Immense delocalization from fractional kinetics

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(Received 13 May 1999)

We observe immense delocalization of the order of 10^9 for a kicked Harper model when a control parameter K is taken to be $K^*=6.349\,972$. This ''magic'' value corresponds to special phase space topology in the classical limit, when a hierarchical self-similar set of sticky islands emerges. The origin of the effect is of the general nature and similar immense delocalization can be found in other systems. [S1063-651X(99)03912-4]

PACS number(s): 05.45.Mt, 05.60.Gg

The problem of quantum delocalization in systems chaotic in the classical limit has attracted much attention in the field of quantum chaos [1,2]. The important problems of transport and spectral features [1–7] considered here reflect the quantum nature of this phenomena. A class of delocalization problems due to classical fractional kinetics has been found recently as well [8–10]. A paradigm of fractional or strange kinetics [12,13] has been related to the unusual behavior of dynamically chaotic trajectories which have fractal coherent structures in phase space and time simultaneously. Dynamics of particles resembles to some extent Lévy-type processes leading to superdiffusion, supermixing, and large long lasting fluctuations. All these ''superfeatures'' can be easily observed when a control parameter of the system, say K, is specially selected with a high accuracy [14,15]. Some recent results show that abnormal properties of chaotic kinetics can be observed in quantized systems [8–10], where coherent space-time structures of trajectories in the classical limit can stimulate delocalization. An experimental evidence of such process has been presented as well [11]. In mesoscopic physics of modern materials [2,7,16,17] the significance of the problem of delocalization cannot be overestimated. For a simplified model, namely, the kicked Harper model, that described such phenomena we show that there exist a special ''magic'' value of the control parameter $K^*=6.349\,972$, for which delocalization effect can be 10^9 times higher as for regular values of K. This value of K^* corresponds to the strongest classical superdiffusion, when trajectories are trapped for an arbitrary long time near the boundaries of the hierarchical self-similar set of islands.

The kicked Harper model (KHM) can be considered as a simplified model for the exploration of chaotic dynamics of Bloch electrons in the presence of a magnetic field and subject to an alternating electromagnetic field. It appears naturally in the chaotic web dynamics corresponding to kicks combined with fourfold symmetry rotation [18]. The variety of interesting phenomena of classical and quantum diffusion as well as localization-delocalization transitions in a wide range of parameters have been observed in Refs. [1–20]. It has been shown recently [21] that KHM describes properly cyclotron resonance in the 2D electron gas in antidot arrays [2,16] and organic metals [17].

We consider KHM in the following symmetrical form:

$$H=K\cos p+K\cos x\,\delta_1(t), \tag{1}$$

where momentum $p=\tilde{h}\hat{n}=-i\tilde{h}(\partial/\partial x)$ and coordinate x are dimensionless canonical pair $[x,p]=i\tilde{h}$, $\tilde{h}$ is the dimensionless Planck constant [22], K is a control parameter of chaos, while $\delta_1(t)=\Sigma_n\delta(t-n)$ is a train of δ kicks with a period $T=1$. In the classical limit this Hamiltonian leads to the map $p_{n+1}=p_n+K\sin x_n$, $x_{n+1}=x_n-K\sin p_{n+1}$, which due to its special symmetry properties has the same phase space topology as the web map: $p_{n+1}=x_n$, $x_{n+1}=-p_n-K\sin p_{n+1}$ [18]. For this symmetrical form quantum dynamics is delocalized, and diffusion takes place [3–5,19] even for small values of K [3].

For the classical counterpart, when K is small, chaotic dynamics is realized inside the narrow stochastic web [18], and most of the phase space is occupied by stable orbits. With K growing, the chaotic region becomes wider and at some condition on K chaos becomes global. For example, when $K=5$, small stability islands are invisible and the dynamics is strongly chaotic [4]. Nevertheless, when K increases, bifurcate reconstruction of phase space with arising of so-called accelerator islands takes place [14,15]. It happens each time when $K>2\pi n$, $n=1,2,\ldots$, within a small interval ΔK [14,15]. While the presence of such islands has a specific interest, more important bifurcations appear for special values of K within ΔK when hierarchical sets of islands-around-islands emerges. These ''magic'' values K^* give rise to strongly anomalous diffusion with long flights of the Lévy type [14,15]. Indeed, at $K=K^*=6.349\,972$ such a bifurcation was observed. The accelerator islands corresponding to forthfold symmetry are shown in Fig. 1(a). Corresponding to this Poincaré section chaotic trajectory with flights in extended phase space is shown in Fig. 1(b). A transport for this value of K^* corresponds to anomalous diffusion in momentum space $\langle p^2\rangle\sim t^{\mu}$, with transport exponent $\mu=1.27$ [14], while for normal diffusion $\mu=1$. The essential increasing of transport due to the flights is shown in Fig. 1(c).

