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Exit time of turbulent signals: A way to detect the intermediate dissipative range

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The exit-time statistics of experimental turbulent data is analyzed. By looking at the exit-time moments (inverse structure functions) it is possible to have a direct measurement of scaling properties of the laminar statistics. It turns out that the inverse structure functions show a much more extended intermediate dissipative range than the structure functions, leading to the first clear evidence of the existence of such a range of scales. [S1063-651X(99)51012-X]

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In isotropic turbulence, the most studied statistical indicators of intermittency are the longitudinal structure functions, i.e., moments of the velocity increments at distance R in the direction of $\hat{\mathbf{R}}$: $S_p(R)=\langle[(\mathbf{v}(\mathbf{x}+\mathbf{R})-\mathbf{v}(\mathbf{x})]\cdot\hat{\mathbf{R}}]^p\rangle$. In typical experiments one is forced to analyze one-dimensional string of data: the output of hot-wire anemometer. In these cases Taylor frozen-turbulence hypothesis is used in order to bridge measurements in space with measurements in time. Within the Taylor hypothesis, one has the large-scale typical time, $T_0=L_0/U_0$, and the dissipative time, $t_d=r_d/U_0$, where U_0 is the large scale velocity field, L_0 is the scale of the energy injection, and r_d is the Kolmogorov dissipative scale. As a function of time increment, τ, structure functions assume the form: $S_p(\tau)=\langle[(v(t+\tau)-v(t)]^p\rangle$. It is well known that for time increment corresponding to the inertial range, $\tau_d\ll\tau\ll T_0$, structure functions develop an anomalous scaling behavior: $S_p(\tau)\sim\tau^{\zeta(p)}$, where $\zeta(p)$ is a nonlinear function, while far inside the dissipative range, $\tau\ll\tau_d$, they show the laminar scaling: $S_p(\tau)\sim\tau^p$.

Beside the huge amount of theoretical, experimental, and numerical studies devoted to the understanding of velocity fluctuations in the inertial range (see [1] for a recent overview), only few attempts, mainly theoretical, have focused on the intermediate dissipation range (IDR), introduced in [2] (see also [3–6]). By IDR we mean the range of scales, $\tau\sim\tau_d$, between the inertial and the dissipative range.

The very existence of the IDR is relevant for the understanding of many theoretical and practical issues. Among them we cite: the modelizations of small scales for optimizing large eddy simulations and the validity of the refined Kolmogorov hypothesis [1].

A nontrivial IDR is connected to the presence of intermittent fluctuations in the inertial range. Namely, anomalous scaling law characterized by the exponents $\zeta(p)$ can be explained by assuming that velocity fluctuations in the inertial range are characterized by a spectrum of different local scaling exponents: $\delta_\tau v=v(t+\tau)-v(t)\sim\tau^h$, with the probability to observe at scale τ a value h given by $P_\tau(h)\sim\tau^{3-D(h)}$. This is the celebrated multifractal picture of the energy cascade which has been confirmed by many independent experiments [1]. The nontrivial dissipative statistics can be explained by defining the dissipative cutoff as the scale where the local Reynolds number is of the order of unity:

$$\mathrm{Re}(\tau_d)=\frac{\tau_d\, v_{\tau_d}}{\nu}\sim O(1). \tag{1}$$

By inverting (1) we obtain a prediction of a fluctuating τ_d: $\tau_d(h)\sim\nu^{1/(1+h)}$, where for the sake of simplicity we have assumed the large scale velocity, U_0, and the outer scale, L_0, both fixed to 1.

In this Rapid Communication we propose, and measure in experimental and synthetic data, a set of observables that are able to highlight the IDR properties. The main idea is to take a one-dimensional string of turbulent data, $v(t)$, and to analyze the statistical properties of the exit times from a set of defined velocity thresholds; roughly speaking, a kind of *inverse* structure function [8] (inverse-SF).

This analysis leads to clear evidence of nontrivial intermittent fluctuations of the dissipative cutoff in turbulent signals. A similar approach has already been exploited for studying the particle separation statistics [7]. Recently, exit-time moments have also been studied to characterize intermittency in the realm of shell models [8].

