Two-particle problem in a nonequilibrium many-particle system

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The two-particle problem within a nonequilibrium many-particle system is investigated in the framework of real-time Green's functions. Starting from the nonequilibrium Bethe-Salpeter equation on the Keldysh contour, a Dyson equation is given for two-time two-particle Green's functions. Thereby the well-known Kadanoff-Baym equations are generalized to the case of two-particle functions. The two-time structure of the equations is achieved in an exact way using the semigroup property of the free-particle propagators. The frequently used Shindo approximation is thus avoided. It turns out that results obtained earlier are valid only in limiting cases of a nondegenerate system or a static interaction, respectively. For the case of thermodynamic equilibrium, the differences to former results obtained for the effective two-particle Hamiltonian are discussed.
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I. INTRODUCTION

This paper is devoted to the kinetic theory of many-particle systems which are able to form bound states. Such systems arise in very different fields, e.g. nuclear, semiconductor and plasma physics, respectively. To be specific, we will consider the case of two-particle bound states, e.g., hydrogenlike atoms or ions. Our focus will be the derivation of a kinetic equation for the distribution function of the (possibly excited) bound states. Starting with papers by, for instance, Waldmann [1], Snider and co-workers [2,3], McLennan and co-workers [4–6], and Klimontovich and Kremp [7], the derivation of kinetic equations has appealed great interest over many years. Usually (see, e.g. the review article by Klimontovich *et al.* [8]) the two-particle density matrix is split into different parts with respect to some projection operator which projects onto the space of the bound states. Often the states are taken to be those of the isolated atom. The diagonal matrix elements are considered to be the distribution function for the respective bound state. However, in a dense system, it is not clear whether the diagonalization of the density matrix with respect to the unperturbed two-particle states is a good approximation.

It is well known that many-particle effects like dynamical screening, self-energies, or phase space occupation may have an influence on the two-particle properties. A unique description of these effects within the investigation of nonequilibrium behavior can be given in the framework of the real-time Green's functions technique. For the single-particle functions, there were derived the Kadanoff-Baym equations for the correlation functions $g_a^{\gtrless}$. In this paper we aim at the derivation of similar equations on the two-particle level.

Some remarks on the bound state problem in equilibrium seem to be useful first. The investigation of bound states in dense systems has been the topic of a lot of papers (for references, see Refs. [9] and [10]). In the framework of the Green's functions method, a proper starting point is the so-called Bethe-Salpeter equation (BSE) for the two-particle causal Green's function [11–13]

$$g_{ab}(12,1'2') = g_a(1,1')g_b(2,2') + i\int d\bar{1}d\bar{2}d\tilde{1}d\tilde{2}\, g_a(1,\bar{1})g_b(2,\bar{2}) \times K_{ab}(\bar{1}\bar{2},\tilde{1}\tilde{2})g_{ab}(\tilde{1}\tilde{2},1'2'). \quad (1)$$

The kernel of this integral equation, the effective interaction K_{ab}, is a four point function and has a dynamical character. This makes the structure complicated: although one is interested only in the two-particle Green's function in the particle-particle channel ($t_1 = t_2$ and $t_1' = t_2'$), the knowledge of a Green's function with three times is enforced in the integral term. In Fourier space (or within the Matsubara technique), this corresponds to the problem that, for the determination of the two-particle Green's function dependent on one frequency, a more general function dependent on two frequencies has to be known. A way out of this dilemma was attempted by applying the Shindo approximation [14], in which a causal quantity with two frequencies is constructed from a quantity with one frequency. Then one obtains a closed equation for the causal Green's function. There are few estimations on the range of validity of this approximation. It is an exact relation for a static interaction K. It has been argued that the Shindo approximation reflects a first order approximation with respect to the retardation of the effective interaction [15,10,16].

With help of this BSE, an effective Schrödinger equation was derived which has some important corrections in comparison with that for an isolated atom: (i) phase space occupation factors, (ii) exchange self-energies (Hartree-Fock), (iii) a dynamically screened effective potential, and (iv) dynamical single-particle self-energy corrections. It has been shown that for localized states there is, to a large extent, a compensation between the effects (i) and (iii) on one side and (ii) and (iv) on the other. It follows that the binding

energies of (at least the low lying) bound states are not changed considerably in comparison with the isolated atom. In contrast, there is a large shift of the continuum edge by the self-energy corrections. This results in a lowering of ionization energies with increasing plasma density and leads, at the end, to the well-known Mott effect. An effective wave equation was solved numerically in Refs. [17,18]; for a discussion of the result, see Kraeft *et al.* [19].

However, the results obtained for the effective Hamiltonian also have some serious shortcomings. There occurs a division by Pauli-blocking factors $1-f_a-f_b$, which causes spurious pole structures for highly degenerate systems. Further, the effective Hamiltonian has static contributions which lack a clear physical interpretation (see, e.g., Ref. [20]).

Another approach was given by Schuck and co-workers [21,22]. They postulated that Dyson equations exist for two-time causal and retarded Green's functions, respectively. Expressions for the self-energy operator (also called the mass operator or effective Hamiltonian) are derived by comparison with the respective equations of motion. Also in this approach, it remains unclear what approximation (if any) is connected with the assumption that such two-time Dyson equations for the investigated functions and the inverse of those functions, respectively, do exist. There also occurs a problem of division by Pauli-blocking factors.

