Stationary periodic and solitary waves induced by a strong short laser pulse

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The propagation of a relativistically intense short laser pulse into an isotropic plasma is described. A kinetic equation for the spectral function of the electromagnetic waves is derived for an arbitrary amplitude pump wave, where the fully relativistic case is considered. The resulting kinetic equation of the spectral function is used along with the set of equations of the plasma to derive a general dispersion relation, where relativistic effects play an important role. In the case of a superstrong short laser pulse, Langmuir waves, with phase velocities larger than the speed of light, and waves of ion-sound type, which are damped only on ions, are found. In addition, for the case when the plasma density along with the mass of the electrons satisfies the ''frozen-in'' condition, stationary nonlinear new type of ion-sound waves are investigated. The mechanism of the emission of these waves is also discussed. [S1063-651X(99)01112-5]

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I. INTRODUCTION

The development of ultraintense short pulse lasers allows exploration of fundamentally new parameter regimes for nonlinear laser-plasma interaction. In fact, a number of experiments have been carried out in which plasmas are irradiated by laser beams with intensities up to 10^{19} W/cm^2. At such intensities the electron quiver velocity rapidly approaches the speed of light, and a host of phenomena have been predicted such as the parametric resonance in an electron plasma [1], the relativistic wave breaking [2], the formation of types of solitons [3], the relativistic self-focusing [4,5], the generation of ''light wind'' [6], the formation of collisionless shock waves [7], the relativistic modulational and filamentational instabilities [8], and the generation of large amplitude plasma waves (wake fields) [6,9]. Numerous works [10–15] have been devoted to the investigation of relativistically intense EM wave propagation into plasma, with the radiation pressure being larger than the plasma pressure. The above treatments were restricted to the case of monochromatic EM waves. For ultrashort pulses the bandwidth of coherent wave is increasingly broad. Even if the bandwidth may be initially narrow, its spectrum may eventually broaden, either as a result of several kinds of instability processes, or as the result of other nonlinear wave-wave interaction processes. In order to study the interaction of spectrally broad relativistically intense EM waves with a plasma, we adopt the EM spectral intensity [16]. This picture of high-frequency EM processes in a plasma opens a way to the formulation of conceptually new problems in plasma electrodynamics.

In the present paper, we consider a class of problems involving the interaction of relativistically intense nonmonochromatic radiation bunches with a nonmagnetized plasma. The paper is organized as follows. First in Sec. II, starting from Maxwell's equations for the EM field in a relativistic plasma, we derive a general equation for the EM spectral intensity. Then in Sec. III we derive the plasma wave dispersion relation in the presence of the relativistic ponderomotive force and discuss a type of longitudinal plasma waves induced by a strong short pulse laser. In the same section it is shown that the ratio of the plasma density to the mass of the electrons is conserved, or there is a ''frozen-in'' condition in the case of stationary waves. The stationary nonlinear ion-sound waves are discussed in Sec. IV and the velocity of the waves and the maximum potential of the field are defined. Finally, a brief summary and discussion of our results are given in the last section.

II. DERIVATION OF THE KINETIC EQUATION FOR THE PHOTON GAS

We start from Maxwell equations for momentum for a circularly polarized EM wave

$$\nabla^2 p - \frac{\partial^2 p}{\partial t^2} = \frac{n}{\gamma} p, \tag{1}$$

where the following dimensionless quantities have been introduced:

$$p \to \frac{p}{m_0 c}, \quad t \to \omega_{Le} t, \quad \mathbf{r} \to k_p \mathbf{r}, \quad k_p = \frac{\omega_{Le}}{c},$$

$$n \to \frac{n}{n_0}, \quad \gamma = (1+p^2)^{1/2},$$

where ω_{Le} is the electron plasma frequency, associated in the usual way with the mean plasma density n_0, and m_0 is the electron rest mass.

