Electromagnetic convective cells in a nonuniform dusty plasma

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(Received 21 April 1999; revised manuscript received 21 June 1999)

It is noticed that in the presence of a perturbed electron current parallel to an external magnetic field, the dispersion relation of the electrostatic convective cell and the magnetostatic modes are not modified, contrary to the result quoted in the literature. Instead, the electrostatic and magnetostatic modes are coupled if the propagation parallel to the magnetic field is taken into account in both dusty and electron-ion plasmas. [S1063-651X(99)02811-1]

PACS number(s): 52.35.Lv, 52.35.Kt

A few years ago, the convective cell mode was studied in magnetized nonuniform dusty plasma [1]. It was pointed out that in the presence of static dust particles, the electrostatic convective cell mode can have a nonvanishing real part of frequency due to the presence of a density gradient. This is unlike the electron-ion plasma case, in which the electrostatic convective cell mode [2] cannot have a real part of frequency even in the presence of inhomogeneity. On the other hand, for the magnetostatic mode [3] the real part of the frequency becomes finite if the plasma is not homogeneous [4].

Recently it has been shown that the propagation parallel to the external magnetic field can modify the dispersion relation of convective cell modes in nonuniform dusty plasmas even in the electrostatic limit [5]. In our opinion, the electron parallel motion can introduce magnetic-field perturbation and hence in the pure electrostatic limit the electron inertia does not contribute to the low-frequency electrostatic convective cell mode. Therefore, the dispersion relation of the electrostatic convective cell mode is not modified; rather, it couples with the magnetostatic drift mode.

Both the fundamental electrostatic and magnetostatic modes are flutelike, and neither one propagates parallel to the external magnetic field. However, in the plasmas, the parallel component of the wave vector can be nonzero as well. We notice that the oblique propagation introduces a coupling of the electrostatic convective cell with the magnetostatic drift mode.

Let us consider the uniform constant external magnetic field along the z axis ($\mathbf{B}_0=B_0\hat{\mathbf{z}}$) of the inhomogeneous slab of a dusty plasma.

The quasineutrality condition at equilibrium is given by

$$n_{i0}=n_{e0}+Z_d n_{d0}, \tag{1}$$

where the subscripts e, i, and d denote the electron, ion, and dust fluids, respectively, while $n_{\alpha 0}$ (where $\alpha=e,i,d$) is the equilibrium number density which is assumed to be along the negative x axis.

The ions have been assumed to be singly charged and Z_d is the assumed constant charge number on dust particles. For simplicity, charge variation on dust particles is ignored.

In the limit $\rho_e k \ll 1$ (where ρ_e is the electron gyroradius and k is the wave number), the linear velocities of electrons and ions in the perpendicular direction are, respectively, given by [1]

$$\mathbf{V}_{e\perp 1}\approx \mathbf{V}_{E1}=\frac{c}{B_0}\hat{\mathbf{z}}\times\nabla_\perp \phi_1, \tag{2}$$

$$\mathbf{V}_{i\perp 1}\approx \mathbf{V}_{E1}+\mathbf{V}_{p1}=\mathbf{V}_{E1}-\frac{c}{B_o\Omega_i}\partial_t \nabla_\perp \phi_1, \tag{3}$$

where the subscript 1 denotes the linearly perturbed quantities.

If the parallel motion is also taken into account, then the component of the electron equation of motion can be written as

$$\partial_t V_{ez1}=-\frac{e}{m_e}E_{z1}-\frac{e}{cm_e}(\mathbf{V}_{e0}\times\mathbf{B}_1)\cdot\hat{\mathbf{z}}, \tag{4}$$

where $\mathbf{V}_{e0}=-(cT_{e0}/eB_o)\hat{\mathbf{z}}\times\nabla \ln n_{e0}$, c is the speed of light, T_{eo} is the equilibrium electron temperature, e is the electronic charge, and m_e is the mass of an electron.

Let the electric field be expressed in the usual way as

$$\mathbf{E}_1=-\nabla\phi_1-\frac{1}{c}\partial_t A_{z1}\hat{\mathbf{z}}, \tag{5}$$

where $\mathbf{A}_\perp=0$ has been assumed.

The linear perturbations are assumed to be $\propto \exp i(k_y y+k_z z-\omega t)$. Therefore, the parallel components of the equations of motion of hot electrons and cold ions yield, respectively,

$$V_{ez1}=-\frac{ek_z}{m_e\omega}\phi_1+\left(1-\frac{\omega_e^*}{\omega}\right)\frac{e}{cm_e}A_{z1} \tag{6}$$

and

$$V_{iz1}=\frac{ek_z}{m_i\omega}\phi_1-\frac{e}{cm_i}A_{z1}, \tag{7}$$

where $\omega_e^*=\mathbf{k}\cdot\mathbf{V}_{e0_\perp}$.