This bifurcation in classical dynamics affects the quantum transport nature as well. This is described by the time dependent Schrödinger equation. Since the Hamiltonian (1) is periodic in time, the Floquet theory can be applied. The dynamics is determined by an evolution operator upon one

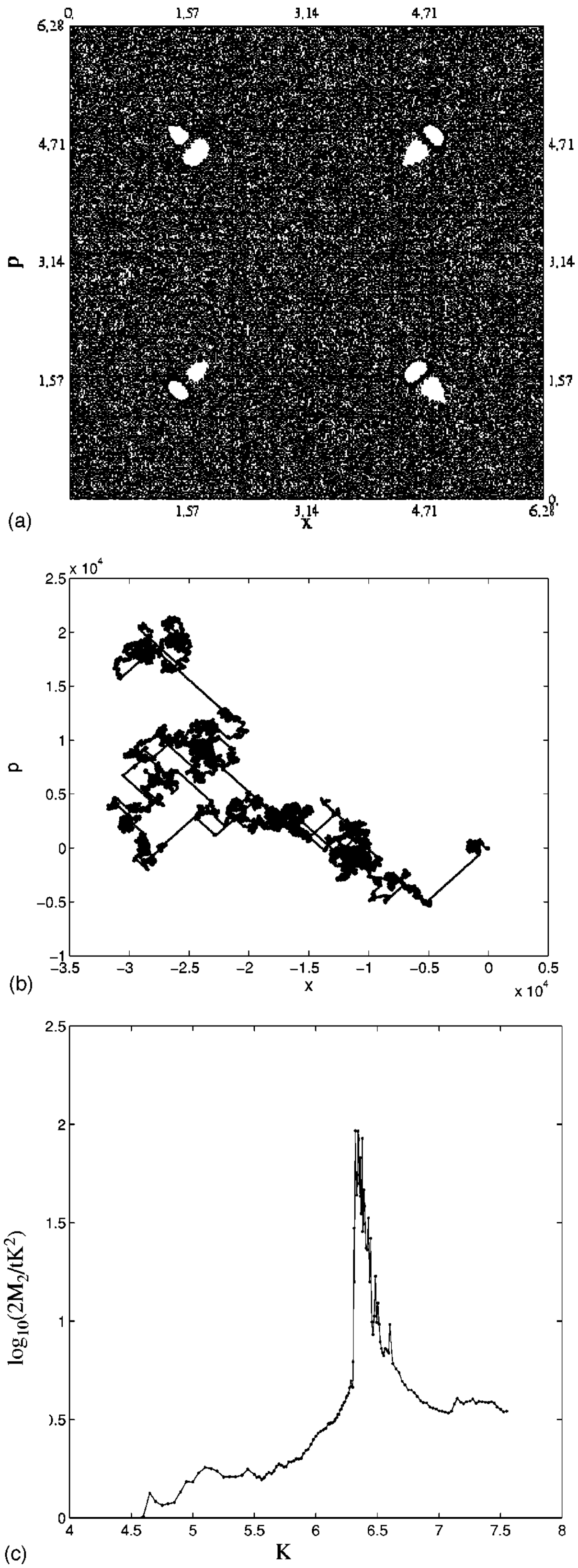

FIG. 1. (a) Properties of the KHM map near the magic $K = K^* = 6.349\,972\cdots$: a trajectory in the phase space for $K = K^*$ after 2×10^5 iterations. The islands corresponds to the acceleration mode. (b) The same as in (a) a trajectory in extended phase space with well resolved flights after 5×10^5 iterations. (c) Normalized kinetic coefficient obtained from $M_2 = \langle p^2(t)\rangle$ for various K near K^* after averaging over 10^4 trajectories for each value of K with 0.5×10^5 iterations. Sharp maximum corresponds to K^*.