In nonequilibrium one can derive an equation of the same structure as in Eq. (1); however, the time integrations then have to be performed on the Keldysh contour. Schäfer *et al.* [23] considered a dynamically screened ladder approximation for the polarizability in a semiconductor within the Keldysh formalism. They gave a formulation for functions depending on three times or—after Fourier transformation—on two frequencies. At the end, however, they used the Shindo approximation for these two-frequency quantities in order to obtain kinetic equations for single-frequency functions.

There were attempts to generalize the Shindo approximation to functions in the time domain [24]. There was also an attempt [25] to generalize the approach of Schuck *et al.* within the nonequilibrium real-time Green's functions method, postulating a Dyson equation for the retarded function $g^R_{ab}(t,t')=\Theta(t-t')(g^>_{ab}-g^<_{ab})$. In both approaches similar results were achieved, and the equilibrium results could be reproduced. Thus the same shortcomings arose.

Within another approach [26], here we will present the solution to this problem. The real-time Green's functions method allows us to describe nonequilibrium systems. Results for thermodynamic equilibium will appear as a special case of the more general equations. In this paper the nonequilibrium Bethe-Salpeter equation is considered in a concrete approximation, the so-called dynamically screened ladder equation. This is the simplest approximation in which the effective interaction is of a dynamical nature. This will enable us to identify the underlying algebraic structures and to keep the equations as simple as possible. The general scheme will be investigated in a forthcoming paper [27].

The structure of this paper is as follows. In Sec. II the scheme of the real-time Green's function method for single-particle Green's functions is summarized. The difficulties of the BSE are discussed in Sec. III. Properties of two-time two-particle correlation and propagator functions, respectively, are given in Sec. IV. The transformation of the BSE into a Dyson equation is shown in Sec. V. The two-time structure will be achieved by applying the semigroup property of the ideal propagators. After that, the algebraic structures turn out to be similar to those of the nonequilibrium Dyson-Keldysh equation in the single-particle case. The thermodynamic equilibrium case is considered in Sec. VI. The structure of the two-particle self-energy which can be understood as an effective Hamiltonian is discussed. The results are compared with the former ones [12,15,10,20]. It will turn out that only in the nondegenerate case and in the static limit is one led to the same results.

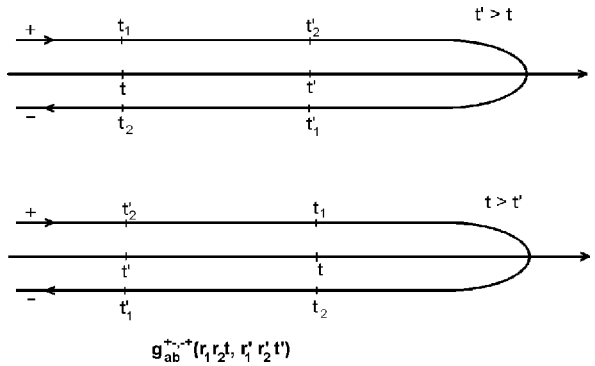

FIG. 1. Time ordering on the Keldysh double time contour for the function $g^{+-,-+}_{ab}$. The ordering is causal on the upper branch, and anticausal on the lower branch.

II. SINGLE-PARTICLE QUANTITIES

Let us briefly summarize the scheme of real-time Green's function technique in the single-particle case. The equations are given on a double-time contour, on the so-called Keldysh contour [28,29]. Working on the Keldysh contour has the advantage that well-developed schemes of functional derivatives and diagrammatic techniques known from equilibrium [30,31] can easily be generalized to nonequilibrium situations; see, e.g., Refs. [29,32–34].

The nonequililibrium Dyson equation on the Keldysh contour reads

$$\underline{g}_a(1,1')=\underline{g}_{a,0}(1,1')+\int_{\mathcal{C}} d\bar{1}d\tilde{1}\,\underline{g}_{a,0}(1,\tilde{1})\underline{\Sigma}_a(\bar{1},\tilde{1})\underline{g}_a(\tilde{1},1'), \tag{2}$$

with $1=\mathbf{r}_1,t_1$ etc., and $g_{a,0}$ being the ideal functions and Σ_a the self-energy, a denotes the species. The time integrations are performed on the Keldysh contour; see Fig. 1. The underlined quantities are matrices containing four functions. One obtains the causal functions for both times on the upper branch of the contour, $g_a=g_a^{++}$, and anticausal ones for both times on the lower branch, $\bar{g}_a=g_a^{--}$. If the first time is on the upper branch and the second one on the lower branch, one obtains $g_a^<=g_a^{+-}$. Fixing the first time on the lower branch and the second time on the upper branch gives $g_a^>=g_a^{-+}$. One can see that these elements are not all independent. It turns out that the equations achieve a more convenient structure if one introduces two other quantities g_a^R and g_a^A, defined by

$$g_a^{R/A}=g_a-g_a^{\lessgtr}=g_a^{\gtrless}-\overline{g}_a. \tag{3}$$

Equation (2) turns into the following form of the nonequilibrium Dyson equation for the correlation function $g_a^<$,

$$\begin{aligned} g_a^<(t,t')&=g_{a,0}^<(t,t')\\ &+\int_{t_0}^{\infty}dt_1\int_{t_0}^{\infty}dt_2[g_{a,0}^<(t,t_1)\Sigma_a^A(t_1,t_2)g_a^A(t_2,t')\\ &+g_{a,0}^R(t,t_1)\Sigma_a^<(t_1,t_2)g_a^A(t_2,t')\\ &+g_{a,0}^R(t,t_1)\Sigma_a^R(t_1,t_2)g_a^<(t_2,t')], \end{aligned} \tag{4}$$

which has to be supplemented by an equation for $g_a^{R/A}$:

$$\begin{aligned} g_a^R(t,t')&=g_{a,0}^R(t,t')\\ &+\int_{t_0}^{\infty}dt_1\int_{t_0}^{\infty}dt_2\, g_{a,0}^R(t,t_1)\Sigma_a^R(t_1,t_2)g_a^R(t_2,t'). \end{aligned} \tag{5}$$

