Equilibrium and oscillations of grains in the dust-plasma crystal

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The dispersion characteristics of the low-frequency mode associated with vertical oscillations of dust grains in a quasi-two-dimensional dust-plasma crystal are calculated, taking into account the dependence of the dust charge on the local sheath potential. It is shown that the equilibrium of the dust grains close to the electrode may be disrupted by large amplitude vertical oscillations. [S1063-651X(99)05111-9]

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I. INTRODUCTION

The formation and properties of strongly coupled dust particle structures (the ''dust-plasma crystals'') observed in plasmas in low-temperature discharges have recently been subjects of increasing interest [1]. The negatively charged dust grains usually levitate in the sheath region near the horizontal negatively biased electrode where there is a balance of gravitational and electrostatic forces on the grains. In this region, the arrangement of the dust particles is also influenced by the strong ion flow [2]. Recently, it was demonstrated that vertical vibrations of dust particles may cause the propagation of specific modes with optical-like dispersion [3]. Observations of vertical motions of the dust are important for diagnostics of processes in the plasma sheath [4], especially in the case of several vertically arranged horizontal layers when vertical oscillations are affected by the parameters of the ion flow [5]. Note that the spontaneous excitation of vertical vibrations of dust grains was recently experimentally observed [6], and driven vertical oscillations were studied in a separate series of experiments [7]. On the other hand, molecular dynamic simulations [8] clearly demonstrate a sequence of phase transitions associated with vertical arrangements of horizontal chains when the strength of the confining (in the vertical dimension) parabolic potential is changed. The vertical rearrangements of the dust grains are directly connected with the possible equilibria of the system.

Previous analytical models considering lattice vibrations [3,5,9], as well as numerical models studying phase transitions [8] in the dust-plasma system, dealt with dust grains of a constant charge. However, it is well known that the charge of dust particles, appearing as a result of electron and ion current onto the grain surfaces, strongly depends on the parameters of the surrounding plasma — see, e.g., Refs. [10,11]. In this paper, we demonstrate that the dependence of the dust particle charge on the sheath parameters has an important effect on the oscillations and equlibrium of dust grains in the vertical plane, leading to a disruption of the equilibrium position of the particle and a corresponding transition to a different vertical arrangement.

II. MODEL EQUATIONS

Consider vertical vibrations of a one-dimensional horizontal chain of particulates of equal mass m_d separated by the distance r_0 in the horizontal direction, see Fig. 1 and Ref. [3] for details.

The charge Q of the dust particles (which is dependent on the sheath parameters, in particular, on the local electric sheath potential) can be found from the condition of a zero total plasma current onto the grain surface:

$$I(Q)=I_e(Q)+I_i(Q)=0. \tag{1}$$

Note that since we are interested in collective processes on the time scale of the characteristic frequencies (of order a few times 10 s^{-1}), which are much lower than the charging frequency [11] (which can be of order 10^5 s^{-1}), we assume that (re)charging of dust grains is practically instantaneous, and we therefore neglect the charging dynamics. The electron and ion currents onto the dust grain are given by

$$I_\alpha(Q)=\sum_\alpha \int e_\alpha f_\alpha \sigma_\alpha(v,Q) v\, d\mathbf{v}. \tag{2}$$

Here, the subscript $\alpha=e,i$ stands for electrons or ions, e_α and f_α are the charge and distribution function of the plasma particles, with $e_e=-e_i\equiv-e$, $v\equiv|\mathbf{v}|$ is the absolute value of the particle speed $\mathbf{v}$, and σ_α is the charging cross section [12]:

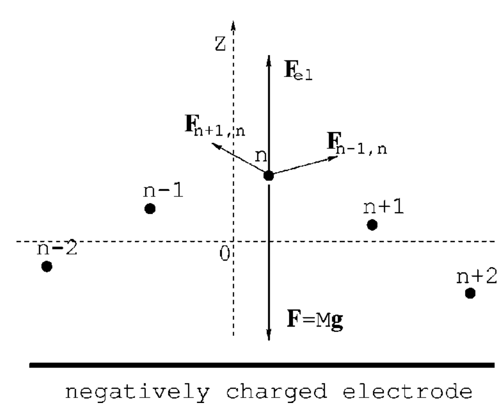

FIG. 1. The considered arrangement and oscillations of dust particles in the vertical z direction.