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We shall consider Eq. (1) at two distinct points and instants of time. Following the procedure described in Ref. [16], we can derive an equation for the correlation function $\langle p(\mathbf{r}_1,t_1)p(\mathbf{r}_2,t_2)\rangle=\Pi(\mathbf{r}_1,t_1,\mathbf{r}_2,t_2)$, where $\langle\cdots\rangle$ denotes ensemble averaging

$$(\nabla_1^2-\nabla_2^2)\Pi(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2)-\left(\frac{\partial^2}{\partial t_1^2}-\frac{\partial^2}{\partial t_2^2}\right)\Pi(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2)=(\rho_1-\rho_2)\Pi(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2), \tag{2}$$

where $\rho=n(\mathbf{r},t)/\gamma(\mathbf{r},t)$.

Introducing new variables

$$\mathbf{R}=\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2),\quad \mathbf{r}=\mathbf{r}_1-\mathbf{r}_2,\quad t=\frac{1}{2}(t_1+t_2),\quad \tau=t_1-t_2, \tag{3}$$

Eq. (2) yields

$$\left(\nabla_{\mathbf{R}}\nabla_{\mathbf{r}}-\frac{\partial^2}{\partial t\,\partial\tau}\right)\Pi(\mathbf{R},\mathbf{r},t,\tau)=\frac{1}{2}(\rho_1-\rho_2)\Pi(\mathbf{R},\mathbf{r},t,\tau), \tag{4}$$

where

$$\rho_1-\rho_2=\rho_1\left(\mathbf{R}+\frac{\mathbf{r}}{2},t+\frac{\tau}{2}\right)-\rho_2\left(\mathbf{R}-\frac{\mathbf{r}}{2},t-\frac{\tau}{2}\right).$$

Performing a Fourier transformation of $\Pi(\mathbf{R},\mathbf{r},t,\tau)$ on the variables $(\mathbf{r},\tau)$ we can introduce the power spectral function $\mathbf{P}(\mathbf{R},t,\mathbf{k},\omega)$ or Wigner representation

$$\mathbf{P}(\mathbf{R},t,\mathbf{k},\omega)=\int d\mathbf{r}\int d\tau\,\Pi(\mathbf{R},t,\mathbf{r},\tau)\exp i(\mathbf{kr}-\omega\tau). \tag{5}$$

We can also write for the momentum autocorrelation function

$$\Pi(\mathbf{R},t)=\langle p^2(\mathbf{R},t)\rangle=\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\omega}{2\pi}\mathbf{P}(\mathbf{R},t,\mathbf{k},\omega). \tag{6}$$

Taking the double Fourier transformation of Eq. (4), we obtain an evolution equation for the power spectral function, in the form

$$\left(\omega\frac{\partial}{\partial t}+\mathbf{k}\cdot\nabla_R\right)\mathbf{P}(\mathbf{R},t,\mathbf{k},\omega)=\frac{1}{2}\int d\mathbf{r}\int d\tau(\rho_1-\rho_2)\Pi(\mathbf{R},t,\mathbf{r},\tau)\exp i(\mathbf{kr}-\omega\tau). \tag{7}$$

Now expanding $(\rho_1-\rho_2)$ in Taylor series we can write

$$\rho_1-\rho_2=\sum_{m=0}^{\infty}\frac{1}{m!}\left(\frac{1}{2}\mathbf{r}\cdot\boldsymbol{\nabla}_R+\frac{1}{2}\tau\frac{\partial}{\partial t}\right)^m\rho[1-(-1)^m]. \tag{8}$$

We can see from the above expansion that only the odd terms in m are nonzero. So, we can choose $m=2l+1$. Finally, we obtain after integration the following equation for the spectral function

$$\left(\omega\frac{\partial}{\partial t}+\mathbf{k}\cdot\nabla_R\right)\mathbf{P}(\mathbf{R},t,\mathbf{k},\omega)=\sum_{l=0}^{\infty}\frac{(-1)^l}{(2l+1)!}\left(\frac{1}{2}\frac{\partial^{2l+1}\rho(\mathbf{R},t)}{\partial\mathbf{R}^{2l+1}}\frac{\partial^{2l+1}\mathbf{P}(\mathbf{R},t,\mathbf{k},\omega)}{\partial\mathbf{k}^{2l+1}}-\frac{1}{2}\frac{\partial^{2l+1}\rho(\mathbf{R},t)}{\partial t^{2l+1}}\frac{\partial^{2l+1}\mathbf{P}(\mathbf{R},t,\mathbf{k},\omega)}{\partial\omega^{2l+1}}\right). \tag{9}$$