In the limit of quasineutrality with the assumption of static dust, one obtains

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$$\partial_t^2 \nabla_\perp^2 \phi_1 + \frac{4\pi ec}{B_0}\frac{\Omega_i^2}{\omega_{pi}^2}\hat{\mathbf{z}}\times\nabla(Zn_{d0})\cdot\nabla\phi_1 - \frac{B_0\Omega_i}{cn_{i0}}\left(\frac{ik_z}{4\pi e}\right) \times\left\{\omega_{pi}^2\left(\frac{k_z}{\omega}\phi_1 - \frac{1}{c}A_{z1}\right) - \omega_{pe}^2\left[-\frac{k_z}{\omega}\phi_1 + \left(1-\frac{\omega_e^*}{\omega}\right)\frac{A_{z1}}{c}\right]\right\} = 0, \tag{8}$$

where $\Omega_\alpha = eB_0/cm_\alpha$ and $\omega_{p\alpha} = 4\pi n_{\alpha 0}e^2/m_\alpha$.

In the limit $A_{z1}=0$ it reduces to equation (65) of [5]. However, we think $A_{z1}\neq 0$ should also be considered if $k_z \neq 0$.

In the low-frequency limit, Ampere's law gives

$$\left(1+\lambda_e^2 k_\perp^2 - \frac{\omega_e^*}{\omega} + \frac{\omega_{pi}^2}{\omega_{pe}^2}\right)\frac{\omega}{ck_z}A_{z1} = \left(1+\frac{\omega_{pi}^2}{\omega_{pe}^2}\right)\phi_1, \tag{9}$$

where $\lambda_e = c/\omega_{pe}$.

If $\phi_1=0$ and $\omega_{pi}^2/\omega_{pe}^2 \to 0$ is assumed, it reduces to the magnetostatic drift mode [4]

$$\omega = \frac{1}{a}\omega_e^*, \tag{10}$$

where $a = 1+\lambda_e^2 k_\perp^2$.

Equations (8) and (9) give the coupled dispersion relation of electromagnetic low-frequency modes in a nonuniform dusty plasma with static dust as

$$[\omega(\omega-\omega_{sv}) - \omega_{cc}^2(1+\alpha_{ei})][a\omega - \omega_e^* + \alpha_{ei}\omega] = -\omega_{cc}^2[\omega-\omega_e^*+\alpha_{ei}\omega][1+\alpha_{ei}], \tag{11}$$

where $\alpha_{ei}=\omega_{pi}^2/\omega_{pe}^2$, $\omega_{sv}=(4\pi ec/B_0)(\Omega_i^2/\omega_{pi}^2)\times[k_y d_x(Z_d n_{d0})/k_\perp^2]$, and $\omega_{cc}^2=(n_{e0}/n_{i0})(k_z^2/k_\perp^2)\Omega_i\Omega_e$.

In the limit $\omega_{pi}^2 \ll \omega_{pe}^2$, Eq. (11) reduces to

$$[\omega(\omega-\omega_{sv}) - \omega_{cc}^2][(1+\lambda_e^2k_\perp^2)\omega - \omega_e^*] = -\omega_{cc}^2[\omega-\omega_e^*]. \tag{12}$$

It should be noticed that the first bracket on the left-hand side is not the dispersion relation of the electrostatic convective cell mode as quoted in [5]. One term on the left-hand side cancels with the right-hand side and it becomes

$$(\omega-\omega_{sv})[(1+\lambda_e^2k_\perp^2)\omega-\omega_e^*] = \lambda_e^2 k_\perp^2 \omega_{cc}^2. \tag{13}$$

This equation shows clearly that $k_z\neq 0$ gives a coupling between the electrostatic convective cell and the magnetostatic modes both in dusty as well as electron-ion plasmas, where $\omega_{sv}=0$.

It is important to note that in the homogeneous plasma Eq. (13) gives the shear Alfven wave dispersion relation as $\omega^2 = V_A^2 k_z^2/1+\lambda_e^2 k_\perp^2$, where $V_A=(B_0^2/4\pi n_0 m_i)^{1/2}$ is the Alfven speed. The parallel electron current introduces the propagation of shear Alfven waves in the system. The linear perturbation cannot remain electrostatic in the presence of parallel electron motion. Therefore, the electrostatic convective cell is not modified due to $k_z\neq 0$, as has been qouted in Ref. [5]. However, the shear Alfven waves can be coupled with electrostatic and magnetostatic modes due to $k_z\neq 0$.

One of us (H.S.) would like to thank the Japan Society for the Promotion of Science (JSPS) for financial support and Professor T. Sato for hospitality at the National Institute for Fusion Science (NIFS), Toki, Japan, where this work was initiated.

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