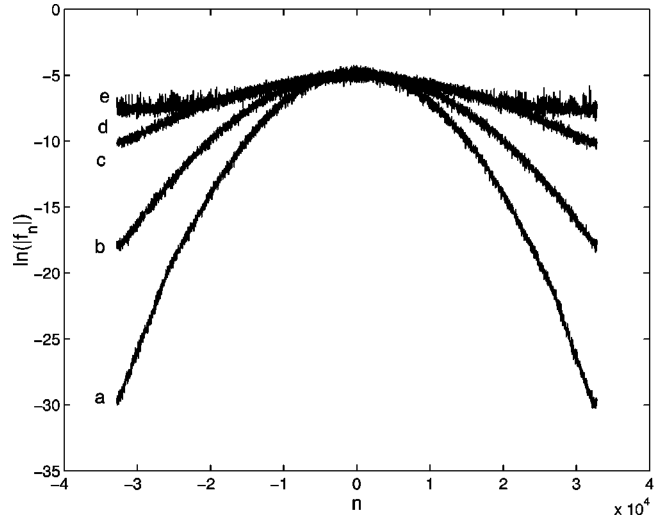

FIG. 2. Level occupation amplitudes $|f_n|$ vs n for various values of K and $\tilde{h} = 0.015\,683$. Data ploted after 200 iterations, and (a) $-K = 5.5$; (b) $-K = 6.0$; (c) $-K = 6.7$; (d) $-K = K^* = 6.349\,972$; (e) $-K = K^*_h = 6.349\,906$.

period $T = 1$. From Eq. (1) we obtain

$$\hat{U}(T=1) = \exp\left\{-i\frac{K}{\tilde{h}}\cos[\tilde{h}(\hat{n}+\theta_x)]\right\}\exp\left\{-i\frac{K}{\tilde{h}}\cos x\right\}, \tag{2}$$

where the quasimomentum θ_x appears here in consequence of the periodicity of the system (1) in x. In the following analysis $\theta_x = 0$. Hence, evolution of the system is determined by the following quantum map for wave functions:

$$\Psi(x,t+T) = \hat{U}\Psi(x,t). \tag{3}$$

In the framework of this map we have studied the time evolution of the moments

$$M_{2m} = \langle p^{2m}(t)\rangle = \tilde{h}^{2m}\langle \hat{n}^{2m}(t)\rangle = \tilde{h}^{2m}\sum_n |f_n(t)|^2 n^{2m},$$

$$\Psi(x,t) = \sum_{n=-\infty}^{\infty} f_n(t)e^{2\pi i n x}, \tag{4}$$

where $m = 1,2,4$ and $|f_n(t)|^2$ is probability of level occupation at time t determining the energy growth spreading over the unperturbed spectrum. The initial occupation value is $f_n(t=0) = \delta_{n,1}$.

For iteration of the map (3) we use the standard technique of the fast Fourier transform [23,24] with up to $N = 2^{18}$ momentum eigenstates. The results of the numerical analysis depicted in Figs. 2–4 show that in the narrow region of values of the parameter K near K^* the quantum dynamics exhibits immense delocalization that differs essentially from the delocalization observed in the previous studies [3–5]. These differences are due to the classical fractional or strange kinetics. In the classical limit hierarchical set of islands for K^* leads to long flights which have ‘‘reduced’’ chaotic properties since the flights correspond to the part of trajectories that stick to the islands boundaries. Reduced

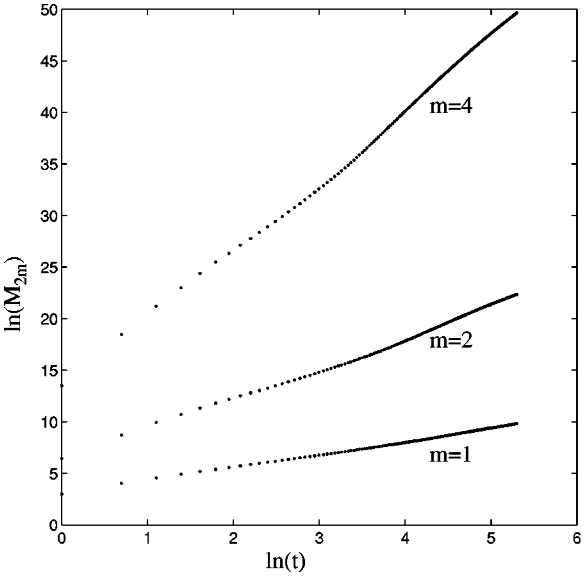

FIG. 3. Log-log plot of the even quantum moments M_{2m}, $m=1,2,4$ vs number of iterations t for $K=K^*$ and $\tilde{h}=0.015\,683$.

