Dynamics of a misaligned astigmatic twisted Gaussian beam in a Kerr-nonlinear parabolic waveguide

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Using the Galerkin criterion in the basis of flexible generalized Gaussian modes, the equations of motion are derived for the parameters of a misaligned astigmatic twisted Gaussian beam in an axially symmetric nonlinear medium. Nontrivial features of the beam dynamics (e.g., phase locking, cycle generation, nonlinear symmetry change) in a parabolic waveguide with Kerr nonlinearity are revealed. [S1063-651X(99)00312-8]

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I. INTRODUCTION

Modeling of axially nonsymmetric beams in axially symmetric media is a necessary part of theoretical study of misaligned lasers and optical waveguides [1–5]. Complex regimes of light beam propagation in an optical fiber have been previously found in the limit of the geometric optics (see Ref. [6]). The wave picture of the beam dynamics can be most generally obtained by means of the direct numerical solution of the relevant boundary problem. This approach is widely used, e.g., in the study of transverse pattern formation [7], spectral characteristics [8], and nonlinear interaction of beams [9]. Recently, explicit solutions for some particular classes of completely integrable optical beam systems have been derived analytically [10]. Our interest will be focused on semianalytic approaches that provide an approximate description of the beam dynamics in terms of a limited set of parameters, depending on the longitudinal coordinate z. Originally aimed at simplifying the calculations, such methods at present seem to be more important as a source of dynamical models demonstrating nontrivial behavior.

In earlier papers [8,11,12] we have suggested an approximate method using Gaussian probe functions whose parameters are determined by Galerkin's criterion in the basis of a small number of flexible Gaussian modes. The procedure yields a set of first-order differential equations that govern the dependence of the beam parameters upon z. This approach is closely related to the well-known method [13,14,5] in which a reduced description of beams is given in terms of the beam intensity moments. In fact, it can be shown that the beam parameters of Refs. [8], [11], and [12] can be expressed via the complex field generalized moments [15] whose definition is analogous to that of the quantum mechanical average. In contrast to the classical method of moments [13,14,5], our approach is equally convenient both for conservative and dissipative nonlinear media.

In Sec. II we extend the technique of Refs. [8], [11], and [12] over the misaligned Gaussian beams of the most general type with astigmatism, shift, deflection, and twist taken into account simultaneously. The resulting set of equations for the beam variables is free of limitations inherent to ray optics [5] and aberrationless theory [3,4]. Considering z as an evolution variable (''time''), we obtain a finite-dimensional nonlinear dynamical model which appears to represent an important but insufficiently studied class of systems with alternating sign of the phase velocity divergence. In Sec. III, we use this model to analyze the complex behavior of a misaligned Gaussian beam in a waveguide medium having parabolic profile of the linear refraction index and Kerr nonlinearity below the self-focusing threshold. In Sec. IV, we discuss the results of testing the method proposed by comparing its results with those of direct numerical integration. We also mention the beam collapse problem and the connection with nonlinear dynamics models relevant to other fields of physics (e.g., quantum optics and statistical mechanics).

II. DERIVATION OF THE DYNAMICAL SYSTEM

We start from the scalar parabolic wave equation governing the complex slow amplitude ψ of the electric field of the beam

$$\hat{H}\psi \equiv \left[4i\frac{\partial}{\partial z}+\nabla_\perp^2+\chi\right]\psi=0, \tag{1}$$

where $\nabla_\perp^2=\partial^2/\partial x^2+\partial^2/\partial y^2$ is the transverse Laplacian responsible for diffraction, the transverse coordinates x,y are scaled to the typical radius a of the beam, the longitudinal coordinate z is scaled to the corresponding diffraction length $L=ka^2/2$, k being the wavenumber. The complex susceptibility $\chi=\chi'+i\chi''$ is, generally, dependent on the coordinates and the field amplitude. We seek the approximate solution of Eq. (1), having the generalized Gaussian form

$$\psi_0=A(z)\exp[-(\eta(z)x'^2+\beta(z)y'^2+i\xi(z)x''^2+i\varepsilon(z)y''^2)], \tag{2}$$

where the coordinates x',y' and x'',y'' are expressed in terms of the laboratory coordinates x,y as

