',t)=\gamma_1(r,t)\rho_{1,1}(r',t), \tag{4}$$

where for $i=1,2$ we define $\gamma_i(r,t)$ according to

$$\gamma_i(r,t)=\rho(t)X_i(r,t). \tag{5}$$

Thus

$$2\int_{E_1}\rho_{2,1}(r'',r,r')w(r')D^d r' \simeq \gamma_1(r,t)\int\rho_{1,1}(r',t)w(r')D^d r'$$

which from Eq. (1) is just $-\gamma_1(r,t)\partial\rho(t)/\partial t$. Finally in $E(r)$, as r is large, in Eq. (3) the correlation $X_1(r,t)$ is unity at the leading order and one can use the approximation

$$\rho_{2,1}(r'',r,r')\simeq\rho^3(t)X_1(r',t)X_2(r'',t) = \rho(t)\gamma_1(r',t)\gamma_2(r'',t).$$

The evolution equation Eq. (2) then becomes

$$-\frac{\partial\rho_{1,1}}{\partial t}(r,t)=w(r)\rho_{1,1}(r,t)-\gamma_1(r,t)\frac{d\rho}{dt}(t)+\rho(t)I_1(r,t), \tag{6}$$

where the integral $I_1(r,t)$ is restricted to region $E(r)$

$$I_1(r,t)=2\int_{E(r)} w(r')\gamma_1(r',t)\gamma_2(r'',t)D^d r'. \tag{7}$$

Expressed in the term of $\gamma_1(r,t)$, Eq. (6) has the following simpler form where the density has been eliminated:

$$-\frac{\partial\gamma_1}{\partial t}(r,t)=w(r)\gamma_1(r,t)+I_1(r,t). \tag{8}$$

We now compute explicitly $I_1(r,t)$ for the dimensions $d=1$, 2, and 3. In one dimension, $E(r)$ is the interval $[A_1B]$, and $r=r'+r''$. This gives a convolution integral

$$I_1(r,t)=2\int_0^r w(r')\gamma_1(r',t)\gamma_2(r-r',t)dr'. \tag{9}$$

In two dimensions, using the coordinates $A_1=(0,0)$, $B=(r,0)$, $A_2=(r'\cos\theta,r'\sin\theta)$ the domain $E(r)$ is the strip $[-\pi/2\leqslant\theta\leqslant\pi/2,\ 0\leqslant r'\leqslant r/\cos\theta]$ and

$$I_1(r,t)=4\int_0^{\pi/2}d\theta\int_0^{r/\cos\theta} w(r')\gamma_1(r',t)\gamma_2(r'',t)r'dr' \tag{10}$$

where

$$r''=[r^2+r'^2-2rr'\cos\theta]^{1/2} \tag{11}$$

In three dimensions, using the frame where $A_1=(0,0,0)$, $B=(0,0,r)$, and $A_2=(r'\sin\theta\cos\varphi,r'\sin\theta\sin\varphi,r'\cos\theta)$ the region $E(r)$ is given by $[0\leqslant\varphi\leqslant 2\pi,\ 0\leqslant\theta\leqslant\pi/2,\ 0\leqslant r'\leqslant r/\cos\theta]$ and finally

$$I_1(r,t)=4\pi\int_0^{\pi/2}\sin\theta d\theta\int_0^{r/\cos\theta}\times w(r')\gamma_1(r',t)\gamma_2(r'',t)r'^2dr', \qquad (12)$$

where r'' is again given by Eq. (11). We thus shall use the representation

$$I_1(r,t)=\int_0^{\pi/2}\lambda_d(\theta)d\theta\int_0^{r/\cos\theta}w(r')\gamma_1(r',t)\gamma_2(r'',t)\times r'^{(d-1)}dr', \qquad (13)$$

where $\lambda_1=2\delta(\theta)$ (the Dirac function), $\lambda_2=4$, $\lambda_3=4\pi\sin\theta$, and r'' given by Eq. (11).

The evolution equation of $\gamma_2(r,t)$ is given by the evolution of $\rho_{2,0}(r,t)$ which is

$$-\frac{\partial\rho_{2,0}}{\partial t}(r,t)=2\int w(r')\rho_{2,1}(r,r',r'')D^dr'. \qquad (14)$$

Applying to this equation the same arguments than previously, one obtains

$$-\frac{\partial\gamma_2}{\partial t}(r,t)=I_2(r,t), \qquad (15)$$

where

$$I_2(r,t)=\int_0^{\pi/2}\lambda_d(\theta)d\theta\int_0^{r/\cos\theta}w(r')\gamma_1(r',t)\gamma_1(r'',t)\times r'^{(d-1)}dr'. \qquad (16)$$

Explicit solutions $\gamma_i(r,t)$ of the system of Eqs. (8)–(15) where $I_i(r,t)$ has the form given in Eqs. (13)–(16) are studied in the next section.