The paper is organized as follows. First we discuss why the the exit-time probability density function is dominated by the IDR. Then, we present the data analysis performed in high-Reynolds number turbulent flows and in synthetic multiaffine signals [9]. Finally, we summarize the evidence supporting a nontrivial IDR and discuss possible further investigations.

Fluctuations of viscous cutoff are particularly important for all those regions in the fluid where the velocity field is locally smooth, i.e., the local fluctuating Reynolds number is small. In this case, the matching between nonlinear and viscous terms happens at scales much larger than the Kolmogorov scale, $\tau_d\sim\nu^{-3/4}$. It is natural, therefore, to look for observables that have been subjected to mainly laminar events. A possible choice is to measure the *exit-time* moments through a set of velocity thresholds. More precisely, given a reference initial time t_0 with velocity $v(t_0)$, we define $\tau(\delta v)$ as the first time necessary to have an absolute variation equal to δv in the velocity data, i.e., $|v(t_0)-v[t_0$

$+\tau(\delta v)]\| = \delta v$. By scanning the whole time series we recover the probability density functions of $\tau(\delta v)$ at varying δv from the typical large scale values down to the smallest dissipative values. Positive moments of $\tau(\delta v)$ are dominated by events with a smooth velocity field, i.e., laminar bursts in the turbulent cascade. Let us define the inverse structure functions as

$$\Sigma_p(\delta v) \equiv \langle \tau^p(\delta v) \rangle. \tag{2}$$

According to the multifractal description we suppose that, for velocity thresholds corresponding to inertial range values of the velocity differences, $\delta_{\tau_d} v \equiv v_m \ll \delta v \ll v_M \equiv \delta_{T_0} v$, the following dimensional relation is valid: $\delta_\tau v \sim \tau^h \rightarrow \tau(\delta v) \sim \delta v^{1/h}$. The probability to observe a value τ for the exit time is given by inverting the multifractal probability, i.e., $P(\tau \sim \delta v^{1/h}) \sim \delta v^{[3-D(h)]/h}$. Made this ansatz, the prediction for the inverse-SF, $\Sigma_p(\delta v)$ evaluated for velocity thresholds within the inertial range is

$$\Sigma_p(\delta v) \sim \int_{h_{min}}^{h_{max}} dh\, \delta v^{[p+3-D(h)]/h} \sim \delta v^{\chi_{sp}(p)}, \tag{3}$$

where the RHS has been obtained by a saddle point,

$$\chi_{sp}(p) = \min_h \{[p+3-D(h)]/h\}. \tag{4}$$

Let us now consider the IDR properties. For each p the saddle point evaluation (4) selects a particular $h = h_s(p)$ where the minimum is reached. Let us also remark that from Eq. (1) we have an estimate for the minimum value assumed by the velocity in the inertial range given a certain singularity h: $v_m(h) = \delta_{\tau_d(h)} v \sim \nu^{h/(1+h)}$. Therefore, the smallest velocity value at which the scaling $\Sigma_p(\delta v) \sim \delta v^{\chi_{sp}(p)}$ still holds depends on both ν and h. Namely, $\delta v_m(p) \sim \nu^{h_s(p)/1+h_s(p)}$. The most important consequence is that for $\delta v < \delta v_m(p)$ the integral (3) is not any more dominated by the saddle point value but by the maximum h value still dynamically alive at that velocity difference, $1/h(\delta v) = -1 - \log(\nu)/\log(\delta v)$. This leads for $\delta v < \delta v_m(p)$ to a pseudoalgebraic law,

$$\Sigma_p(\delta v) \sim \delta v^{\{p+3-D[h(\delta v)]\}/h(\delta v)}. \tag{5}$$

The presence of this p-dependent velocity range, intermediate between the inertial range, $\Sigma_p(\delta v) \sim \delta v^{\chi_{sp}(p)}$, and the far dissipative scaling, $\Sigma_p(\delta v) \sim \delta v^p$, is the IDR signature. Then, it is easy to show that inverse-SF should display an enlarged IDR. Indeed, for the usual *direct* structure functions the saddle point $h_s(p)$ value is reached for $h < 1/3$. This pushes the IDR to a range of scales very difficult to observe experimentally [4]. On the other hand, as regards the inverse-SF, the saddle point estimate of positive moments is always reached for $h_s(p) > 1/3$. This is an indication that we are probing the laminar part of the velocity statistics. Therefore, the presence of the IDR must be felt much earlier in the range of available velocity fluctuations. Indeed, if $h_s(p) > 1/3$, the typical velocity field at which the IDR shows up is given by $\delta v_m(p) \sim \nu^{h_s(p)/[1+h_s(p)]}$, that is much larger than the Kolmogorov value $\delta v_{r_d} \sim \nu^{1/4}$. In Fig. 1 we plot $\Sigma_1(\delta v)$ evaluated on a string of high-Reynolds number experimental