$$x'=[x-x_I(z)]\cos\varphi(z)+[y-y_I(z)]\sin\varphi(z);$$

$$y'=-[x-x_I(z)]\sin\varphi(z)+[y-y_I(z)]\cos\varphi(z); \tag{3}$$

$$x''=[x-x_p(z)]\cos\theta(z)+[y-y_p(z)]\sin\theta(z);$$

$$y''=-[x-x_p(z)]\sin\theta(z)+[y-y_p(z)]\cos\theta(z). \tag{4}$$

From Eqs. (2), (3), and (4) it follows that $|A(z)|^2 = |\psi_0|^2|_{x'=0,y'=0} \equiv I(z)$ is the maximal intensity of the beam, $\eta(z)$ and $\beta(z)$ are the inverse square dimensions of the beam spot, $\xi(z)$ and $\varepsilon(z)$ define the principal values of the wave front curvature. The elliptical beam spot is shifted by $x_I(z), y_I(z)$ from the laboratory z axis, its principal axes being turned by the angle $\varphi(z)$ with respect to the laboratory xOy frame. The wave front is a second-order surface shifted by $x_p(z), y_p(z)$ and turned by the angle $\theta(z)$ with respect to the laboratory frame. Thus, the set of z-dependent parameters $I,\eta,\beta,\xi,\varepsilon,\varphi,\theta,x_I,y_I,x_p,y_p$ plus the phase $\arg(A)$ forms a complete list of the beam variables. To eliminate singularities, we introduce the average beam slope angles

$$\alpha(z) = \xi((x_p - x_I)\cos\theta + (y_p - y_I)\sin\theta), \quad \gamma(z) = \varepsilon((x_p - x_I)\sin\theta - (y_p - y_I)\cos\theta),$$

instead of the original wave front shift variables (x_p, y_p) that may be infinitely large as the wave front curvature tends to zero.

The statement that a certain function ψ is an exact solution of Eq. (1) is, obviously, equivalent to the statement that the function $\hat{H}\psi$ is orthogonal to a complete orthonormal basis in the Hilbert space with the scalar product defined as an integral over the beam cross section: $\langle\phi|\psi\rangle \equiv \int \phi^* \psi dS$. If the basis is reduced to a finite set of functions, this orthogonality condition provides a criterion for choosing the best approximate solution from a given class of functions (Galerkin criterion). Following the approach of Refs. [8], [11], and [12], we choose the basis to be a set of generalized Gaussian modes $\langle mn| \equiv |m\rangle_x |n\rangle_y$, where

$$|0\rangle_x = N_x \exp[-\eta x'^2 - i\xi x''^2];$$

$$|0\rangle_y = N_y \exp[-\beta y'^2 - i\varepsilon y''^2];$$

$$|1\rangle_x = x'\sqrt{4\eta}|0\rangle_x; \quad |1\rangle_y = y'\sqrt{4\beta}|0\rangle_y;$$

$$|2\rangle_x = \frac{1-4\eta x'^2}{\sqrt{2}}|0\rangle_x; \quad |2\rangle_y = \frac{1-4\beta y'^2}{\sqrt{2}}|0\rangle_y; \ldots \tag{5}$$

whose basic mode is proportional to the probe function ψ_0 (2), N_x, N_y being the normalization coefficients. The modes (5) are orthonormal and z dependent via the parameters $\eta,\beta,\xi,\varepsilon$ as well as via the shift and rotation of the coordinate frames $x'O'y'$ and $x''O''y''$. Using this basis we apply the Galerkin criterion to the probe function ψ_0. Each of the orthogonality conditions

$$\langle mn|\hat{H}\psi_0\rangle = 0 \tag{6}$$

with $m,n = 0,1,2,\ldots$ yields a complex equation that involves the beam variables and their first-order derivatives with respect to z. The integration is simplified if performed in the coordinate frame $x'O'y'$. For linear homogeneous media, $\hat{H}\psi_0$ is a quadratic polynomial in x', y' that makes the integration straightforward. Generally, χ depends upon the transverse coordinates due to the medium inhomogeneouity and/or nonlinearity, so that the corresponding integrals of χ enter the equations and should be calculated separately. To get a closed set of equations for the 12 beam variables it appears to be enough to use six orthogonality conditions (6) with $(m,n) = (0,0),(0,1),(1,0),(1,1),(0,2),(2,0)$. Solving these equations with respect to the derivatives, we finally get:

$$\frac{dI}{dz} = I\left(\xi + \varepsilon - \frac{\langle 20|\chi''|00\rangle}{2\sqrt{2}} - \frac{\langle 02|\chi''|00\rangle}{2\sqrt{2}} - \frac{\langle 00|\chi''|00\rangle}{2}\right); \tag{7}$$

$$\frac{d\eta}{dz} = \eta\left[2(\xi\cos^2\delta + \varepsilon\sin^2\delta) - \frac{1}{\sqrt{2}}\langle 20|\chi''|00\rangle\right]; \tag{8}$$