III. LINEAR LONGITUDINAL PLASMA WAVES. FROZEN-IN CONDITION

We now consider the propagation of small perturbations in such a plasma. To this end, we linearize Eq. (9) with respect to the perturbations, which are represented as

$$\rho=\rho_0+\delta\rho\exp i(\mathbf{q}\cdot\mathbf{R}-\Omega t),$$
$$\mathbf{P}(\mathbf{R},t,\mathbf{k},\omega)=\mathbf{P}_0(\mathbf{k},\omega)+\delta\mathbf{P}\exp i(\mathbf{q}\cdot\mathbf{R}-\Omega t). \tag{10}$$

The result is

$$(\mathbf{q}\cdot\mathbf{k}-\Omega\omega)\delta\mathbf{P}=\delta\rho\sum_{l=0}^{\infty}\frac{1}{(2l+1)!}\frac{1}{2^{2l+1}}\times\left(\mathbf{q}\boldsymbol{\nabla}_k+\Omega\frac{\partial}{\partial\omega}\right)^{2l+1}\mathbf{P}_0(\mathbf{k},\omega) \tag{11}$$

or after summation we obtain the following relation:

$$(\mathbf{qk}-\Omega\omega)\delta\mathbf{P}=\delta\rho\left\{\mathbf{P}_0^+\left(\mathbf{k}+\frac{\mathbf{q}}{2},\omega+\frac{\Omega}{2}\right)-\mathbf{P}_0^-\left(\mathbf{k}-\frac{\mathbf{q}}{2},\omega-\frac{\Omega}{2}\right)\right\}. \tag{12}$$

Then from Eq. (6) we have for the perturbation of Π,

$$\delta\Pi=\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\omega}{2\pi}\frac{\mathbf{P}_0^+-\mathbf{P}_0^-}{\mathbf{q}\cdot\mathbf{k}-\Omega\omega}\delta\rho \tag{13}$$

and for $\delta\rho$ we can write

$$\delta\rho=\frac{\delta n}{\gamma_0}-\frac{1}{2\gamma_0^3}\delta\Pi. \tag{14}$$

From Eqs. (13) and (14) follows the relation between $\delta\Pi$ and δn

$$\left\{1+\frac{1}{2\gamma_0^3}\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\omega}{2\pi}\frac{\mathbf{P}_0^+-\mathbf{P}_0^-}{\mathbf{q}\cdot\mathbf{k}-\Omega\omega}\right\}\delta\Pi=\frac{\delta n}{\gamma_0}\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\omega}{2\pi}\frac{\mathbf{P}_0^+-\mathbf{P}_0^-}{\mathbf{q}\cdot\mathbf{k}-\Omega\omega}. \tag{15}$$

In the absence of the density perturbation δn we get from Eq. (15) the dispersion relation due to relativistic self-modulation

$$1+\frac{1}{2\gamma_0^3}\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\omega}{2\pi}\frac{\mathbf{P}_0^+-\mathbf{P}_0^-}{\mathbf{qk}-\Omega\omega}=\mathbf{0}. \tag{16}$$

Equation (16), as well as the case with $\delta n\neq 0$, has been studied in Ref. [8] for monochromatic waves.