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$$\sigma_\alpha=\pi a^2\left(1-\frac{2e_\alpha Q}{am_\alpha v^2}\right) \quad \text{if} \quad \frac{2e_\alpha Q}{am_\alpha v^2}<1,$$

$$\sigma_\alpha=0 \quad \text{if} \quad \frac{2e_\alpha Q}{am_\alpha v^2}\geqslant 1, \tag{3}$$

where a is the radius of the dust particle and m_α is the electron or ion mass. The last inequality in Eq. (3) gives a restriction on the electron charging velocities for the negatively charged dust particles that are assumed here.

The electrons are assumed to be Boltzmann distributed, and we have the electron current (for a negative charge on the dust)

$$I_e(Q)=-\sqrt{\pi/2}ea^2n_0\sqrt{\frac{T_e}{m_e}}\exp\left(\frac{Qe}{aT_e}+\frac{e\varphi}{T_e}\right), \tag{4}$$

where $\varphi=\varphi(z)$ is the external potential in the sheath relative to the potential in the bulk plasma ($z\to\infty$), and T_e and n_0 are the electron temperature (in energy units) and electron density in the bulk plasma. We neglect possible changes of the electron temperature in the plasma sheath.

We are assuming the discharge pressure to be low enough that ion collisions with neutrals and other species can be neglected (this corresponds to experiments in a low-pressure discharge where spontaneous excitation of vertical vibrations was observed [6]). Thus in contrast to the electron distribution, we consider collisionless, ballistic ions within the sheath with the distribution function $f_i\propto\delta(\mathbf{v}_\perp)\delta(v_z-v_i(z))$, where $v_i(z)$ is the ion streaming velocity at the distance z from the electrode. The intergrain distance is assumed not less than the Debye length, so that the ion trajectory is affected by only a single grain. Thus the ion current onto the dust grain is given by

$$I_i=\pi a^2 en_i(z)v_i(z)\left(1-\frac{2eQ(z)}{am_iv_i^2(z)}\right). \tag{5}$$

Here, the ion velocity can be determined from the energy balance equation, and the ion density follows from the continuity equation assuming the ion losses to dust to be negligible:

$$v_i(z)=v_0\left(1-\frac{2e\varphi(z)}{m_iv_0^2}\right)^{1/2}, \quad n_i=n_{0i}v_0/v_i(z), \tag{6}$$

where v_0 and $n_{0i}=n_0$ are the ion velocity and density far from the sheath ($z\to\infty$).

Thus the charge of a dust particle in the sheath region is determined by Eq. (1), i.e., by the equation

$$\sqrt{2\pi}\left(1-\frac{2eQ(z)}{am_iv_0^2[1-2e\varphi(z)/m_iv_0^2]}\right)=\frac{v_s}{v_0}\sqrt{\frac{m_i}{m_e}}\exp\left[\frac{e}{T_e}[\varphi(z)+Q(z)/a]\right], \tag{7}$$

where $v_s^2=T_e/m_i$ is the squared ion sound speed. We note that from Eq. (7) the charge can become zero for a strong enough sheath potential, such that the ion current dominates, a result obtained also in Ref. [14]. This means that the dust particle cannot levitate and must fall onto the electrode.