$$\frac{d\beta}{dz} = \beta\left[2(\xi\sin^2\delta + \varepsilon\cos^2\delta) - \frac{1}{\sqrt{2}}\langle 02|\chi''|00\rangle\right]; \tag{9}$$

$$\frac{d\xi}{dz} = \xi^2 - \eta^2\cos^2\delta - \beta^2\sin^2\delta + \frac{1}{\sqrt{2}}[\eta\langle 20|\chi'|00\rangle\cos^2\delta + \beta\langle 02|\chi'|00\rangle\sin^2\delta] + \frac{\sqrt{\eta\beta}}{2}\langle 11|\chi'|00\rangle\sin 2\delta; \tag{10}$$

$$\frac{d\varepsilon}{dz} = \varepsilon^2 - \eta^2\sin^2\delta - \beta^2\cos^2\delta + \frac{1}{\sqrt{2}}[\eta\langle 20|\chi'|00\rangle\sin^2\delta + \beta\langle 02|\chi'|00\rangle\cos^2\delta] - \frac{\sqrt{\eta\beta}}{2}\langle 11|\chi'|00\rangle\sin 2\delta; \tag{11}$$

$$\frac{d\varphi}{dz} = \sin 2\delta\frac{\eta+\beta}{2(\eta-\beta)}(\varepsilon - \xi) + \frac{\sqrt{\eta\beta}}{2(\eta-\beta)}\langle 11|\chi''|00\rangle; \tag{12}$$

$$\frac{d\theta}{dz} = \sin 2\delta\frac{\beta^2-\eta^2}{2(\xi-\varepsilon)} - \frac{\sqrt{\eta\beta}}{2(\xi-\varepsilon)}\cos 2\delta\langle 11|\chi'|00\rangle - \frac{\sin 2\delta}{2\sqrt{2}(\xi-\varepsilon)}[\beta\langle 02|\chi'|00\rangle - \eta\langle 20|\chi'|00\rangle]; \tag{13}$$

$$\frac{dx_I}{dz} = \alpha\cos\theta + \gamma\sin\theta + \frac{\sin\varphi}{4\sqrt{\beta}}\langle 01|\chi''|00\rangle - \frac{\cos\varphi}{4\sqrt{\eta}}\langle 10|\chi''|00\rangle; \tag{14}$$

$$\frac{dy_I}{dz} = \alpha\sin\theta - \gamma\cos\theta - \frac{\cos\varphi}{4\sqrt{\beta}}\langle 01|\chi''|00\rangle - \frac{\sin\varphi}{4\sqrt{\eta}}\langle 10|\chi''|00\rangle; \tag{15}$$

$$\frac{d\alpha}{dz}=-\gamma\frac{d\theta}{dz}+\frac{\sqrt{\eta}}{4}\cos\delta\langle 10|\chi'|00\rangle-\frac{\sqrt{\beta}}{4}\sin\delta\langle 01|\chi'|00\rangle$$
$$-\frac{\xi}{4\sqrt{\beta}}\sin\delta\langle 01|\chi''|00\rangle+\frac{\xi}{4\sqrt{\eta}}\cos\delta\langle 10|\chi''|00\rangle; \quad (16)$$

$$\frac{d\gamma}{dz}=\alpha\frac{d\theta}{dz}-\frac{\sqrt{\eta}}{4}\sin\delta\langle 10|\chi'|00\rangle-\frac{\sqrt{\beta}}{4}\cos\delta\langle 01|\chi'|00\rangle$$
$$-\frac{\varepsilon}{4\sqrt{\beta}}\cos\delta\langle 01|\chi''|00\rangle-\frac{\varepsilon}{4\sqrt{\eta}}\sin\delta\langle 10|\chi''|00\rangle. \quad (17)$$

Here $\delta=\varphi-\theta$ is the angle of the beam spot rotation with respect to the wave front. The equation for $\arg(A)$ is omitted since this variable does not enter all other equations and, therefore, does not affect the dynamics of the system [16].

The validity of the generalized Gaussian approximation has been tested by comparison of the results with those of direct integration of the paraxial wave equation using different numerical schemes [23,24]. In the discussion we shall dwell on this problem for more detail.