We now define the relativistic expression for the ponderomotive force

$$\mathbf{F}=-\nabla\gamma=-\nabla[1+\Pi(\mathbf{R},t)]^{1/2}. \tag{17}$$

After linearization of this equation with respect to the perturbation we have

$$\mathbf{F}=-\frac{1}{2\gamma_0}\nabla\,\delta\Pi\,\exp i(\mathbf{q}\cdot\mathbf{R}-\Omega t), \tag{18}$$

or using Eq. (15) we obtain

$$\mathbf{F}=-\frac{1}{2\gamma_0^2}\frac{\int d\mathbf{k}/(2\pi)^3\int(d\omega/2\pi)(\mathbf{P}_0^+-\mathbf{P}_0^-)/(\mathbf{qk}-\Omega\omega)}{1+(1/2\gamma_0^3)\int d\mathbf{k}/(2\pi)^3\int(d\omega/2\pi)(\mathbf{P}_0^+-\mathbf{P}_0^-)/(\mathbf{qk}-\Omega\omega)}\cdot\nabla\,\delta n\,\exp i(\mathbf{q}\cdot\mathbf{R}-\Omega t). \tag{19}$$

Some interesting relativistic features follow from the expression of the ponderomotive force (19). First, in the case when the dominator goes to zero $\mathbf{F}$ increases, or $\delta n\to 0$. Second, when the integral in the dominator becomes much greater than unity, we have

$$\mathbf{F}=-\gamma_0\nabla\,\delta n\,\exp i(\mathbf{q}\cdot\mathbf{R}-\Omega t). \tag{20}$$

This expression of the ponderomotive force coincides formally with the gasdynamic force, only instead of the temperature we have $m_0\gamma_0c^2$ in Eq. (20), and it exists only for the relativistic motion of the electrons in a superstrong short pulse laser.

Now, if we write kinetic equations for electrons and ions with the ponderomotive force (17) and linearize them, taking into account the relation (15), we obtain the general dispersion relation in dimensional form. The result is

$$\varepsilon\left(1+\frac{\omega_{pe}^2}{2\gamma_0^2}\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\omega}{2\pi}\frac{\mathbf{P}_0^+-\mathbf{P}_0^-}{\mathbf{qk}c^2-\Omega\omega}\right)+(1+\delta\varepsilon_i)\,\delta\varepsilon_e q^2c^2\frac{1}{2\gamma_0^2}\times\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\omega}{2\pi}\frac{\mathbf{P}_0^+-\mathbf{P}_0^-}{\mathbf{q}\cdot\mathbf{k}c^2-\Omega\omega}=0, \tag{21}$$

where

$$\varepsilon=1+\delta\varepsilon_e+\delta\varepsilon_i,\qquad \delta\varepsilon_\alpha=\frac{4\pi e^2}{q^2}\int\frac{(\mathbf{q}\partial f_{0\alpha}/\partial\mathbf{p})}{\Omega-\mathbf{q}\cdot\mathbf{v}}d\mathbf{p},$$

$$\omega_{pe}^2=\frac{\omega_{Le}^2}{\gamma_0}.$$

Equation (21) has several complex solutions for Ω, resulting in different types of instability. But here we focus our attention on the case of the propagation of a stationary longitudinal wave in a plasma due to a strong laser pulse. Such a possibility exists, if the condition

$$\frac{\omega_{pe}^2}{2\gamma_0^2}\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\omega}{2\pi}\frac{\mathbf{P}_0^+-\mathbf{P}_0^-}{\mathbf{qk}c^2-\Omega\omega}\gg 1 \tag{22}$$

is satisfied. In this case the dispersion relation (21) is reduced to the form

$$(1+\delta\varepsilon_i)\left(1+\delta\varepsilon_e\frac{q^2c^2}{\omega_{pe}^2}\right)+\delta\varepsilon_e=0. \tag{23}$$

This dispersion relation describes the propagation of a stationary longitudinal wave in the presence of relativistically intense EM waves.

Let us now consider some special limits. First, in the case when only electrons participate in the oscillation, i.e., $\delta\varepsilon_i=0$, for $\Omega\gg qv_{tre}$, where $v_{tre}=(T_e/m_0)^{1/2}$, we obtain from Eq. (23)

$$\Omega^2=\omega_{pe}^2+q^2c^2. \tag{24}$$

This is a Langmuir wave due to strong relativistic effects. The physical interpretation for Eq. (24) is that the strong ponderomotive force not only leads to the separation of charge and creation of the longitudinal self-consistent field, but also generates the dispersion term q^2c^2, which is due to the strong coupling of EM waves with the electrons.