III. SCALING FORM OF THE EVOLUTION EQUATIONS

The reaction radius $R(t)$, defined by $tw(R(t))=1$, i.e., $R(t)=r_0\log t$ when $w(r)=e^{-r/r_0}$, is such that at large time no interacting particles survive if their separation r is smaller than $R(t)$. (In the following we assume that $r_0=1$.) In this regime it is thus convenient to use a scaled variable $x=r/R(t)$ and to see if Eqs. (8)–(15) admit asymptotic solutions of the form

$$\gamma_i(r,t)=\rho(t)X_i(r,t)=h_i(x)/R^\alpha(t), \qquad (17)$$

where, according to previous results, the exponent α is expected to be equal to $d/2$. Imposing the condition $X_i(r\to\infty,t)=1$ in the definition (17) gives the density and the pair correlation functions in term of $h_i(x)$ according to

$$\rho(t)=\lim_{x\to\infty}h_1(x)/R^\alpha(t)=\lim_{x\to\infty}h_2(x)/R^\alpha(t), \qquad (18)$$

$$X_i(r=xR(t),t)=h_i(x)/h(\infty). \qquad (19)$$

These expressions evidently imply that $h_i(x)$ are non-negative functions with the same asymptotic limit $h(\infty)$.

To study $h_i(x)$, we begin by inserting Eq. (17) into Eq. (8) and Eq. (15). Since $\partial x/\partial t=-x/t\log t$, one easily finds

$$-\frac{\partial\gamma_i}{\partial t}(r,t)=x^{1-\alpha}\frac{d}{dx}(x^\alpha h_i(x))/tR^\alpha(t)\log t. \qquad (20)$$

To express the integrals $I_i(r,t)$ appearing in Eq. (13) and Eq. (16) we change the variables according to $r=xR(t)$, $r'=yR(t)$, and $r''=zR(t)$ where from Eq. (11) z is defined by

$$z=[x^2+y^2-2xy\cos\theta]^{1/2}. \qquad (21)$$

Thus $\gamma_i(r',t)\gamma_j(r'',t)=h_i(y)h_j(z)R^{-2\alpha}(t)$ and the integrals become $I_i(r,t)=R^{d-2\alpha}(t)\int_0^{\pi/2}\lambda_d(\theta)d\theta J_i(\theta,x,t)$ with

$$J_i(\theta,x,t)=\int_0^{x/\cos\theta}t^{-y}h_1(y)h_k(z)y^{d-1}dy, \qquad (22)$$

where here and in the following the index k has the value 2 for $i=1$ and the value 1 for $i=2$. We have used the relation $w(yR(t))=t^{-y}$, which also gives $w(r)\gamma_i(r,t)=t^{-x}h_i(x)R^{-\alpha}(t)$. If we multiply both sides of Eqs. (8) and (15) by $tR^\alpha(t)\log t$, we obtain

$$x^{1-\alpha}\frac{d}{dx}(x^\alpha h_1(x))=t^{1-x}h_1(x)\log t+tR^{d-\alpha}(t)\log t\int_0^{\pi/2}\lambda_d(\theta)d\theta J_1(\theta,x,t), \qquad (23)$$

$$x^{1-\alpha}\frac{d}{dx}(x^\alpha h_2(x))=tR^{d-\alpha}(t)\log t\int_0^{\pi/2}\lambda_d(\theta)d\theta J_2(\theta,x,t). \qquad (24)$$

The left-hand side of Eq. (23) is time-independent, and this has to be the case for each member of the right-hand side, since they are non-negative. The first term implies that

$$h_1(x)=0 \quad \text{for} \quad 0\leq x\leq 1. \qquad (25)$$

For the second term, we have to evaluate the leading term at large time of the integral $J_1(\theta,x,t)$. Equation (25) indicates that the range of the y integration in Eq. (22) starts at $y=1$ and that this value dominates the integral due to the factor t^{-y}. We thus change variables in Eq. (22) and use u defined by $y=1+u/\log t$, in such a way that $t^{-y}dy=e^{-u}du/(t\log t)$. Assuming that in the vicinity of $y=1$ the function $h_1(y)$ has the behavior

$$h_1(y)\approx\mu(y-1)^\beta, \qquad (26)$$

where μ and β are constants to be fixed latter on, one obtains that at large time

$$J_i(\theta,x,t)\simeq\mu\Gamma(1+\beta)h_k([x^2+1-2x\cos\theta]^{1/2})/t\log^{1+\beta}t \qquad (27)$$