Here only the functions's dependence on times was written explicitly in order to save space. Often the initial time is considered in the limit $t_0\to-\infty$. The quantity g_a^A is connected with g_a^R by $g_a^A(\mathbf{r}t,\mathbf{r}'t')=[g_a^R(\mathbf{r}'t',\mathbf{r}t)]^\dagger$.

In the following sections the one-particle self-energy will be needed in a special approximation which is called the V^S approximation (and often also called the GW approximation). This approximation which involves the dynamically screened potential, allows one to describe the influence of a plasma on the particles' propagation in it. Here one has

$$\underline{\Sigma}_a(1,1')=\underline{\Sigma}_a^H+i\underline{V}_{aa}^S(1,1')\underline{g}_a(1,1'), \tag{6}$$

with Σ^H being the Hartree self-energy. As for the Green's functions, there is a set of functions describing the dynamically screened interaction $\underline{V}_{ab}^S$, e.g.,

$$\begin{aligned} V_{ab}^S(t,t')&=V_{ab}\delta(t-t')+\Theta(t-t')V_{ab}^{S>}(t,t')\\ &+\Theta(t'-t)V_{ab}^{S<}(t,t'), \end{aligned} \tag{7}$$

and, further, $V_{ab}^S+\overline{V}_{ab}^S=V_{ab}^{S>}+V_{ab}^{S<}$, and $V_{ab}^{R/A}=V_{ab}^S-V_{ab}^{S\lessgtr}$. Here the correlation functions are defined by

$$V_{ab}^{S\gtrless}(t,t')=\sum_{c,d}V_{ac}L_{cd}^{\gtrless}(t,t')V_{db}, \tag{8}$$

where $L^{\gtrless}$ are the correlation functions of density fluctuations

$$\begin{aligned} iL_{ab}^>(\mathbf{r}_1t_1,\mathbf{r}_2t_2)&=\langle\delta\hat{\rho}_a(\mathbf{r}_1,t_1)\delta\hat{\rho}_b(\mathbf{r}_2,t_2)\rangle,\\ iL_{ab}^<(\mathbf{r}_1t_1,\mathbf{r}_2t_2)&=\langle\delta\hat{\rho}_b(\mathbf{r}_2,t_2)\delta\hat{\rho}_a(\mathbf{r}_1,t_1)\rangle, \end{aligned} \tag{9}$$

with $\delta\hat{\rho}_a(\mathbf{r},t)=\Psi_a^\dagger(\mathbf{r},t)\Psi_a(\mathbf{r},t)-\langle\Psi_a^\dagger(\mathbf{r},t)\Psi_a(\mathbf{r},t)\rangle$, and $\Psi_a^\dagger$ and Ψ_a are creation and annihilation operators of second quantization obeying the known commutation relations. It follows that, e.g., $V_{ab}^{S<}(1,1')=V_{ba}^{S>}(1',1)$ and $V_{ab}^S(1,1')=V_{ba}^S(1',1)$, but $V_{ab}^{SR}(1,1')=V_{ba}^{SA}(1',1)$.

III. BETHE-SALPETER EQUATION

The two-particle Green's function is determined by the so-called Bethe-Salpeter equation

$$\begin{aligned} g_{ab}(12,1'2')&=g_a(1,1')g_b(2,2')\\ &+i\int d\overline{1}d\overline{2}d\widetilde{1}d\widetilde{2}g_a(1,\overline{1})g_b(2,\overline{2})\\ &\times K_{ab}(\overline{12},\widetilde{\overline{12}})g_{ab}(\widetilde{\overline{12}},1'2'), \end{aligned} \tag{10}$$

in which, by introduction of the effective interaction kernel K_{ab}, a closed equation is formally achieved for the four-point function. Here the kernel K_{ab} is the sum of all diagrams irreducible with respect to a cutting of two single-particle lines. The Bethe-Salpeter equation can be understood to hold in various contexts: for the ground state $T=0$), for the imaginary-time equilibrium Green's functions in the Matsubara technique, or for the real-time Green's functions on the Keldysh contour.

The properties of a pair of particles should follow from this equation in the so-called particle-particle channel. If one considers the causal two-particle Green's function in this channel ($t_1=t_2=t;t_1'=t_2'=t'$), one has

$$\begin{aligned} &i^2g_{ab}(r_1r_2t,r_1'r_2't')\\ &\quad=\theta(t-t')\langle\Psi_a(\mathbf{r}_1,t)\Psi_b(\mathbf{r}_2,t)\Psi_b^\dagger(\mathbf{r}_2',t')\Psi_a^\dagger(\mathbf{r}_1',t')\rangle\\ &\qquad+\theta(t'-t)\langle\Psi_a^\dagger(\mathbf{r}_1',t')\Psi_b^\dagger(\mathbf{r}_2',t')\\ &\qquad\times\Psi_b(\mathbf{r}_2,t)\Psi_a(\mathbf{r}_1,t)\rangle\\ &\quad=\theta(t-t')i^2g_{ab}^>+\theta(t'-t)i^2g_{ab}^<. \end{aligned} \tag{11}$$