III. DYNAMICS OF THE BEAM VARIABLES IN A PARABOLIC WAVEGUIDE

Consider, for example, a transparent waveguide medium with parabolic profile of linear refraction index and Kerr nonlinearity

$$\chi'=\chi_0\left(1-\frac{x^2+y^2}{R_0^2}\right)+\chi_{NL}|\psi|^2, \quad \chi''=0.$$

In this case, the integration over x',y' to calculate the matrix elements $\langle nm|\chi'|kl\rangle$ in the right-hand sides of Eqs. (7)–(17) is easily performed analytically. We get

$$\langle 01|\chi'|00\rangle=\frac{\chi_0}{R_0^2\sqrt{\beta}}(x_I\sin\varphi-y_I\cos\varphi);$$

$$\langle 10|\chi'|00\rangle=-\frac{\chi_0}{R_0^2\sqrt{\eta}}(x_I\cos\varphi+y_I\sin\varphi);$$

$$\langle 11|\chi'|00\rangle=0; \quad \langle 02|\chi'|00\rangle=\frac{\chi_0}{R_0^2}\frac{1}{2\sqrt{2}\beta}+I\frac{\chi_{nl}}{4\sqrt{2}};$$

$$\langle 20|\chi'|00\rangle=\frac{\chi_0}{R_0^2}\frac{1}{2\sqrt{2}\eta}+I\frac{\chi_{nl}}{4\sqrt{2}}. \quad (18)$$

Equations (7)–(17) define a finite-dimension dynamical system, which is obviously nonlinear, even when $\chi_{nl}=0$. To decide formally whether this system is conservative or dissipative, let us consider the divergence of the phase velocity vector F' whose coordinates are the right-hand sides of Eqs. (7)–(17):

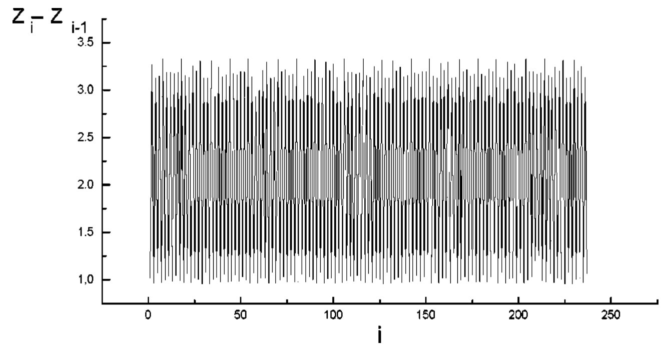

FIG. 1. On the evaluation of the phase velocity divergence: separation between the neighboring roots z_i of the equation $\int_0^z \mathrm{div}(F')dz=0$ versus i.

$$\mathrm{div}(F')=\mathrm{Sp}(J)=5\varepsilon+5\xi+\cos 2\delta\left\langle\frac{\eta+\beta}{\eta-\beta}(\varepsilon-\xi)-\frac{\beta^2-\eta^2}{\xi-\varepsilon}+\frac{\beta-\eta}{8(\varepsilon-\xi)}I\chi_{\mathrm{nl}}\right\rangle,$$

where J is the Jacobian matrix.

To demonstrate the long-trace behavior of $\mathrm{div}(F')$ we plotted the distance between the adjacent ith and $i+1$th zeros of the integral $\int_0^z \mathrm{div}(F')dz$, evaluated numerically, versus i (Fig. 1). The complicated periodical behavior of the integral is evidence for the zero mean value of the divergence, and, therefore, the dynamical system is conservative.

Our choice of the susceptibility allows a substantial reduction of the system dimension from eleven to seven due to the separation of the variables x_i,y_i,α,γ that describe the misalignment of the beam. Moreover, Eqs. (14) and (15) can be easily solved analytically and yield harmonic oscillations having the period $T_1=4\pi R_0/\sqrt{\chi_0}$. Further reduction to five dimensions occurs under the initial condition of $\delta=0$. Then, φ and θ are constants that means invariant orientation of the propagating beam spot and phase front. In this case, the equations for α,γ possess harmonic solutions with the same period T_1. Physically, it means that in a linear parabolic waveguide, the projection of the beam spot center trajectory on the transverse plane is, generally, an ellipse.

A. Stationary solutions

The conditions of the stationary beam propagation corresponding to nonlinear waveguide modes can be written as follows:

$$\delta=0, \quad \eta=\beta=I\chi_{nl}/16+\sqrt{I^2\chi_{nl}^2/16^2+\chi_0/(4R_0^2)},$$

$$\varepsilon=\xi=0.$$

In this case the dimension of the system is reduced to five, which allows an analytical study of its stability.