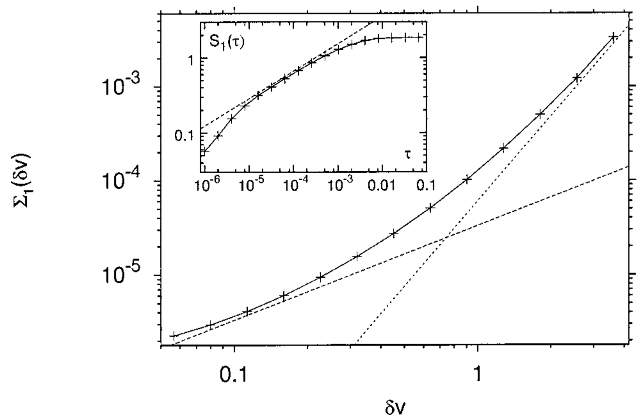

FIG. 1. Inverse-SF $\Sigma_1(\delta v)$. The straight line shows the dissipative range bahavior (dashed line) $\Sigma_1(\delta v) \sim \delta v$, and the inertial range nonintermittent behavior (dotted line) $\Sigma_1(\delta v) \sim (\delta v)^3$. The inset shows the direct structure function $S_1(\tau)$ with superimposed the intermittent slope $\zeta(1) = 0.39$.

data as a function of the available range of velocity thresholds δv. This data set has been measured in a wind tunnel at $Re_\lambda \sim 2000$.

Let us first make a technical remark. If one wants to compare the predictions (3) and (5) with the experimental data, it is necessary to perform the average over the time-statistics in a weighted way. This is due to the fact that by looking at the exit-time statistics we are not sampling the time-series uniformly, i.e., the higher the value of $\tau(\delta v)$ is, the longer it is detectable in the time series. Let us call $\tau_1(\delta v), \tau_2(\delta v), \ldots, \tau_N(\delta v)$ the string of exit time values obtained by analyzing the velocity string data consecutively for a given δv. N is the number of times for which $\delta_\tau v$ reaches a given threshold. It is easy to realize [10] that the sequential time average of any observable based on exit-time statistics, $\langle \tau^p(\delta v) \rangle_t \equiv (1/N) \Sigma_{i=1}^N \tau_i^p$, is connected to the uniformly-in-time multifractal average, $\langle (\cdot) \rangle \equiv \int dh(\cdot)$, by the relation

$$\langle \tau^p(\delta v) \rangle = \sum_{i=1}^N \tau_i^p \frac{\tau_i}{\sum_{j=1}^N \tau_j} = \frac{\langle \tau^{p+1} \rangle_t}{\langle \tau \rangle_t}, \tag{6}$$

where $\tau_i / \Sigma_{j=1}^N \tau_j$ takes into account the nonuniformity in time. Let us now go back to Fig. 1. One can see that the scaling is very poor. Indeed, it is not possible to extract any quantitative prediction about the inertial range slope. For this reason, we have only drawn the dimensional non-intermittent slope and the dissipative slope as a possible qualitative references. On the other hand, (inset of Fig. 1) the scaling behavior of the direct structure functions $\langle |\delta v(\tau)| \rangle \sim \tau^{\zeta(1)}$ is quite clear in a wide range of scales. This is a clear evidence of IDR's contamination into the whole range of available velocity values for the Inverse-SF cases. Similar results (not shown) are found for higher orders Σ_p structure functions.