On the right hand side of Eq. (10), however, there occurs a function depending on three times $\overline{t_1}$, $\overline{t_2}$, and t', which is enforced by the dynamical character of K_{ab}. This Green's function consists of six different correlation functions:

$$\begin{aligned} &i^2g_{ab}(\overline{r_1}\overline{t_1}\overline{r_2}\overline{t_2};r_1'r_2't')\\ &\quad=\theta(\overline{t_1}-\overline{t_2})\theta(\overline{t_2}-t')\langle\Psi_a\Psi_b\Psi_b^\dagger\Psi_a^\dagger\rangle\\ &\qquad\pm\theta(\overline{t_2}-\overline{t_1})\theta(\overline{t_1}-t')\langle\Psi_b\Psi_a\Psi_b^\dagger\Psi_a^\dagger\rangle\\ &\qquad+\theta(\overline{t_1}-t')\theta(t'-\overline{t_2})\langle\Psi_a\Psi_b^\dagger\Psi_a^\dagger\Psi_b\rangle\\ &\qquad+\theta(\overline{t_2}-t')\theta(t'-\overline{t_1})\langle\Psi_b\Psi_a^\dagger\Psi_b^\dagger\Psi_a\rangle\\ &\qquad\pm\theta(t'-\overline{t_1})\theta(\overline{t_1}-\overline{t_2})\langle\Psi_a^\dagger\Psi_b^\dagger\Psi_a\Psi_b\rangle\\ &\qquad+\theta(t'-\overline{t_2})\theta(\overline{t_2}-\overline{t_1})\langle\Psi_a^\dagger\Psi_b^\dagger\Psi_b\Psi_a\rangle. \end{aligned} \tag{12}$$

Only a static interaction in Eq. (10) would enforce $\overline{t_1}=\overline{t_2}$, and the function would turn into the two-time causal one [Eq. (11)].

In principle, one could of course try to solve the Bethe-Salpeter equation for a function depending on three times (equivalent to a function depending on two frequencies), and then extract from this the information one is interested in. This, however, seems to be too complicated.

In a number of papers attempts were made to work with equations which involve two-time functions exclusively. This was achieved in two ways. The first approach [12,13] uses the so-called Shindo approximation within the Matsubara technique, in which the two-frequency function is constructed from the single-frequency causal Green's function. This is possible in an exact way for a statical interaction, and therefore it was argued that for arbitrary K_{ab} this would be correct in first order with respect to the retardation. In the other approach [21], a closed equation for the causal two-time or single-freqency function, respectively, is postulated. The effective Hamiltonian (two-particle self-energy) is then determined by comparison with equations of motion.

However, enforcing a closed equation for combinations of the correlation functions $g_{ab}^{\gtrless}$ defined in Eq. (11) means that the information contained in the correlation functions $\langle \Psi_a \Psi_b^\dagger \Psi_a^\dagger \Psi_b \rangle$ and $\langle \Psi_b \Psi_a^\dagger \Psi_b^\dagger \Psi_a \rangle$ [third and fourth terms in Eq. (12)] is neglected. Therefore, such a closed equation for the causal two-time Green's function can exist only in an approximate way. In Sec. V we will show that closed equations do exist only for other combinations of correlation functions.

IV. TWO-PARTICLE GREEN'S FUNCTIONS DEPENDING ON TWO TIMES

In analogy to the single-particle case, one is interested in obtaining information on the statistical properties, carried by the two-particle density matrix, as well as information on the two-particle dynamics, i.e., spectral information. Below we will see that it is not a trivial question to ask which quantity carries this information.

One of the quantities of interest is the following two-time correlation function

$$g_{ab}^{<}(\mathbf{r}_1\mathbf{r}_2 t,\mathbf{r}_1'\mathbf{r}_2' t') = \frac{1}{i^2}\langle \Psi_a^\dagger(\mathbf{r}_1',t')\Psi_b^\dagger(\mathbf{r}_2',t')\Psi_b(\mathbf{r}_2,t)\Psi_a(\mathbf{r}_1,t)\rangle. \tag{13}$$

In the case $t'=t$, the quantity $i^2 g_{ab}^{<}$ is just the two-particle density matrix $\rho_{ab}(\mathbf{r}_1\mathbf{r}_2\mathbf{r}_1'\mathbf{r}_2',t)$.

There are some other two-particle correlation functions in which the two creation operators have the time $t_1'=t_2'=t'$, whereas the two annihilation operators have the time $t_1=t_2=t$. Although the physical times t_1 and t_2 as well as t_1' and t_2' are equal, there are still 16 possibilities to fix the times t_1, t_2, t_1', and t_2' on the upper and lower branches of the Keldysh contour. Fixing the times t_1 and t_2 on the upper branch of the Keldysh contour and t_1' and t_2' on the lower branch, one obtains the correlation function $g_{ab}^{++,--}=g_{ab}^{<}$ defined in Eq. (13). Three other important cases are the following:

$$g_{ab}^{+-,-+}(\mathbf{r}_1\mathbf{r}_2 t,\mathbf{r}_1'\mathbf{r}_2' t') = \Theta(t-t')\frac{1}{i^2}\langle \Psi_a^\dagger(\mathbf{r}_1',t')\Psi_b(\mathbf{r}_2,t) \times \Psi_a(\mathbf{r}_1,t)\Psi_b^\dagger(\mathbf{r}_2',t')\rangle + \Theta(t'-t)\frac{1}{i^2}\langle \Psi_b(\mathbf{r}_2,t) \times \Psi_a^\dagger(\mathbf{r}_1',t')\Psi_b^\dagger(\mathbf{r}_2',t')\Psi_a(\mathbf{r}_1,t)\rangle, \tag{14}$$

$$g_{ab}^{-+,+-}(\mathbf{r}_1\mathbf{r}_2 t,\mathbf{r}_1'\mathbf{r}_2' t') = \Theta(t-t')\frac{1}{i^2}\langle \Psi_b^\dagger(\mathbf{r}_2',t')\Psi_a(\mathbf{r}_1,t)\Psi_b(\mathbf{r}_2,t) \times \Psi_a^\dagger(\mathbf{r}_1',t')\rangle + \Theta(t'-t)\frac{1}{i^2}\langle \Psi_a(\mathbf{r}_1,t)\Psi_b^\dagger(\mathbf{r}_2',t') \times \Psi_a^\dagger(\mathbf{r}_1',t')\Psi_b(\mathbf{r}_2,t)\rangle, \tag{15}$$

$$g_{ab}^{++,--}(\mathbf{r}_1\mathbf{r}_2 t,\mathbf{r}_1'\mathbf{r}_2' t') = \frac{1}{i^2}\langle \Psi_a(\mathbf{r}_1,t)\Psi_b(\mathbf{r}_2,t) \times \Psi_b^\dagger(\mathbf{r}_2',t')\Psi_a^\dagger(\mathbf{r}_1',t')\rangle. \tag{16}$$

To give an example, the time ordering on the Keldysh contour is shown for $g_{ab}^{+-,-+}$ in Fig. 1. All other functions $g_{ab}^{\alpha\beta,\gamma\delta}$ with greek indices equal to ''+'' or ''−'' can be expressed in terms of the six correlation functions involved in Eqs. (13) and (14)–(16).

We define the following retarded and advanced quantities:

$$G_{ab}^{R}(\mathbf{r}_1\mathbf{r}_2 t,\mathbf{r}_1'\mathbf{r}_2' t') \equiv \Theta(t-t')i(g_{ab}^{++,--}-g_{ab}^{+-,-+}-g_{ab}^{-+,+-}+g_{ab}^{--,++}) = \Theta(t-t')\frac{1}{i}\langle [\Psi_a^\dagger(\mathbf{r}_1',t'),[\Psi_b^\dagger(\mathbf{r}_2',t'),\Psi_b(\mathbf{r}_2,t) \times \Psi_a(\mathbf{r}_1,t)]_-]_{\mp}\rangle, \tag{17}$$

$$G_{ab}^{A}(\mathbf{r}_1\mathbf{r}_2 t,\mathbf{r}_1'\mathbf{r}_2' t') \equiv \Theta(t'-t)(-i)(g_{ab}^{++,--}-g_{ab}^{+-,-+}-g_{ab}^{-+,+-}+g_{ab}^{--,++}) = \Theta(t'-t)\frac{1}{-i}\langle [\Psi_a(\mathbf{r}_1,t),[\Psi_b(\mathbf{r}_2,t),\Psi_b^\dagger(\mathbf{r}_2',t') \times \Psi_a^\dagger(\mathbf{r}_1',t')]_-]_{\mp}\rangle. \tag{18}$$

In order to achieve the nested commutator structures, we used the fact that operators with equal times can be interchanged according to the well-known relations. Interestingly enough, these nested structures were found also by Rajagopal and Majumdar in their analysis of double dispersion relations for the two-frequency causal Matsubara Green's function (Appendix II of Ref. [11]). We showed in Appendix C of Ref. [35] how the function G_{ab}^{A} is connected in thermodynamic equilibrium with the analytic continuation of the two-frequency Matsubara Green's function. The functions $G_{ab}^{R/A}$ have the following properties.

(i) They are connected by Hermitian conjugation

$$G^R_{ab}(\mathbf{r}_1\mathbf{r}_2t,\mathbf{r}_1'\mathbf{r}_2't')=[G^A_{ab}(\mathbf{r}_1'\mathbf{r}_2't',\mathbf{r}_1\mathbf{r}_2t)]^\dagger. \tag{19}$$

(ii) Both functions have the property of crossing symmetry, i.e.,

$$G^{R/A}_{ab}(\mathbf{r}_1\mathbf{r}_2t,\mathbf{r}_1'\mathbf{r}_2't')=G^{R/A}_{ba}(\mathbf{r}_2\mathbf{r}_1t,\mathbf{r}_2'\mathbf{r}_1't'). \tag{20}$$

(iii) The inhomogeneity in the equations of motion for these functions consists of δ functions only (without Pauli-blocking terms). This is easy to see from Eqs. (17), and (18). Derivation of the Heaviside function gives a δ function $\delta(t-t')$, and the commutation relations for the field operators lead to $[\delta(\mathbf{r}_1-\mathbf{r}_1')\delta(\mathbf{r}_2-\mathbf{r}_2')\pm\delta_{ab}\delta(\mathbf{r}_1-\mathbf{r}_2')\delta(\mathbf{r}_2-\mathbf{r}_1')]$.