The eigenvalues of the Jacobian matrix on the stationary solution are

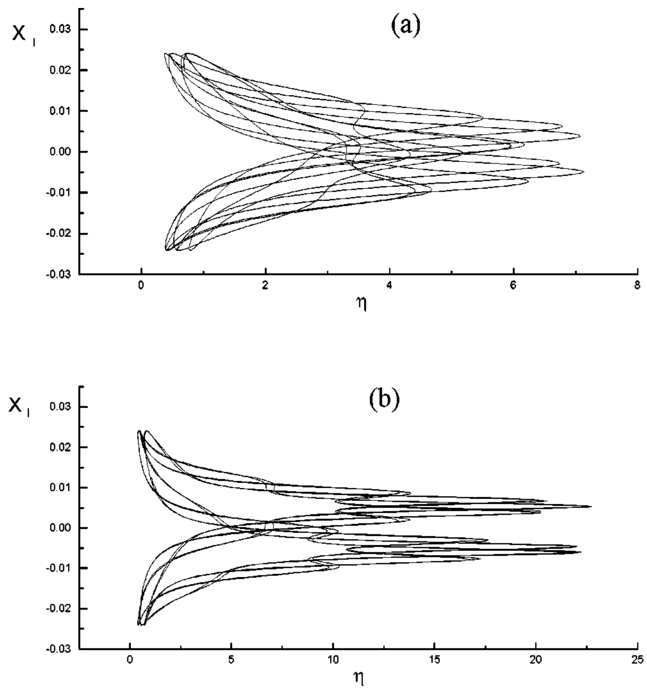

FIG. 2. Phase shift irregularity effect (a) and phase locking of dynamical variables (η, x_I) at $\chi_{nl}=0.07$ (a), 0.09 (b).

$$\lambda_1=0$$

$$\lambda_{2,3}=\pm i\sqrt{\chi_0}/R_o$$

$$\lambda_{4,5}=\pm i\left(\frac{I\chi_{nl}\eta}{4}+\frac{\chi_0}{R_0^2}\right)^{1/2}. \tag{19}$$

The real parts of all eigenvalues (Lyapunov exponents) are zero. This corresponds to a Hopf-like bifurcation in which the stationary points of the phase space lose their stability by expelling a trajectory. Below we shall prove that this trajectory is a stable orbit when χ_{nl} is zero.

B. Periodical and quasiperiodical solutions

Consider a linear waveguide $(\chi_{nl}=0)$. In general, the stability analysis of periodical solutions requires explicit calculation of the monodromy matrix Y. However, we can avoid this calculation by making use of the numerical observation that the period T of an oscillating solution of Eqs. (7)–(17) is equal to the period T_{lin} of the corresponding solution g_{lin} of the linearized equations. The monodromy matrix Y can be found from the relation $g_{\text{lin}}(z+T)=Y(T)g_{\text{lin}}(z)$. Since we observed $T=T_{\text{lin}}$ in all numerous examples considered, we can conclude that Y is a unit matrix. Hence, the multipliers of the periodical solution are equal to one and the phase trajectory is a stable orbit.

In low-dimensional systems, it is well known that for a ''supercritical'' Hopf bifurcation, the eigenvalues have the structure $\lambda=\pm i\omega$, where ω is the frequency of the oscillations arising. In our case for $\chi_{nl}=0$ the expressions (19) yield the period $T_2=2\pi R_0/\sqrt{\chi_0}$. Note, that $2T_2=T_1$, where T_1 is the period of the harmonic motion of the beam as a whole. We can summarize that in a linear waveguide, the beam dynamics is characterized by a superposition of stable nonharmonic oscillations of the inner variables (intensity, spot dimensions, wave front curvature, etc.) with the period T_2 and stable harmonic oscillations of the misalignment variables with the period $T_1=2T_2$.