The sheath potential can be found from Poisson's equation [13]:

$$\frac{d^2\varphi(z)}{dz^2}=4\pi en_0\left[\exp\left(\frac{e\varphi(z)}{T_e}\right)-\left(1-\frac{2e\varphi(z)}{m_iv_0^2}\right)^{-1/2}\right], \tag{8}$$

where we neglect the total charge contributed by the dust grains (i.e., assuming the dust number density to be small), in contrast to Ref. [14], where the space charge of the dust grains is included, but the mutual interaction of the grains is neglected. The collisionless sheath structure depends on the ballistic equations (6) for the ions. The case of higher pressure, with a sheath structured by ions drifting through the background neutrals, has been considered in Ref. [15]. Eq. (8) can be integrated once to give [applying the boundary conditions $E(z=\infty)=\varphi(z=\infty)=0$]

$$\left(\frac{d\varphi(z)}{dz}\right)^2=8\pi n_0T_e\left\{\exp\left(\frac{e\varphi(z)}{T_e}\right)-1+\frac{v_0^2}{v_s^2}\left[\left(1-\frac{2e\varphi(z)}{T_e}\frac{v_s^2}{v_0^2}\right)^{1/2}-1\right]\right\}. \tag{9}$$

Assuming the electrode has a potential of -4 V, typical of dust plasma experiments [6,7], Eq. (9) can be numerically integrated to give the dependence of the potential, and thence of the sheath electric field, see Fig. 2.

The dependences of the Mach number $M\equiv v_i(z)/v_s$ of the ion flow, found from Eq. (6) and the dust grain charge, found using Eq. (7), on the distance from the electrode, for the potential and field distributions of Fig. 2, are presented in Fig. 3. If the electrode potential becomes even more negative, and a dust grain is very close to the electrode, its charge can become zero and, possibly, positive; for real conditions this only means that the particle cannot levitate at this distance.

III. EQUILIBRIUM OF THE DUST GRAIN

For a particle levitating in the sheath field, the sheath electrostatic force on the grain is given by

$$F_{\rm el}=Q(z)E(z). \tag{10}$$

Note that the force (10) includes the z dependence of the grain charge Q, since we assume instantaneous transfer of charge onto and off the dust grain at any grain position in the sheath, such that Eq. (7) is always satisfied. The balance of forces in the vertical direction also includes the gravitational force $F_g=m_dg$, so that for equilibrium of the grain

$$Q(z)E(z)=m_dg. \tag{11}$$

Solution of this equation together with the charging equation (7) gives the dependence of the charge of the grain, levitating in the sheath electric field, as a function of its size, as shown in Fig. 4. Note that there are no equilibrium solutions for a

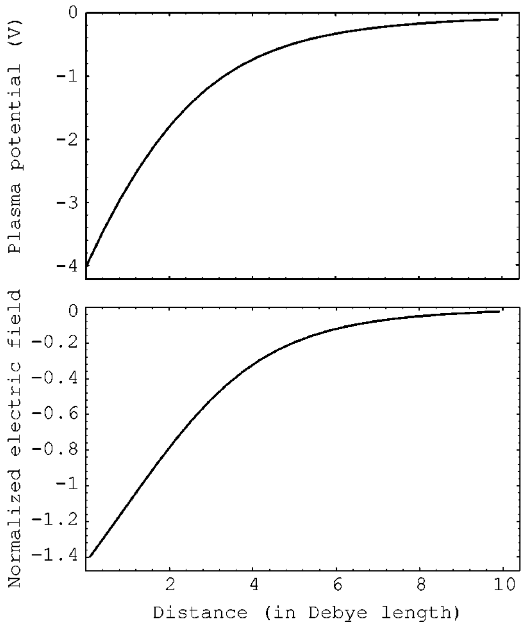

FIG. 2. The dependence of the plasma potential $\varphi(z)$ and the electric field $\mathcal{E}_z(z)=E_z(z)/\sqrt{4\pi n_0 T_e}=E_z(z)T_e/e\lambda_{D0}$ on the distance $h=z/\lambda_{D0}$ from the electrode. Here we have $\lambda_{D0}=3\times10^{-2}$ cm, $T_e=1$ eV, $M_0=v_0/v_s=1$, and $m_i/m_e=40\times10^3$.