(iv) For vanishing interaction between particles a and b, the correlation functions in Eqs. (17) and (18) can be contracted into products of single-particle correlation functions $g^{\gtrless}$; one obtains

$$\begin{aligned}G^R_{ab}(\mathbf{r}_1\mathbf{r}_2t,\mathbf{r}_1'\mathbf{r}_2't')&=ig^R_a(\mathbf{r}_1t,\mathbf{r}_1't')g^R_b(\mathbf{r}_2t,\mathbf{r}_2't')\\&\pm\delta_{ab}ig^R_a(\mathbf{r}_1t,\mathbf{r}_2't')g^R_b(\mathbf{r}_2t,\mathbf{r}_1't'),\end{aligned} \tag{21}$$

with the retarded single-particle function g^R_a defined by Eq. (3).

(v) The difference of the retarded and the advanced functions defines a spectral function

$$\begin{aligned}&A_{ab}(\mathbf{r}_1\mathbf{r}_2t,\mathbf{r}_1'\mathbf{r}_2't')\\&\quad=iG^R_{ab}(\mathbf{r}_1\mathbf{r}_2t,\mathbf{r}_1'\mathbf{r}_2't')-iG^A_{ab}(\mathbf{r}_1\mathbf{r}_2t,\mathbf{r}_1'\mathbf{r}_2't')\\&\quad=i^2[g^{++,--}_{ab}-g^{+-,-+}_{ab}-g^{-+,+-}_{ab}+g^{--,++}_{ab}],\end{aligned} \tag{22}$$

which, in the case of equal times ($t=t'$), gives

$$\begin{aligned}A_{ab}(\mathbf{r}_1\mathbf{r}_2t,\mathbf{r}_1'\mathbf{r}_2't)&=\delta(\mathbf{r}_1-\mathbf{r}_1')\delta(\mathbf{r}_2-\mathbf{r}_2')\\&\pm\delta_{ab}\delta(\mathbf{r}_1-\mathbf{r}_2')\delta(\mathbf{r}_2-\mathbf{r}_1'),\end{aligned} \tag{23}$$

which corresponds to a sum rule in frequency space, $\int d\omega/(2\pi)A_{ab}(\omega,t)=1$.

We will see in Sec. V that the two-time functions defined by Eqs. (13)–(16) and (17)–(18) will be the constituents of the algebraic structure of a two-particle nonequilibrium Dyson equation.

V. TRANSFORMATION OF THE BETHE-SALPETER EQUATION INTO A DYSON EQUATION

The present section, together with the foregoing, is the most important part of this paper. The Bethe-Salpeter equation will be considered here in a concrete approximation: for the effective interaction kernel K_{ab}, the dynamically screened potential V^S_{ab} is taken. This is the simplest approximation in which K_{ab} has a dynamical character which will allow us to identify the general algebraic structure of a two-particle Dyson equation.

In this approximation one has on the Keldysh contour

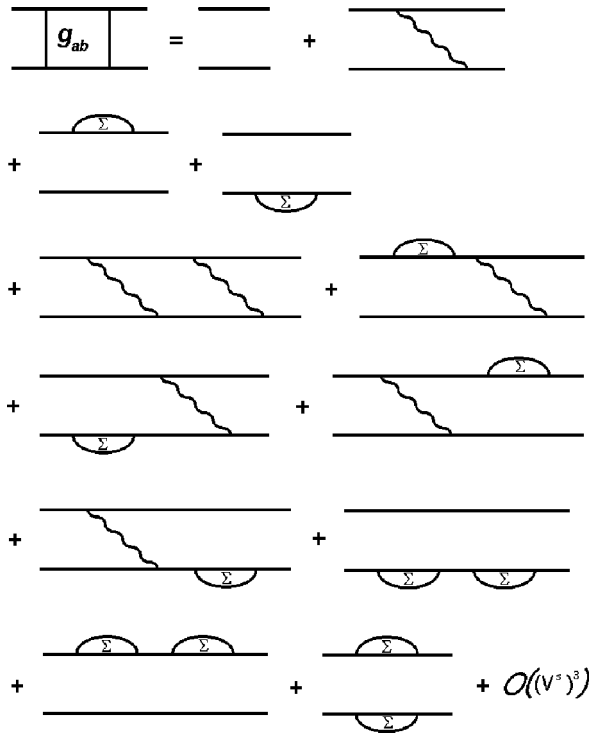

FIG. 2. Diagrammatic expansion of the dynamically screened ladder equation up to second order.

$$\begin{aligned}\underline{g}_{ab}(t_1t_2,t_1't_2')&=\underline{g}_a(t_1,t_1')\underline{g}_b(t_2,t_2')\\&\quad+\int_{\mathcal{C}}d\overline{t}_1\,d\overline{t}_2\,\underline{g}_a(t_1,\overline{t}_1)\underline{g}_b(t_2,\overline{t}_2)\\&\quad\times i\underline{V}^S_{ab}(\overline{t}_1,\overline{t}_2)\underline{g}_{ab}(\overline{t}_1\overline{t}_2,t_1't_2').\end{aligned} \tag{24}$$

Iteration of this integral equation leads to ladder-type terms. This BSE (24) will be considered in the following in the special cases $t_1=t_2=t$ and $t_1'=t_2'=t'$.

Below the dynamically screened ladder equation as a special approximation of the BSE is transformed into a Dyson equation in which the occurring two-particle Green's functions and two-particle self-energy functions are dependent on two times only. For this purpose, the perturbation expansion of the BSE (24) is considered in the diagrammatic form shown in Fig. 2. It is analyzed first for $g^<_{ab}$; details are presented for different orders of V^S_{ab} in the Appendix.