In order to better understand the scaling properties of $\Sigma_p(\delta v)$ we investigate a synthetic multiaffine field obtained by combining successive multiplications of Langevin dynamics [9]. The advantage of using a synthetic field is that

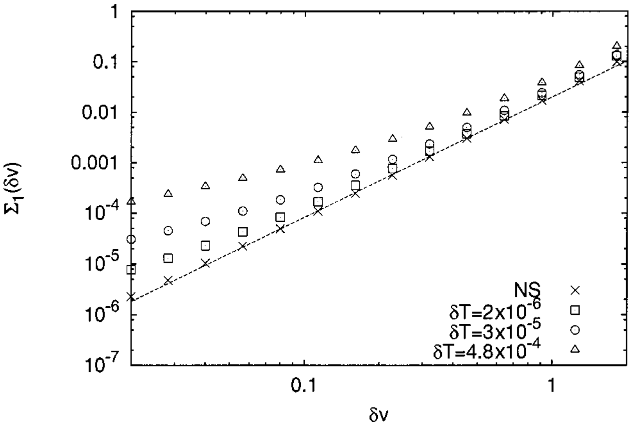

FIG. 2. Inverse-SF $\Sigma_1(\delta v)$ vs δv for the synthetic signals not smoothed (NS) and smoothed with time windows: $\delta T = 4.8\times10^{-4}$, 3×10^{-5}, 2×10^{-6}, the straight line is obtained from the inverse multifractal prediction (4).

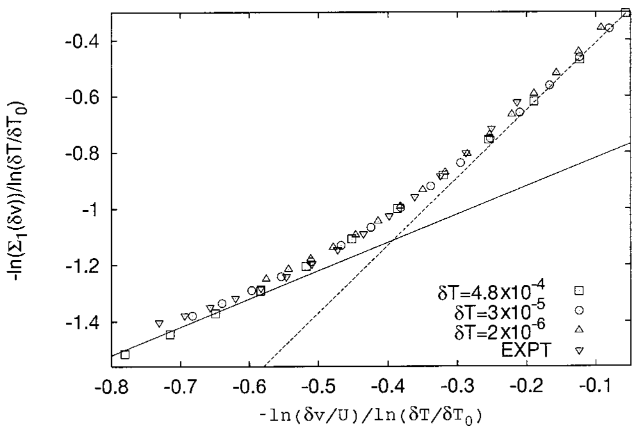

FIG. 3. Data collapse of the inverse-SF, $\Sigma_1(\delta v)$, obtained by the rescaling (7) for the smoothed synthetic signals (with time windows: $\delta T = 4.8\times10^{-4}$, 3×10^{-5}, 2×10^{-6}) and the experimental data (EXPT). The two straight lines have the dissipative (solid line) and the inertial range (dashed line) slope.

one can control analytically the scaling properties of direct structure functions in order to have the same scaling laws observed in experimental data. An IDR can be introduced in the synthetic signals by smoothing the original dynamics on a moving time-window of size δT. Imposing a smoothing time-window is equivalent to fixing the Reynolds numbers, Re$\sim\delta T^{-4/3}$. The purpose to introduce this stochastic multiaffine field is twofold. First we want to reach high Reynolds numbers to test the inverse-multifractal formula (4). Second, we want to test that the very extended IDR observed in the experimental data, see Fig. 1, is also observed in this stochastic field. This would support the claim that the experimental result is the evidence of an extended IDR.

In Fig. 2 we show the inverse-SF, $\Sigma_1(\delta v)$, measured in the multiaffine synthetic signal at high-Reynolds numbers. The observed scaling exponent, $\chi(1)$, is in agreement with the prediction (4). The same agreement also holds for higher moments. In Table I, we compare the best fit to the $\Sigma_p(\delta v)$ measured on the synthetic field with the inversion formula (4). As for the comparison between the theoretical expectation (4) and the synthetic data let us note the following points. First, in [9] it was proved that the signal possesses the given direct-structure functions exponents for positive moments, i.e., the $\zeta(p)$ exponents are in a one-to-one correspondence with the $D(h)$ curve for $h<1/3$. Nothing was proved for observables feeling the $h>1/3$ interval and therefore the agreement between the inversion formula (4) and the numerical results cannot be found analytically. Second, because the synthetic signal is defined by using Langevin processes, the less singular h-exponents expected to contribute to the saddle-point (4) is $h=0.5$. Therefore, the theoretical prediction, $\chi_{sp}(q)$, in Table I has been obtained by imposing $h_{max}=0.5$.