$>a_{\max}=3.75\,\mu$m. Below, by considering dust vertical vibrations, we will see that in fact disruption of the equlibrium happens for a slightly lesser size $a_{\rm cr}<a_{\max}$. For the levitating dust particle, there is a one-to-one correspondence of its size to its equilibrium position of levitation in the sheath, as shown in Fig. 5. Note that the equilibrium solutions for positions closer than $z_{\min}=1.64\lambda_{D0}$, where the grain has a critical size $a=a_{\rm cr}$, are unstable to vertical oscillations, as shown in the following section.

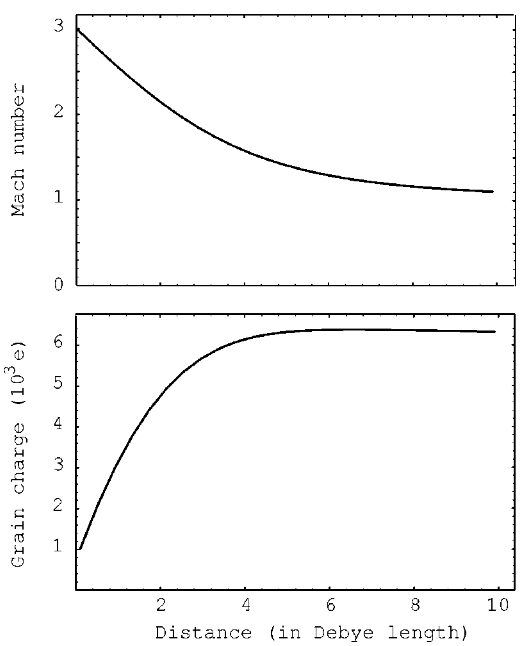

FIG. 3. The dependence of the Mach number of the ion flow $M(z)\equiv v_i(z)/v_s$ and the charge of the dust grain $q(z)=-[Q(z)/e]\times10^{-4}$ on the distance $h=z/\lambda_{D0}$ from the electrode. Here we have $M_0=v_0/v_s=1$, $\lambda_{D0}=3\times10^{-2}$ cm, $T_e=1$ eV, and $m_i/m_e=40\times10^3$, and $a=0.35\times10^{-3}$ cm.

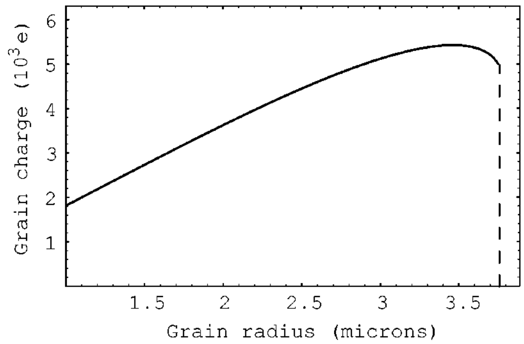

FIG. 4. The dependence of the charge $q=-(Q/e)\times10^{-3}$ of the dust grain, levitating in the sheath electric field, on its size. Here we have $M_0=v_0/v_s=1$, $\lambda_{D0}=3\times10^{-2}$ cm, $T_e=1$ eV, $m_i/m_e=40\times10^3$, $\rho=1\,\mathrm{g/cm^3}$, and $a_{\max}=0.375\times10^{-3}$ cm.