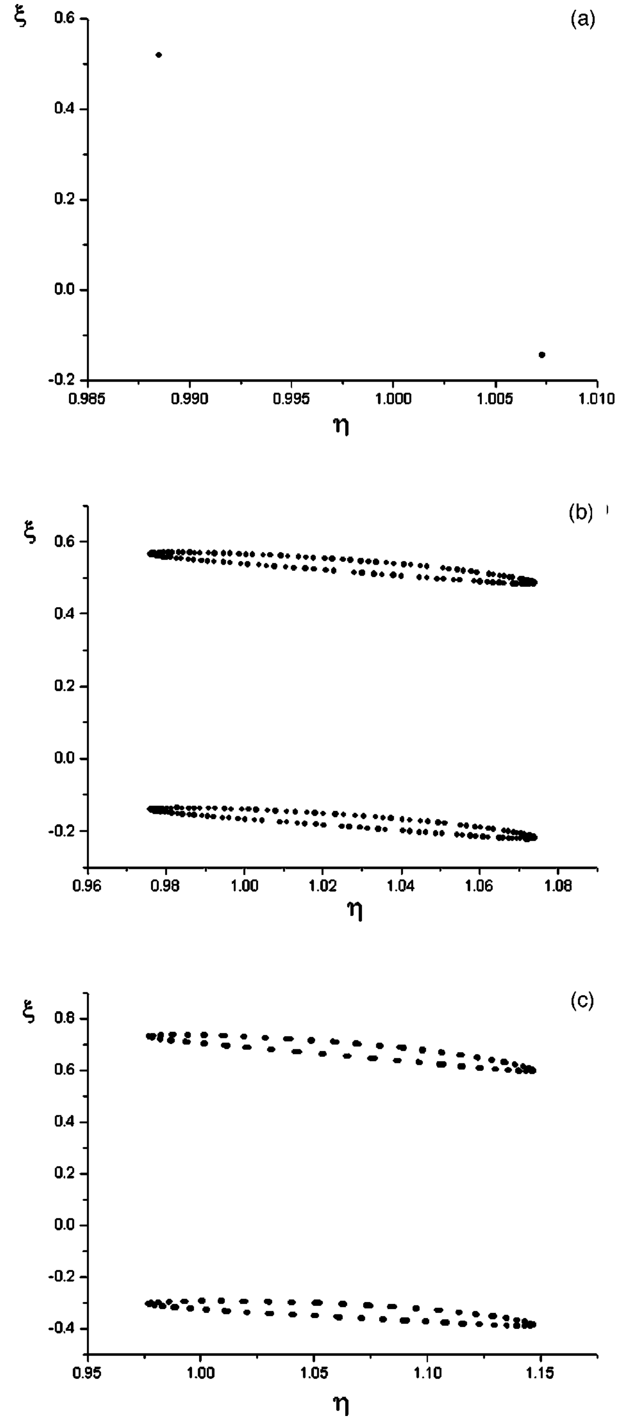

FIG. 3. Poincaré section in (η,ξ) projection. Hopf bifurcation (a), (b) and mutual phase locking between I, η, and ξ (c).

Under the condition $\chi_{nl}\neq 0$ (nonlinear waveguide) the eigenvalues $\lambda_{4,5}$ (19) become dependent on the dynamical variables. In this case, a quasiperiodical regime takes place. This is a manifestation of nonlinear interaction (cross-modulation) of the dynamical variables due to which the new frequencies arise that are not integer multiples of each other.

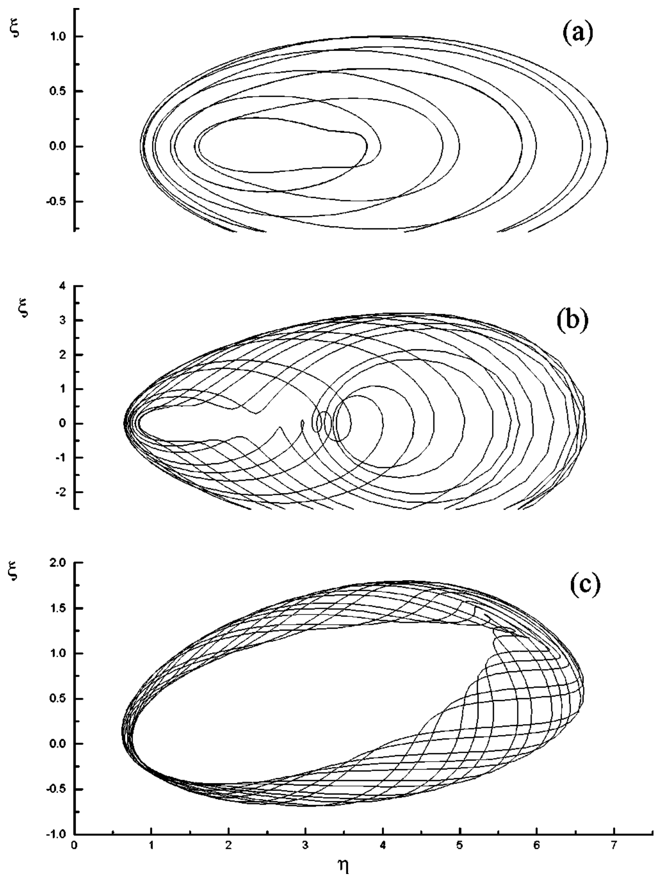

FIG. 4. Birth of cycles in the section (η,ξ) for zero initial twist (a), (b) and degeneration of this effect for the beam with twist (c).

To visualize and interpret the nonlinear dynamics of the beam variables we used the Poincaré map and phase portrait methods. In all examples presented below, the Poincaré section surface has been taken as $I=\mathrm{const}$ for the sake of clearer interpretation. Generally, the birth of a quasiperiodical regime corresponds to a bifurcation of a periodic orbit into an invariant manifold. In the Poincaré map this can be seen as a Hopf-like bifurcation of the stationary points [Figs. 3(a) and 3(b)].