Next in the case, when $\delta n_i\neq 0$, two frequency ranges can be considered for Ω. One is $kv_{tri}\ll\Omega\ll kv_{tre}$, and the other is $kv_{tre}\ll\Omega\ll\omega_{pe}$. For both cases we obtain from Eq. (23) a type of ion-sound solution

$$\Omega=\left(\frac{m_0\gamma_0}{m_i}\right)^{1/2}\frac{qc}{(1+q^2c^2/\omega_{pe}^2)^{1/2}}=\frac{qc_s}{(1+q^2c^2/\omega_{pe}^2)^{1/2}}. \tag{25}$$

It is clear that now the characteristic length of the inhomogeneity is comparable to c/ω_{pe}, but not to the electron Debye length as we have for the ion-sound wave without a laser pulse. As follows from Eq. (25) the maximum value of the frequency is ω_{pi}, similar to the result obtained in Ref. [17]. We specifically note here that $c_s = c(m_0\gamma_0/m_i)^{1/2}$ now depends not only on the mass of the particles, but also on the intensity of the laser pulse $[\gamma_0=(1+\Pi_0)^{1/2}]$. Therefore, these waves in an experiment exhibit frequencies depend on the intensity of the laser pulse and the sort of gas. The relativistic modes in particle-in-cell simulations were reported in Ref. [18].

We now try to physically understand existence of the solutions (24) and (25). First note that for the stationary case when the laser pulse propagates with a constant velocity $\mathbf{v} = (\mathbf{k}c^2/\omega)[\mathbf{P}(\mathbf{R},t,\mathbf{k},\omega)=\mathbf{P}(\mathbf{k},\omega,R-vt)]$, the result (20) for the ponderomotive force can be obtained without the linearization of Eq. (9). In this case, the left-hand side of Eq. (9) becomes zero and one of the solutions of Eq. (9) is

$$\rho = \frac{n(\mathbf{R},t)}{\gamma(\mathbf{R},t)} = \text{const}, \tag{26}$$

or equivalently

$$\frac{n}{m_e(\gamma)} = \text{const},$$

which shows that the plasma density and mass of the electrons satisfy a ''frozen-in'' condition. This condition implies a localization of the energy of the laser pulse in the region of high plasma density. The solution identical with Eq. (26) was shown in Ref. [19], considering the strong EM wave propagation in an electron-positron plasma. In the case when expression (26) is valid, we obtain a simple expression for the ponderomotive force from Eq. (17) for arbitrary variation of the density

$$\mathbf{F} = -m_0\gamma_0 c^2 \boldsymbol{\nabla}\frac{n}{n_0}. \tag{27}$$

One can simply show that if the ''frozen-in'' condition (26) is fulfilled, the hydrodynamic equations [16], the equation of motion, and the equation of continuity for electrons become linear and for an arbitrary variation of the electron density we have the following linear equation:

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{pe}^2 - c^2\boldsymbol{\nabla}^2\right)\frac{n-n_0}{n_0} = 0. \tag{28}$$

This equation shows that for plane waves one can obtain the same dispersion relation as Eq. (24). It is important to emphasize that Eq. (1) with condition (26) becomes a linear equation and EM wave momentum with arbitrary power will spread out in a plasma.

IV. STATIONARY PERIODIC AND SOLITARY WAVES

In this section, we consider the propagation of stationary nonlinear ion-sound waves, when the phase velocity of a type of ion-sound waves is large compared with the electron thermal velocity and the plasma density along with the mass of the electrons satisfies the ''frozen-in'' condition (26). To describe the one-dimensional motion of the electrons and ions of such waves, we employ hydrodynamic equations with a self-consistent field:

$$\frac{\partial p_e}{\partial t} = e\frac{\partial\phi}{\partial x} - m_0 c^2\gamma_0 \frac{\partial}{\partial x}\frac{n}{n_0}, \tag{29}$$

$$m_i\left(\frac{\partial}{\partial t} + u_i\frac{\partial}{\partial x}\right)u_i = -e\frac{\partial\phi}{\partial x}, \tag{30}$$