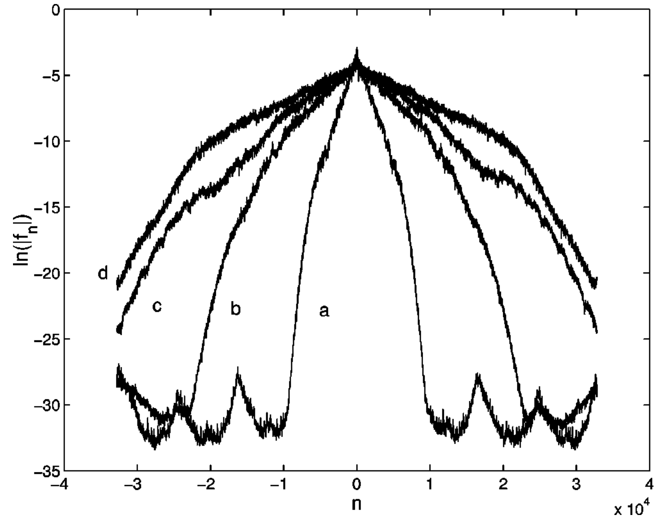

FIG. 4. Level occupation amplitudes $|f_n|$ vs n for various values of $\tilde{h}$ and $K=K_h^*$ after 2600 iterations and (a) $-K_h^*=6.3072$, $\tilde{h}=0.4023$; (b) $-K_h^*=6.3388$, $\tilde{h}=0.2052$; (c) $-K_h^*=6.3436$; $\tilde{h}=0.1547$; (d) $-K_h^*=6.3459$; $\tilde{h}0.1241$.

chaos increases delocalization in the quantum case. The level of the chaos reduction depends on how regular is the islands hierarchy. In Fig. 1(a) dark strips correspond to the stickiness of trajectories. The smaller are islands in their hierarchy set, the smaller the strip of sticky domain [14,15], the smaller the level of randomness. That is why we need a fairly accurate tuning of the control parameter to observe the effect the immense delocalization.

Comparing curves 1 and 4 in Fig. 2 we observe effect of delocalization of order of $e^{22}\sim 10^{9.57}$ of the level occupation amplitude. Curve 5 corresponds to the renormalized $K_h^*=2K^*\sin(\tilde{h}/2)/\tilde{h}$ [19] and shows a small increase of the effect. Increasing of the transport exponent μ for anomalous diffusion in both classical and quantum dynamics is another manifestation of the role of sticky islands hierarchy. The moments plot in Fig. 3 demonstrates the existence of anomalous exponents μ from Eq. (4): $M_{2m}\sim t^{\mu_m}$ with different values of μ in different time windows similarly to the classical one [14,15]. The values of μ_m where obtained within time interval $t>100$. The strong intermittent character of μ_m as a function of m: $\mu_1=\mu=1.4$, $\mu_2=3.3>2\mu$, $\mu_4=6.9>4\mu$ can be a result of short time iteration of Eq. (3) with $t\leqslant 200$. These values of the transport exponent μ show strong superdiffusion which even exceeds the classical one. We observed also that the values of μ grow as K approaches to the ''magic'' value K^*.

Finally, in Fig. 4 we display a level of delocalization for $K=K^*$ for various values of the Planck constant $\tilde{h}$. The smaller is $\tilde{h}$, i.e., the more the system is classical, the stronger the delocalization. Again, comparing the occupation amplitudes $|f_n|$ for curves 1 and 4 at $n=1\times 10^4$, we find that the difference between them is of order of 10^{10}. It shows that the delocalization for ''magic'' values K^* has classical nature.

We conclude by a comment that observation of strong delocalization of the order of 10^5 for the standard map [9] is of the same nature, although it was obtained for not small $\tilde{h}$. We conjecture that a similar delocalization can be found in any other physical system if the corresponding magic value of the control parameter is discovered. A proper adjusting of the control parameter to obtain strongly anomalous transport can be interesting for the realization of mesoscopic devices with effective controlling of conductivity.