The results presented below were obtained under the following initial conditions:

$$I=50,\quad \eta=0.7,\quad \beta=0.5,\quad \varepsilon=-0.4,\quad \xi=-0.1,$$

$$x_I=0.02,\quad y_I=0.01,\quad \alpha=0.01,\quad \gamma=-0.01.$$

We fixed the linear waveguide parameters as $\chi_0=5$, $R_0=1.5$ and varied the Kerr coefficient χ_{nl}.

We start from demonstrating the typical effects that arise under the initial condition $\delta(0)=0$. As mentioned above, in this case $\delta(z)=0$ for any z, i.e., the beam has no twist. Sweeping the parameter χ_{nl} across the interval (0.08, 0.095) we found a number of nonlinear resonances where the manifestations of phase locking involving two or more variables could be seen. For couples of variables, this effect is easily detected by means of phase portraits. Figures 2(a) and 2(b) shows the most informative projection of the phase trajectory which reveals the mutual phase locking between the misalignment variables (x_I,y_I,α,γ) and the variables $(\eta,\beta,\varepsilon,\xi)$ corresponding to the ''inner'' degrees of freedom of the beam.

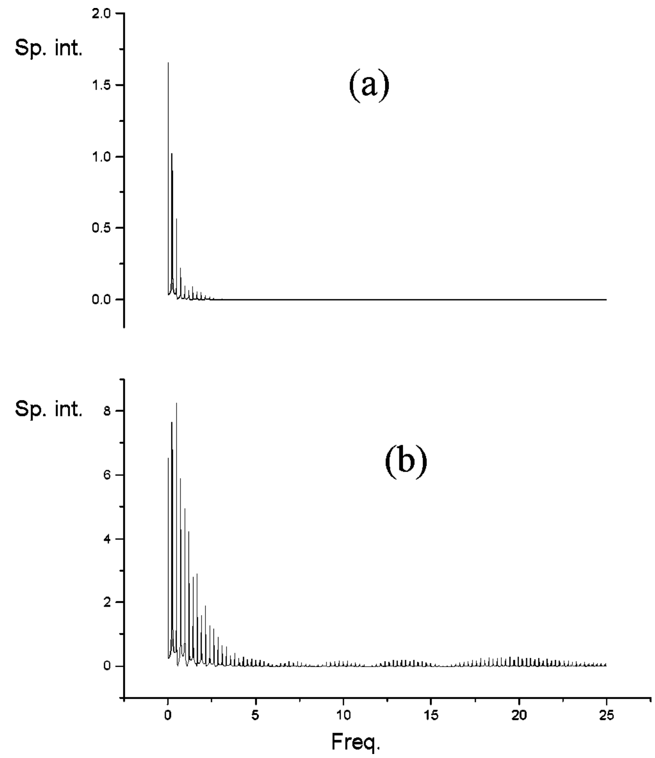

FIG. 5. Spectral representation of η at $\chi_{nl}=0.08$ (a),0.1 (b).

Since the Poincaré section surface may be chosen in an arbitrary way, one can observe the mutual phase locking for more than two dynamical variables. A transformation of the everywhere dense invariant manifold into an approximately closed orbit is displayed on the Poincaré map as a considerable reduction of the number of distinct points (see, for example, Fig. 3) provided that the number of intersections is fixed. In the vicinity of the phase-locking point one can observe nonstability of the relative phase shift of the variables η and x_I [Fig. 2(a)].

As χ_{nl} increases, the mutual modulation of the variables (η,ξ) [or (β,ε)] leads to the generation of a cycle. This phenomenon can be clearly seen, for example, in the projection of the phase trajectory onto the plane (η,ξ), presented in Figs. 4(a) and 4(b).

To make the nature of cycle generation more clear we used the spectral representation of η (Fig. 5). A shift of the maximum of spectral intensity distribution is seen when χ_{nl} grows.

Now consider a more general case of a beam with nonzero twist $(\delta\neq 0)$. Formally, one could expect more complexity in the beam behavior due to the additional degree of freedom. However, it is not so, and the dynamics of the beam actually appears to be even more simple than in the previous case of $\delta=0$. Our previous conclusions concerning the phase locking effect remain valid, while in some other phenomena under discussion a qualitative difference appears. In particular, the comparison of Figs. 2(a) and 6 shows that in the twisted beam, the irregularity of the phase shift disappears. Our calculations proved this statement to hold at any physically reasonable values of nonlinear refraction index.