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}nu_e = 0, \tag{31}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}n_i u_i = 0. \tag{32}$$

Here u_i, n_i are the ion velocity and density, respectively, and ϕ is the electrostatic potential which is coupled with the electron and ion densities through the Poisson equation

$$\frac{\partial^2\phi}{\partial x^2} = 4\pi e(n-n_i). \tag{33}$$

Equations (29)–(33) are a closed set of equations describing the propagation of one-dimensional waves including solitary waves we are interested. In this case we can let all quantities depend on coordinates and time as $x-vt$, where v is constant. From Eqs. (29)–(32) the following expressions for electron and ion densities are obtained:

$$\frac{n}{n_0} = 1 + \frac{e\phi}{m_0\gamma_0 c^2}, \tag{34}$$

$$\frac{n_i}{n_0} = \left(1 - \frac{2e\phi}{m_i v^2}\right)^{-1/2}. \tag{35}$$

Substituting these expressions for the densities into the Poisson equation, we get

$$\frac{\partial^2\phi}{\partial x^2} = 4\pi e n_0\left[1 + \frac{e\phi}{m_0\gamma_0 c^2} - \left(1 - \frac{2e\phi}{m_i v^2}\right)^{-1/2}\right]. \tag{36}$$

Now we first consider the case when $e\phi \ll m_i v^2/2$, i.e., stationary waves with weak nonlinearity. In this case the last term in Eq. (36) can be expanded in a power series and we obtain

$$\frac{\partial^2\phi}{\partial x^2} = \frac{\omega_{pi}^2}{v^2}\left(\frac{v^2}{c_s^2} - 1\right)\phi - \frac{3}{2}\frac{\omega_{pi}^2}{v^2}\frac{e\phi^2}{m_i v^2}. \tag{37}$$

If we neglect the last term in Eq. (37), there are two possibilities in the linear approximation. The first is the propagation of ion-sound waves with the velocity given in Eq. (25), when $m_i v^2 < m_0\gamma_0 c^2$. In the second case when the opposite inequality is valid, we obtain a type of the Debye potential with the characteristic scale length

$$r_D=\frac{c}{\omega_{pe}(1-c_s^2/v^2)^{1/2}}. \qquad (38)$$

This expression shows that the effect of the Coulomb field extends to a distance of the order of r_D, which plays a role of the Debye screening distance.

Now let us consider the structure of a solitary wave, for this it is necessary that $m_i v^2 > m_0\gamma_0 c^2$. Then the solution of Eq. (37) is

$$e\phi=\frac{m_i v^2(v^2/c_s^2-1)}{ch^2(\omega_{pi}/v)(v^2/c_s^2-1)^{1/2}(x-vt)} \qquad (39)$$

and

$$\frac{n}{n_0}=1+\frac{v^2}{c_s^2}\left(\frac{v^2}{c_s^2}-1\right)\frac{1}{ch^2(\omega_{pi}/v)(v^2/c_s^2-1)^{1/2}(x-vt)}. \qquad (40)$$

Since we have supposed that $e\phi \ll m_i v^2/2$, from Eq. (39) it is clear that $|v^2/c_s^2-1|\ll 1$. The relation between the propagation velocity v and the maximum amplitude $\phi_{\max}$ of the wave, can be obtained from Eq. (39),

$$v^2=c_s^2+\frac{e\phi_{\max}}{m_i} \qquad (41)$$

and now for ϕ we have

$$\phi=\frac{\phi_{\max}}{ch^2(\omega_{pi}/c_s^2)(e\phi_{\max}/m_i)^{1/2}(x-vt)}. \qquad (42)$$

We see that $n>n_0$ and $n_i>n_0$, since $\phi>0$. A solitary wave in a quasiequilibrium plasma is, therefore, always a compressional wave.