IV. VERTICAL OSCILLATIONS

Considering now the equilibrium and oscillations of a horizontal line of dust grains, we note that the parallel (to the electrode) component of the interaction force acting on the particle at the position n due to the particle at the position $n-1$ (see Fig. 1) can be written according to the Debye law as

$$\mathbf{F}_{\parallel,n,n-1}=\frac{\mathbf{R}_{\parallel}Q_n(z_n)Q_{n-1}(z_{n-1})}{|\mathbf{R}|^3}\left[1+\frac{|\mathbf{R}|}{\lambda_D(z_{n-1})}\right]\times\exp\left[-\frac{|\mathbf{R}|}{\lambda_D(z_{n-1})}\right], \quad (12)$$

where $\mathbf{R}$ is the vector from the $n-1$ position to the n position, and $\mathbf{R}_{\parallel}$ is its component parallel to the electrode, so that $|\mathbf{R}|^2=|\mathbf{R}_{\parallel}|^2+(z_n-z_{n-1})^2$, and λ_D is the Debye screening length of the dust charge Q by plasma particles (in our case $\lambda_D(z)=\lambda_{De}(z)=[T_e/4\pi n(z)e^2]^{1/2}$). Note that Q and λ_D are functions of the vertical position of the dust particles. The component of the interparticle force in the z direction is given by

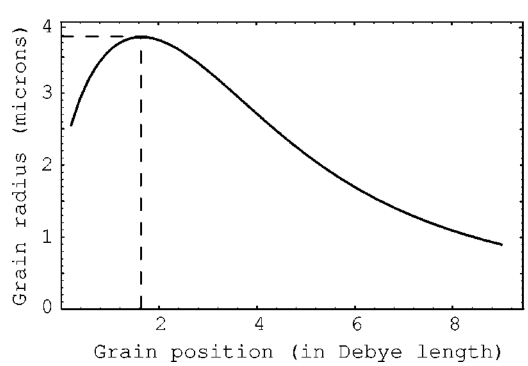

FIG. 5. The dependence of the position $h=z/\lambda_{D0}$ of the dust grain, levitating in the sheath electric field, on its size. The sheath plasma parameters are the same as in Fig. 4; the position, corresponding to $a_{\rm cr}$, is $z_{\min}=1.64\lambda_{D0}$.

$$F_{z,n,n-1}=\frac{(z_n-z_{n-1})Q_n(z_n)Q_{n-1}(z_{n-1})}{|\mathbf{R}|^3}\times\left[1+\frac{|\mathbf{R}|}{\lambda_D(z_{n-1})}\right]\exp\left[-\frac{|\mathbf{R}|}{\lambda_D(z_{n-1})}\right]. \quad (13)$$

Assuming only nearest neighbor interactions, a small perturbation δz of the equilibrium $z=z_0$, given by the force balance (11), gives the equation of motion for vertical oscillations of the grains in the linear approximation

$$m_d\frac{d^2\delta z_n}{dt^2}=-\gamma\delta z_n+\frac{Q^2(z_0)}{r_0^3}e^{-\frac{r_0}{\lambda_D(z_0)}}\times\left[1+\frac{r_0}{\lambda_D(z_0)}\right](2\,\delta z_n-\delta z_{n-1}-\delta z_{n+1}), \quad (14)$$

where r_0 is the particle separation in the horizontal plane and the coupling constant is

$$\gamma=EQ\frac{dE}{d\varphi}+E^2\frac{dQ}{d\varphi}. \quad (15)$$

Note that all the derivatives (as functions of the sheath potential φ and the particle charge Q) in Eq. (14) can be found analytically from Eqs. (7) and (9). Substituting $\delta z_n \sim \exp(-i\omega t+iknr_0)$ into Eq. (14), we obtain the expression for the frequency of the mode associated with the vertical vibrations at the position z_0:

$$\omega^2=-\frac{4Q^2(z_0)\exp[-r_0/\lambda_D(z_0)]}{m_d r_0^3}\left[1+\frac{r_0}{\lambda_D(z_0)}\right]\times\sin^2\frac{kr_0}{2}+\frac{\gamma}{m_d}. \quad (16)$$

This mode has an optical-mode-like dispersion similar to that studied earlier [3] in the case of a constant grain charge. We see that with charge variation taken into account, the characteristics of the mode are strongly affected by the sheath parameters, in particular, by the sheath potential. Note that this equilibrium is stable only when the last term on the right hand side of Eq. (16) dominates over the first one. The case when both terms are equal to each other, corresponds to the phase transition associated with the vertical rearrangement of the type $N\rightarrow N+1$, where N is the number of the one-dimensional chains in the vertical dimension. This type of transition was clearly observed in numerical experiments employing molecular dynamics simulations [8].