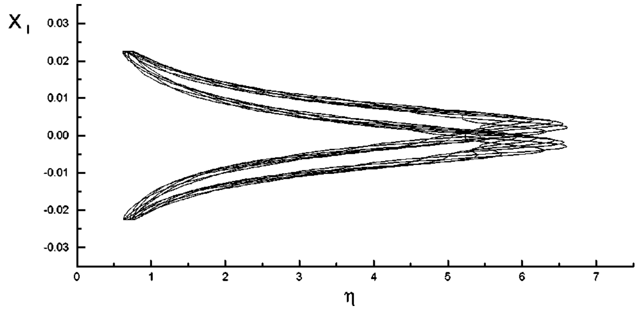

FIG. 6. Twisted beam: the irregularity of the phase shift between η and x_I disappears.

It was also shown that at $\delta \neq 0$, no cycle generation takes place under the same conditions as considered above. Instead one can observe only a weak amplitude modulation of the variables [see Fig. 4(c) for (η,ξ)].

IV. DISCUSSION

The approximation of beams by best-fit generalized Gaussian functions has, obviously, a limited area of application. These functions are known to be exact solutions of the paraxial wave equation only in linear homogeneous media and linear parabolic waveguides. Generally, the beam, even initially single Gaussian, acquires distortions while propagating through a nonlinear and/or nonparabolic medium due to the contribution of higher-order waveguide modes [25]. To estimate these distortions, we solved Eq. (1) numerically [23] using the spectral method [7]. At each step of the propagation problem, the transverse field distribution was decomposed in terms of more than 200 fixed Gauss-Laguerre radial and azimuthal modes of the linear waveguide. Diffraction and refraction were taken into account by means of the split-step scheme. One of the criteria for the integral evaluation of the quality of Gaussian field approximation was based on its mean-square deviation (MSD) from the result of the direct numerical solution, i.e., the square field difference averaged over the beam cross section. For typical $\chi_{nl} = 0.06$, the ratio of MSD to the peak intensity was found to be less than 5 percent [23]. We have also shown that under the conditions of the present study, the non-Gaussian distortions of the beam profile cause minor changes in the values and dynamics of the beam spot dimensions and beam rotation angle.

Our next test numerical experiment [24] has been performed using the multiple-coordinate splitting, implicit absolutely stable scheme and complex scaling providing the correct transverse boundary conditions. The aim of the study was to test the Gaussian flexible-mode approximation [Eqs. (7)–(17)] in nonparabolic waveguides with possible leakage of radiation. In the presence of leakage, the limitations of the method described here are particularly strong. However, we have found many situations when, in spite of the substantial non-Gaussian distortions of the field profile, the dynamics of the beam variables, derived from the results of the direct numerical solution, is very close to that given by the approximate model (7)–(17) of the present paper. In particular, this is true in the vicinity of stationary points. Thus, we have strong enough evidence to conclude that in moderate-nonlinear media, the generalized Gaussian approximation can be used to describe the global beam behavior.

In Kerr-nonlinear media, the beams transversely limited in two dimensions can collapse when the beam power exceeds the threshold value [17]. In all the examples considered above, the beam power was taken to be lower than the self-focusing threshold. We performed special calculations to observe the starting phase of the collapse at higher beam powers. Small-scale self-focusing, which is known to cause transverse beam instability in Kerr media [18,19], could not be considered here since the original equations do not take the fluctuations into account.

Thus, we have generalized the previously proposed approach of Refs. [8], [11], and [12] to derive a finite-dimensional dynamical model of misaligned beam propagation. The approximate reduction of the wave boundary problem to a Cauchy problem for a finite set of ordinary differential equations, made it possible to prove some general dynamical properties and to study important regimes analytically. Simple numerical calculations allowed us to trace the long-term evolution of the beam variables and to reveal nontrivial effects, such as phase locking, cycle generation, etc.

From the dynamical point of view, this study may be considered as an application to an optical system of the ideas developed earlier for multifrequency interaction in statistical mechanics [20] and in the dynamics of coupled nonlinear oscillators [21,22]. Among other closely related problems, we would like to mention the evolution of coherent states in quantum optics [26–30], since the coordinate representation of a coherent state is nothing but a Gaussian with variable parameters. These parameters also satisfy nonlinear evolution equations, notwithstanding that the fundamental equations of quantum mechanics are linear.

It seems interesting to extend this study over a wider class of waveguide systems including the media with different linear refraction profiles and nonlinear susceptibilities. In particular, we plan similar investigations of beam dynamics in dissipative media.

ACKNOWLEDGMENTS

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