TABLE I. Comparison between $\chi_{syn}(p)$ measured in the synthetic signal and the multifractal prediction (4). The synthetic signal has a $D(h)$ function which leads to the same set of experimental $\zeta(p)$ for direct structure functions.

p	1	2	3	4	5
$\chi_{syn}(p)$	2.32(4)	4.40(8)	6.38(8)	8.3(1)	10.1(2)
$\chi_{sp}(p)$	2.32	4.34	6.34	8.35	10.35

Let us now go back to the most interesting question about the statistical properties of the IDR. In order to study this question we have smoothed the stochastic field, $v(t)$, by performing a running-time average over a time-window, δT. Then we compare Inverse-SF scaling properties at varying Reynolds numbers, i.e. for different dissipative cutoff: Re $\sim\delta T^{-4/3}$.

The expression (5) predicts the possibility to obtain a data collapse of all curves with different Reynolds numbers by rescaling the inverse-SF as follows [2,3]:

$$-\frac{\ln[\Sigma_p(\delta v)]}{\ln(\delta T/\delta T_0)} \quad \text{vs} \quad -\frac{\ln(\delta v/U)}{\ln(\delta T/\delta T_0)}, \tag{7}$$

where U and δT_0 are adjustable dimensional parameters. Within the same experimental (or synthetic) set up they are Reynolds number independent (i.e., δT independent).

The rationale for the rescale (7) stems from the observation that, in the IDR, $h_s(p)$ is a function of $\ln(\delta v)/\ln(\nu)$ only. Therefore, identifying Re$\propto\nu^{-1}$, the relation (7) directly follows from (5). This rescaling was originally proposed as a possible test of IDR for direct structure functions in [2] but, as already discussed above, for the latter observable it is very difficult to detect any IDR due to the extremely small scales involved [4].

Figure 3 shows the rescaling (7) of the Inverse-SF, $\Sigma_1(\delta v)$, for the synthetic field at different Reynolds numbers and for the experimental signals. As it is possible to see, the data-collapse is very good for both the synthetic and experimental signal. This is a clear evidence that the poor scaling range observed in Fig. 1 for the experimental signal can be explained as the signature of the IDR. The same behavior holds for higher moments (not shown).

It is interesting to remark that for a self-affine signal $[D(h)=\delta(h-1/3)]$, the IDR is highly reduced and the inverse-SF, scaling trivially as $\Sigma_p(\delta v)\sim(\delta v)^{3p}$, do not bring any new information.

In conclusions we have shown that exit-time moments, $\Sigma_p(\delta v)$, are dominated by the laminar part of the energy cascade: they depend only on the part of $D(h)$ which falls to the right of its maximum , i.e., $h>1/3$. These h's values are not testable by the direct structure functions. Inverse-SF are the natural tool to test any model concerning velocity fluctuations less singular than the Kolmogorov value $\delta v \sim \tau^{1/3}$.

By analyzing high-Reynolds data and synthetic fields, we have proved that the extension of the IDR for $\Sigma_p(\delta v)$ is magnified. The rescaling (7) based on the assumption (1) gives a good data collapse for different Reynolds numbers. This is a clear evidence of the IDR.

Many questions are still open. First, the analysis of a wider set of experimental data could make it possible to quantify the agreement of the data-collapse with the prediction based on Eqs. (1) and (5). Indeed, it is easy to realize that, by using different parameterization for the onset of the viscous range, one would have predicted the existence of an extended IDR for $\Sigma_p(\delta v)$ but with a slightly different rescaling procedure [5]. The quality of experimental data available to us is not high enough to distinguish between the two different predictions. Analyzing different experimental datasets, at different Reynolds numbers, could also make it possible to better explore $D(h)$ for $h>1/3$. This is an important question which opens a new problem. Indeed, doubts about the universality of these $D(h)$ values may be raised on the basis of the usual energy cascade picture. For example, as discussed above, in the Langevin synthetic-data a good agreement between the multifractal prediction and the numerical data is obtained by imposing $h_{max}=0.5$, similarly in true turbulent data other h_{max} values could appear depending on the physical mechanism driving the energy transfer at large scales.

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