We should comment that the appearance of the accelerator mode islands is a result of a bifurcation and topological reconstruction of the phase space. Another result of this reconstruction is a change of cantori properties. We may expect that simultaneously with emerging of a hierarchical set of islands, a hierarchical set of cantori has to appear. This leads us to the conclusion that the problem of quantum delocalization can be also considered from the point of view of a corresponding restructure of quantum barriers as it was mentioned in Ref. [25]. Unfortunately, we do not know too much about the cantori bifurcation and their topological reconstruction.

We thank M. Edelman for help in preparing Fig. 1. This research was supported by the U.S. Departments of Navy Grants and by U.S. Department of Energy. A.I. was also supported by the Israel Science Foundation and by the Niedersachsen Ministry of Science, Germany.

[1] See for review T. Geisel, R. Ketzmerick, and G. Petschel, in *Quantum Chaos: Between Order and Disorder*, edited by G. Casati and B. V. Chirikov (Cambridge, Cambridge University Press, 1996); p. 633; R. Artuso, G. Casati, F. Borgonovi, L. Rebuzzini, and I. Guarneri, Int. J. Mod. Phys. B **8**, 207 (1994).

[2] R. Fleishmann, T. Geisel, R. Ketzmerick, and G. Petschel, Physica D **86**, 171 (1995).

[3] R. Lima and D. Shepelyansky, Phys. Rev. Lett. **67**, 1377

(1991).
[4] T. Geisel, R. Ketzmerick, and G. Petschel, Phys. Rev. Lett. **67**, 3635 (1991).
[5] R. Artuso, F. Borgonovi, I. Guarneri, L. Rebuzzini, and G. Casati, Phys. Rev. Lett. **69**, 3302 (1992).
[6] G. Petschel and T. Geisel, Phys. Rev. Lett. **71**, 239 (1993).
[7] R. Ketzmerick, K. Kruse, and T. Geisel, Phys. Rev. Lett. **80**, 137 (1998).
[8] L. Bonci, P. Grigolini, A. Laux, and R. Roncaglia, Phys. Rev. A **54**, 112 (1996).
[9] B. Sundaram and G. M. Zaslavsky, Phys. Rev. E **59**, 72 311 (1999).
[10] D. Kusnezov, A. Bulgac, and G. D. Dang, Phys. Rev. Lett. **82**, 1136 (1999).
[11] B. G. Klappauf, W. H. Oskay, D. A. Steck, and M. G. Raizen, Phys. Rev. Lett. **81**, 4044 (1999).
[12] G. M. Zaslavsky, in *Topological Aspects of the Dynamics of Fluids and Plasmas*, edited by H. K. Moffat, G. M. Zaslavsky, P. Comte, and M. Tabor (Kluwer, Dordrecht, 1992), p. 481.
[13] M. F. Shlesinger, G. M. Zaslavsky, and J. Klafter, Nature (London) **363**, 31 (1993).
[14] G. M. Zaslavsky and B. A. Niyazov, Phys. Rep. **283**, 73 (1997).
[15] G. M. Zaslavsky, M. Edelman, and B. A. Niyazov, Chaos **7**, 1 (1997).
[16] E. Vasiliadou, *et al.*, Phys. Rev. B **52**, R8658 (1995).
[17] J. Singleton, *et al.*, Phys. Rev. Lett. **68**, 2500 (1992).
[18] G. M. Zaslavsky, M. Yu. Zakharov, R. Z. Sagdeev, D. A. Usikov, and A. A. Chernikov, Sov. Phys. JETP **64**, 294 (1986).
[19] G. P. Berman, V. Yu. Rubaev, and G. M. Zaslavsky, Nonlinearity **4**, 543 (1991).
[20] I. Dana, Phys. Rev. Lett. **73**, 1609 (1994).
[21] A. Iomin and S. Fishman, Phys. Rev. Lett. **81**, 1921 (1998).
[22] D. R. Hofstadter, Phys. Rev. B **14**, 2239 (1976).
[23] D. R. Grempel, S. Fishman, and R. E. Prange, Phys. Rev. A **29**, 1639 (1984).
[24] S. Fishman, D. R. Grempel, and R. E. Prange, Phys. Rev. A **36**, 289 (1987).
[25] T. Geisel, G. Radons, and J. Rubner, Phys. Rev. Lett. **57**, 2883 (1986).