Inserting this relation in the right-hand sides of Eq. (23) (second term) and Eq. (24), their time independence implies that

$$\alpha+\beta=d. \qquad (28)$$

For $x>1$, we thus obtain the following equation to determine the function $h_1(x)$:

$$x^{1-\alpha}\frac{d}{dx}(x^\alpha h_1(x))=\mu\Gamma(1+\beta)\int_0^{\pi/2}\lambda_d(\theta)h_2 \times([1+x^2-2x\cos\theta]^{1/2})d\theta, \quad (29)$$

the function $h_2(x)$ being determined for $x\geqslant 0$ by Eq. (24) which becomes

$$x^{1-\alpha}\frac{d}{dx}(x^\alpha h_2(x))=\mu\Gamma(1+\beta)\int_0^{\pi/2}\lambda_d(\theta)h_1([1+x^2 -2x\cos\theta]^{1/2})d\theta. \quad (30)$$

Among the solutions of these equations we have to select those which have the same asymptotic behavior $h_1(\infty)=h_2(\infty)=h(\infty)$. This constraint determines μ, since equating the two members of Eq. (29) or Eq. (30) in the limit $x\to\infty$, one obtains $\alpha h(\infty)=\mu\Gamma(1+\beta)h(\infty)\int_0^{\pi/2}\lambda_d(\theta)d\theta$. These values are

$$d=1,\quad \mu=\alpha/2\Gamma(1+\beta);\quad d=2,$$
$$\mu=\alpha/2\pi\Gamma(1+\beta);\quad d=3,\quad \mu=\alpha/4\pi\Gamma(1+\beta). \quad (31)$$

Finally, the functions $h_i(x)$ are fixed for $x>\delta_{1i}$ by the system

$$x^{1-\alpha}\frac{d}{dx}(x^\alpha h_i(x))=\int_0^{\pi/2}\mu_d(\theta)h_k([1+x^2-2x\cos\theta]^{1/2})d\theta, \quad (32)$$

with $\mu_1(\theta)=\alpha\delta(\theta)$, $\mu_2(\theta)=2\alpha/\pi$, and $\mu_3(\theta)=\alpha\sin\theta$. We study the solutions of these equations in the next section, but at this stage of our analysis, we have to point out that the exponent α is not yet fixed. This is a major difference with the $A+A\to 0$ annihilation where imposing scaling implies $\alpha=d$.

IV. STUDY OF THE SCALING SOLUTIONS

In one dimension, the defining system is

$$x^{1-\alpha}\frac{d}{dx}(x^\alpha h_i(x))=\alpha h_k(x-1),$$
$$h_1(x=1^+)=\alpha(x-1)^\beta/2\Gamma(1+\beta). \quad (33)$$

As $h_1(x)$ vanishes for $0\leqslant x\leqslant 1$, this system with $i=2$ determines $h_2(x)$ for $0\leqslant x\leqslant 2$ according to

$$h_2(x)=C/x^\alpha. \quad (34)$$

In Eq. (34) C is a constant which is fixed by the constraint on $h_1(x=1^+)$ since with $i=1$ Eq. (33) gives $\alpha h_1(x)+xh_1'(x)=\alpha C(x-1)^{-\alpha}$.

At the value $x=1$ it gives $C=1/2\Gamma(\beta)$. More generally the functions $h_i(x)$ can be obtained step by step by integrations on intervals of unit length, the integration constants being fixed by continuity requirements.

It is moreover possible to explicitly solve the system through a Laplace transform. Let $g_\pm(x)=h_2(x)\pm h_1(x)$ which from Eq. (33) are solutions of

$$\alpha g_\pm(x)+xg_\pm'(x)=\pm\alpha g(x-1). \quad (35)$$

For their Laplace transform we choose $G_\pm(p)=\int_1^\infty e^{-px}g_\pm(x)dx$ to avoid the singularity at $x=0$. Then

$$\int_1^\infty e^{-px}g_\pm(x-1)dx=e^{-p}G_\pm(p)+e^{-p}\int_0^1 e^{-px}Cx^{-\alpha}dx, \quad (36)$$

where we have used that on the range $0\leqslant x\leqslant 1$, $g_\pm(x)=h_2(x)=Cx^{-\alpha}$. In the same way, an integration by parts gives

$$\int_1^\infty e^{-px}g_\pm'(x)dx=pG_\pm(p)-Ce^{-p}. \quad (37)$$

Taking the p derivative of the two members of the previous relation and using Eq. (36) we finally obtain the equations defining the functions $G_\pm(p)$:

$$p\frac{d}{dp}G_\pm(p)+(\beta\pm\alpha e^{-p})G_\pm(p)=\Phi_\pm(p), \quad (38)$$

where $\Phi_\pm(p)=-Ce^{-p}(1\pm\alpha p^{-\beta}\int_0^p e^{-x}x^{-\alpha}dx)$.