Turning now to the study of Eq. (36), we integrate it once to obtain

$$E^2(\phi)=\left(\frac{\partial\phi}{\partial x}\right)^2=4\pi e n_0\left\{2\phi+\frac{e\phi^2}{m_0\gamma_0 c^2}+\frac{2}{e}m_i v^2\left(1-\frac{2e\phi}{m_i v^2}\right)^{1/2}\right\}+A. \qquad (43)$$

Various periodic waves can now be found depending on the choice of the integration constant A. In the case when ϕ and $\partial\phi/\partial x\to 0$ at $|x-vt|\to\infty$, we have $A=-8\pi n_0 m_i v^2$. This case corresponds to a solitary wave. We find the equation which determines the potential ϕ as a function of the coordinates and time

$$x-vt=\pm\int\frac{d\phi}{[E^2(\phi)]^{1/2}}. \qquad (44)$$

The velocity of propagation of this wave v, as a function of the maximum amplitude of the wave $\phi_{\max}$, is found from Eq. (43) by writing $\partial\phi/\partial x=0$ at $\phi=\phi_{\max}$, i.e.,

$$2e\phi_{\max}+\frac{e^2\phi_{\max}^2}{m_0\gamma_0 c^2}+2m_i v^2\left\{\left(1-\frac{2e\phi_{\max}}{m_i v^2}\right)^{1/2}-1\right\}=0, \qquad (45)$$

and from here we obtain

$$v^2=c_s^2\left(1+\frac{e\phi_{\max}}{2m_0\gamma_0 c^2}\right)^2. \qquad (46)$$

We note here that Eq. (45) has a solution only when $\phi_{\max}$ is not too large. From Eq. (45) it follows that the maximum possible value of the amplitude of the ion-sound wave can be determined from the relation $m_i v^2/2=e\phi_{\max}$, because ions can no longer move across the potential barrier. Solving this equation together with Eq. (46) we obtain $e\phi_{\max}=2m_0\gamma_0 c^2$ and for the velocity of the stationary ion-sound solitary wave $v=2c_s$.

V. SUMMARY AND DISCUSSION

We have investigated the propagation of a relativistically intense short laser pulse into an unmagnetized plasma. Starting from the fully relativistic equations, we have derived a general kinetic equation for the photon gas. This is valid for waves with a large spectral width. The relativistic expression for the ponderomotive force is also derived and some interesting relativistic features are discussed. The kinetic equation was used to derive the plasma wave dispersion relation and the propagation of stationary longitudinal waves in the presence of relativistically intense EM waves is studied. Due to strong relativistic effects a novel Langmuir wave with phase velocities larger than the speed of light and waves of the ion-sound type, which are damped only on ions, are found. In addition, for the case when the plasma density along with the mass of the electrons satisfies the ‘‘frozen-in’’ condition, stationary periodic and solitary waves are studied. The relation between the wave amplitude and its propagation velocity is derived. The possible mechanism of the emission of a new type of ion-sound waves may be attributed to the laser pulse acceleration due to the plasma inhomogeneity [20]. In this case one can find an explicit form of the density distribution in emission of ion sound, as has been shown in Ref. [20]. These dynamics of the plasma under the relativistically intense electromagnetic waves may be relevant in the study of the contemporary problems of laser-matter interaction such as the fast ignitor concept [21]. In this scheme, short pulse laser energy deposition efficiency and spacetime characteristics are essentially important, since the deposited laser energy has to be efficiently transferred to dense plasmas. In this context, it is necessary to understand how a relativistically intense short pulse laser propagates into overdense plasmas through long scale underdense plasma. As investigated in this work, in the case of a superstrong short pulse laser a type of ion-sound waves can be generated. Which can be of great importance for the heating of plasma. This wave is intensity dependent, therefore it can be observed in experiments and possibly be used as a diagnostic of ultraintense short pulse laser propagation. Langmuir waves with phase velocities larger than the phase velocity of the laser pulse can also exist due to strong relativistic effects, as is found in this paper. We specifically note that these waves are potential candidates for ultrahigh gradient electron acceleration. The development and study of feasibility of the laser-induced electron acceleration phenomenon based on the above disclosed Langmuir waves will be discussed in a separate paper. Finally, the theory developed in this paper should also be the case for relativistic astrophysical objects such as pulser atmosphere.

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