We search for (and, indeed, find) the structures [cf. the corresponding equations for the single-particle functions, Eqs. (4) and (5)]

$$\begin{aligned}g^<_{ab}&=\mathcal{G}^<_{ab}+\mathcal{G}^R_{ab}[V_{ab}+\Sigma^R_{ab}]g^<_{ab}+\mathcal{G}^R_{ab}\sigma^<_{ab}G^A_{ab}\\&\quad+\mathcal{G}^<_{ab}[V_{ab}+\Sigma^A_{ab}]G^A_{ab},\end{aligned} \tag{25}$$

$$G^A_{ab}=\mathcal{G}^A_{ab}+\mathcal{G}^A_{ab}[V_{ab}+\Sigma^A_{ab}]G^A_{ab}, \tag{26}$$

with the definitions $\mathcal{G}^<_{ab}(t,t')=g^<_{a,0}(t,t')g^<_{b,0}(t,t')$, $\mathcal{G}^R_{ab}(t,t')=ig^R_{a,0}(t,t')g^R_{b,0}(t,t')$, and $\mathcal{G}^A_{ab}(t,t')=(-i)g^A_{a,0}(t,t')g^A_{b,0}(t,t')$. All quantities in the above equations depend on two times only. The integration of intermediate times runs in the interval $[t_0,\infty]$ as in Eqs. (4) and (5).

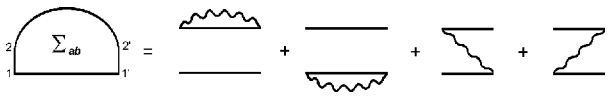

FIG. 3. Diagrammatic structure of the two-particle self-energy in first order in V^S.

The zeroth order of $g^<_{ab}$ with respect to the two-particle self-energy is $g^{<(0)}_{ab}=\mathcal{G}^<_{ab}$; first and second orders are given by

$$g^{<(1)}_{ab}=\mathcal{G}^R_{ab}[V_{ab}+\Sigma^R_{ab}]\mathcal{G}^<_{ab}+\mathcal{G}^R_{ab}\sigma^<_{ab}\mathcal{G}^A_{ab} +\mathcal{G}^<_{ab}[V_{ab}+\Sigma^A_{ab}]\mathcal{G}^A_{ab}, \tag{27}$$

$$\begin{aligned} g^{<(2)}_{ab}=&\mathcal{G}^R_{ab}[V_{ab}+\Sigma^R_{ab}]\mathcal{G}^R_{ab}[V_{ab}+\Sigma^R_{ab}]\mathcal{G}^<_{ab} \\ &+\mathcal{G}^R_{ab}[V_{ab}+\Sigma^R_{ab}]\mathcal{G}^R_{ab}\sigma^<_{ab}\mathcal{G}^A_{ab} \\ &+\mathcal{G}^R_{ab}[V_{ab}+\Sigma^R_{ab}]\mathcal{G}^<_{ab}[V_{ab}+\Sigma^A_{ab}]\mathcal{G}^A_{ab} \\ &+\mathcal{G}^R_{ab}\sigma^<_{ab}\mathcal{G}^A_{ab}[V_{ab}+\Sigma^A_{ab}]\mathcal{G}^A_{ab} \\ &+\mathcal{G}^<_{ab}[V_{ab}+\Sigma^A_{ab}]\mathcal{G}^A_{ab}[V_{ab}+\Sigma^A_{ab}]\mathcal{G}^A_{ab}. \end{aligned} \tag{28}$$

The self-energy functions $\sigma^<_{ab}$ and Σ^R_{ab}, respectively, are then identified by comparison with the expansion terms of the ladder equation (Fig. 2).

All functions in the above equations (25) and (26) are understood to depend on two times. The key idea in order to achieve such a two-time structure of the equations is to use the semigroup properties of the ideal single-particle propagators $g^R_{a,0}$ and $g^A_{a,0}$ (the time-local Hartree-Fock self-energy could also be included). In particular, for any time $\overline{t}$ with $t>\overline{t}>t'$ one has the following relation:

$$g^R_{a,0}(\mathbf{r}_1t,\mathbf{r}_1't')=i\int d^3r_2\, g^R_{a,0}(\mathbf{r}_1t,\mathbf{r}_2\overline{t})g^R_{a,0}(\mathbf{r}_2\overline{t},\mathbf{r}_1't'). \tag{29}$$

There is no time integration in the above equation. Analogously, for the advanced function with $t<\overline{t}<t'$, one has (integration with respect to $\mathbf{r}_2$ suppressed) $g^A_{a,0}(t,t')=(-i)g^A_{a,0}(t,\overline{t})g^A_{a,0}(\overline{t},t')$.