It then appears that the solutions can be found by the standard techniques and are

$$G_\pm(p)=p^{-\beta}\left\{\exp(\pm\alpha E_1(p)) -\Gamma^{-1}(\beta)\int_0^p e^{-x}x^{-\alpha}dx\right\}\Big/2, \quad (39)$$

where $E_1(p)$ is the exponential integral

$$E_1(p)=\int_p^\infty e^{-y}dy/y\simeq-\gamma-\log p \quad \text{as} \quad p\to 0 \quad (40)$$

and $\gamma\simeq 0.57721\ldots$ is the Euler constant.The asymptotic values for $x\to\infty$, which follow from the usual relation $g_\pm(\infty)=\lim[pG_\pm(p),p\to 0]$, are

$$g_+(\infty)=\exp(-\alpha\gamma)/2,\quad g_-(\infty)=0. \quad (41)$$

We have already derived these results, for the case $\alpha=\beta=1/2$, in our previous work [11]. In the present approach all the solutions, which fulfill the relation $\alpha+\beta=1$ but where $\alpha\neq\beta$ have to be rejected on physical grounds. In a true simulation of the process, as in Refs. [4,7], some initial conditions are taken into account and lead in any case to a definite value $\alpha=d/2$. Here, since only asymptotics conditions are used self-consistently we propose to select the physical solutions by imposing short-range order for the two-particle correlation functions. A simple constraint is then provided by $G_-(p=0)=\int_1^\infty(h_2(x)-h_1(x))dx$ which is a constant when the correlations saturate their common limit suffitiently quick, as suggested by the experimental data. Since our solution behaves as $G_-(p=0)\simeq p^{\alpha-\beta}\exp(\alpha\gamma)/2$, we obtain the wanted result $\alpha=\beta=1/2$. The asymptotic value $h(\infty)=\exp(-\gamma/2)/4=0.187327\ldots$ appears then to be quickly reached: $h_1(3)=0.187$; $h_2(3)=0.188$.

In higher dimensions, we cannot exactly solve the system, but we know from our previous work [12] that its solutions can be found by iteration. Assuming $\alpha=\beta$ we find $\alpha=\beta=d/2$ and we must recover the density and pair correlation functions already numerically given in the work of Refs. [4,7], where the scaling properties are now apparent.

V. CONCLUSION

We recall that the Kirkwood superposition approximation has been shown to give a precise description of the static annihilation process. We have shown in this work the particular scaling form of the large time limit of the density and the correlations implied by this assumption. It must be stressed, as it is clear from our derivation, that the asymptotic regime is independent of the initial density. The role of the reaction radius $R(t)$ appears clearly: since $h_1(x)$ vanishes for $x\leqslant 1$, there are no interactive AB pairs with a pair separation smaller than $R(t)$. Since $h_1(x)$ is nontrivial on the range $1\leqslant x \lessapprox 3$ self-organization effects appear on the range $R(t)\leqslant r \lessapprox 3R(t)$. The correlation function for like particles is given by $h_2(x)$ which decreases strongly from $x=0$ where it is singular to $x\gtrsim 3$ where it reaches its asymptotic value: this is the clustering effect.

These results thus extend the properties found for the AA static annihilation in a way generally expected.

[1] G. Oshanin, S. Burlatsky, E. Clement, D. S. Graff, and L. M. Sander, J. Phys. Chem. **98**, 7390 (1994).

[2] N. A. Efremov, S. G. Kulikov, R. I. Personov, and Yu. V. Romanovskii, Chem. Phys. **128**, 9 (1988).

[3] B. Bonnier and R. Brown, Phys. Rev. E **55**, 6661 (1997).

[4] H. Schnörer, V. Kuzovkov, and A. Blumen, J. Chem. Phys. **92**, 2310 (1990).

[5] M. Tachiya and A. Mozumder, Chem. Phys. Lett. **28**, 87 (1974).

[6] S. F. Burlatsky and A. I. Cheroustan, Phys. Lett. **56A**, 145 (1990).

[7] H. Schnörer, V. Kuzovkov, and A. Blumen, Phys. Rev. Lett. **63**, 805 (1989).

[8] V. Kuzovkov and E. Kotomin, Rep. Prog. Phys. **51**, 1479 (1988).

[9] J. G. Kirkwood, J. Chem. Phys. **3**, 300 (1935).

[10] B. Bonnier, R. Brown, and E. Pommiers, J. Phys. A **28**, 5165 (1995).

[11] B. Bonnier and E. Pommiers, Phys. Rev. E **52**, 5873 (1995).

[12] B. Bonnier, Phys. Rev. E **58**, 5424 (1998).