It also important to note that the second term on the right hand side of Eq. (15) is negative and becomes dominant for larger dust size. The function $\gamma(a)$ is presented in Fig. 6. Consider the oscillation of an isolated dust grain ($r_0 \gg \lambda_D$), so the first term on the right hand side of Eq. (16) is negligible. We see that for $a>a_{\rm cr}=3.72\ \mu$m, the coupling constant is negative and therefore no oscillations are possible. This case corresponds to an instability of the equlibrium levitation in the sheath field because the heavy (large) particle is positioned too close to the electrode where the charging by plasma electrons is insufficient because of the electron density depletion.

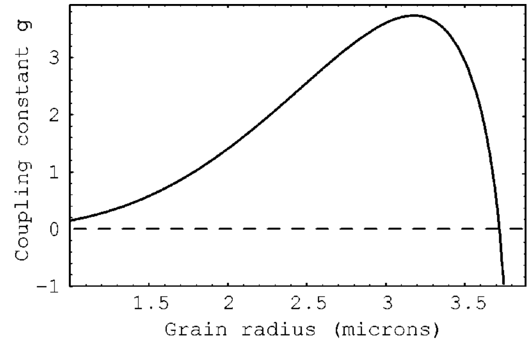

FIG. 6. The dependence of the coupling constant γ of the dust vertical vibration on the dust size. The critical radius when $\gamma=0$ is $a_{\rm cr}=0.372\times10^{-3}$ cm.

The equilibrium charge as well as sheath potential and electric field at the dust grain, and thence the derivatives in Eq. (16), can be found by solving Eqs. (7) and (11) simultaneously. For example, with the parameters $\lambda_{D0}=3\times10^{-2}$ cm, $T_e=1$ eV, $M_0=v_0/v_s=1$, $m_i/m_e=40\times10^3$, $\rho=1\,$g/cm^3, and $a=0.35\times10^{-3}$ cm, the resulting characteristic long-wavelength frequency is

$$f_{ch}(k=0)=\frac{1}{\sqrt{2\pi}}\sqrt{\frac{\gamma}{m_d}}\approx 12.2\ \ \mathrm{Hz}, \quad (17)$$

and the equilibrium charge is $q=-(Q/e)\times10^{-3}\approx5.4$.

In general, the characteristic frequency of the dust vertical vibration is a function of the dust size, as shown in Fig. 7 for an isolated grain; the frequency becomes zero for $a=a_{\rm cr}=3.72\ \mu$m. Note that in the case of a horizontal chain of interacting dust particles, with non-negligible negative first term on the right hand side of Eq. (16), the oscillation frequency can become zero (and hence the equilibrium can be disrupted) for even smaller dust sizes than $a_{\rm cr}$.

V. ENERGY OF THE GRAIN

It is also instructive to find the total potential energy, relative to the electrode position, of a single dust particle of given size at the position z in the sheath electric field:

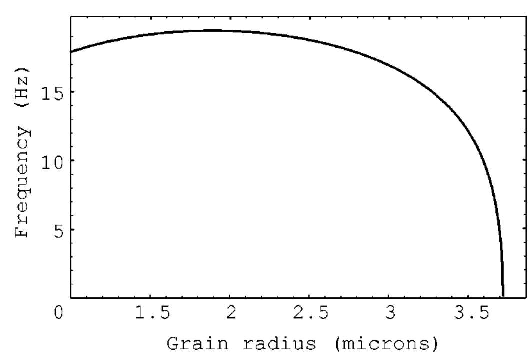

FIG. 7. The dependence of the characteristic frequency f_{ch} (in Hz) of the oscillation of dust on the size a (in microns) of the dust grain. The critical dust radius is $a_{\rm cr}=0.372\times10^{-3}$ cm.

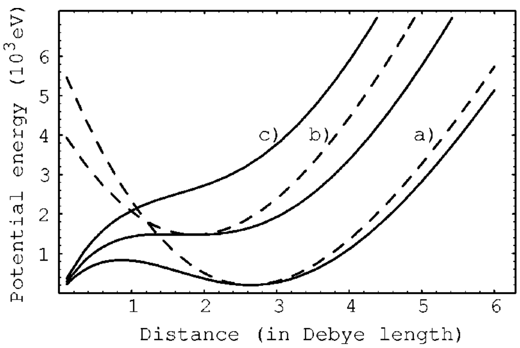

FIG. 8. The total interaction energy $U_{\rm tot}$ as a function of the distance $h=z/\lambda_D$ from the electrode for the different sizes of a dust particle: (a) $a=0.35\times10^{-3}$ cm; (b) $a=a_{\rm cr}=0.372\times10^{-3}$ cm; and (c) $a=0.4\times10^{-3}$ cm. The dashed lines correspond to the case of a charge constant at the equilibrium (or marginal equilibrium) position: (a) $Q=-5.42e\times10^3$ and (b) $Q=-4.93e\times10^3$.

$$U_{\rm tot}(z)=-\int_0^z dz'[Q(z')E(z')-m_d g]. \qquad (18)$$

Note that the total energy in this case contains not only the electrostatic energy $Q(z)\varphi(z)$, but also terms associated with $dQ/d\varphi$ which represent, because of the openness of the system, the work of external forces which change the dust charge. Solving Eqs. (7) and (9), we can find the dependence of the total potential energy on the distance from the electrode, as shown in Fig. 8. For comparison, we also plot in Fig. 8 the energy in the case of a constant Q placed at the same equilibrium (or marginal equlibrium) position. We see that the potential always has a minimum for the case of $Q=$const, but in the case of a variable charge there can be a maximum and a minimum, corresponding to the two equilibrium positions found in Sec. III. The minimum (the stable equilibrium) disappears if $a>a_{\rm cr}$ [the curve (c) in Fig. 8]. A similar result has been found for the collisional sheath case in Ref. [15]. Other effects which have been neglected here, such as an electron temperature increasing towards the electrode, may serve to increase the negative charge on a grain, and so preserve an equilibrium. The critical radius can be found by solving Eqs. (7) and (11) simultaneously with the condition $\omega(k=0)=0$. For the parameters considered here, for $a=a_{\rm cr}=0.372\times10^{-3}$ cm, the minimum disappears. This is close to the critical radius observed experimentally [6].

Thus for the collisionless sheath, for a less than the critical radius, there is an unstable equilibrium position deep inside the sheath, and a stable equilibrium position closer to the presheath, just as for the collisional sheath [15]. For a greater than the critical radius there is no equilibrium position. Vertical oscillations about the stable equilibrium, with frequencies given by Eq. (16), may develop high amplitudes (because of an instability in the background plasma [6] or a driving force [7]). This may lead to a fall of the oscillating grain onto the electrode when the potential barrier [see the curve (a) in Fig. 8] is overcome. Such a disruption of the dust motion has been observed experimentally [6,7].

VI. CONCLUSION

We have demonstrated that the charge, position, and spectrum of vertical oscillations in the one-dimensional chain of dust grains levitating in a collisionless sheath field of a horizontal negatively biased electrode strongly depend on the parameters of the sheath, in particular, the sheath potential. The dependence of the particle charge on the potential is crucial for the characteristics of the mode associated with vertical vibrations as well as for the equilibrium of the dust particles. Large amplitude vertical oscillations of the dust grains, with frequencies derived here, may be responsible for experimentally observed disruptions of the equilibrium of the dust crystal as well as with numerically demonstrated phase transitions associated with vertical rearrangements of the grains.

ACKNOWLEDGMENT

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