For the ideal one-particle correlation functions $g^{\gtrless}_{a,0}$, there follows

$$\begin{aligned} &g^{\gtrless}_{a,0}(t,t')=ig^R_{a,0}(t,\overline{t})g^{\gtrless}_{a,0}(\overline{t},t') \quad \text{for} \quad t>\overline{t}, \\ &g^{\gtrless}_{a,0}(t,t')=(-i)\,g^{\gtrless}_{a,0}(t,\overline{t})g^A_{a,0}(\overline{t},t') \quad \text{for} \quad \overline{t}<t', \\ &g^{\gtrless}_{a,0}(t,t')=(-i^2)g^R_{a,0}(t,\tilde{t})g^{\gtrless}_{a,0}(\tilde{t},\overline{t})g^A_{a,0}(\overline{t},t') \\ &\qquad \text{for} \quad t>\tilde{t} \quad \text{and} \quad \overline{t}<t'. \end{aligned} \tag{30}$$

Proceeding in the manner presented in the Appendix, and comparing the results with the anticipated structure [Eqs. (25) and (26)], we obtain the following expression for the retarded self-energy function:

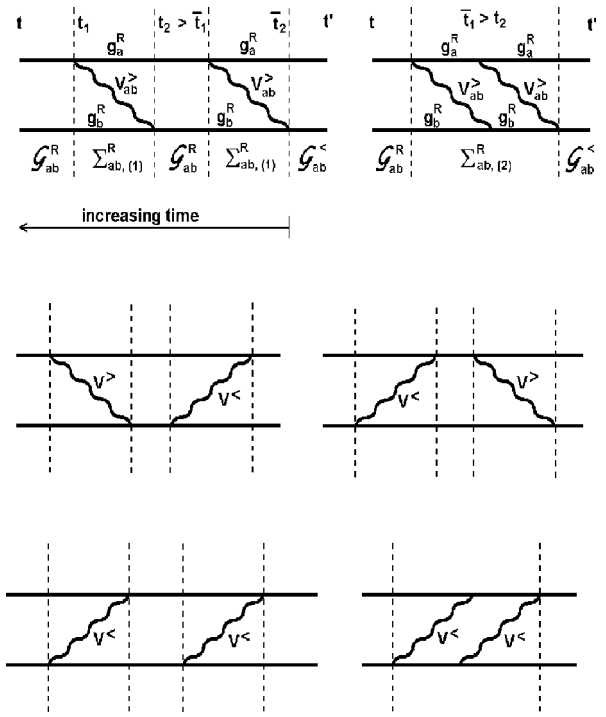

FIG. 4. Evaluation of the ladder terms with two rungs. The first, third, fourth, and fifth terms are reducible, i.e., there are two successive self-energy insertions of first order. The second and last terms, however, are not reducible: they are two-particle self-energies of second order in V^S.

$$\begin{aligned} \Sigma^R_{ab}(t,t')=&i\Sigma^R_a(t,t')g^R_{b,0}(t,t')+i\Sigma^R_b(t,t')g^R_{a,0}(t,t') \\ &+ig^R_{b,0}(t,t')iV^{S>}_{ab}(t,t')g^R_{a,0}(t,t') \\ &+ig^<_{b,0}(t,t')iV^{SR}_{ab}(t,t')g^R_{a,0}(t,t') \\ &+ig^R_{a,0}(t,t')iV^{S<}_{ab}(t',t)g^R_{b,0}(t,t') \\ &+ig^<_{a,0}(t,t')iV^{SA}_{ab}(t',t)g^R_{b,0}(t,t'), \end{aligned} \tag{31}$$

where the one-particle self-energies have to be used in first order of the dynamically screened potential V^S [cf. Eq. (6)], i.e.,

$$\begin{aligned} \Sigma^R_a(t,t')=&\Sigma^H_a(t)\,\delta(t-t')+iV^{S>}_{aa}(t,t')g^R_a(t,t') \\ &+iV^{SR}_{aa}(t,t')g^<_a(t,t'). \end{aligned} \tag{32}$$

The correlation function $\sigma^<_{ab}$ is found to be

$$\begin{aligned} \sigma^<_{ab}(t,t')=&\Sigma^<_a(t,t')g^<_{b,0}(t,t')+\Sigma^<_b(t,t')g^<_{a,0}(t,t') \\ &+g^<_{b,0}(t,t')iV^{S<}_{ab}(t,t')g^<_{a,0}(t,t') \\ &+g^<_{a,0}(t,t')iV^{S>}_{ab}(t',t)g^<_{b,0}(t,t'), \end{aligned} \tag{33}$$

where the single-particle self-energy function $\Sigma^<_a$ is given by $\Sigma^<_a(t,t')=iV^{S<}_{aa}(t,t')g^<_a(t,t')$.

The diagrammatic structure of the two-particle self-energy functions is shown in Fig. 3. Primarily, these functions consist of naked lines because we worked in first order with respect to the dynamically screened potential. However,

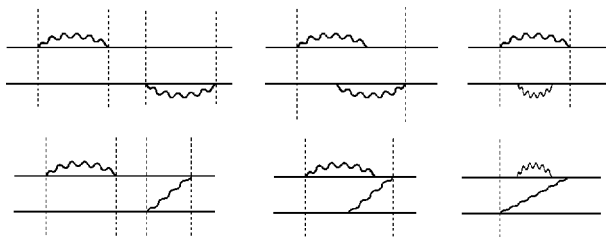

FIG. 5. Here two other types of second-order terms are shown: in the first row two single-particle self-energies are combined, and in the second row one single-particle self-energy is combined with one interaction between the particles. Depending on how the times overlap, there are three different kinds of diagrams.

all diagrams necessary to dress the lines could be found in higher orders of the expansion; see Appendix A 2 and especially Fig. 5.

The self-energy functions $\Sigma_{ab}^{R/A}$ and $\sigma_{ab}^{<}$ are functionals of single-particle Green's functions. That is the reason why it is sufficient to consider the two equations (25) and (26) in order to determine the correlation function $g_{ab}^{<}$. In higher approximations, the two-particle self-energy is also expected to be a functional of two-particle correlation functions. Then one would need also the equations for the other three quantities defined in Eqs. (14)–(16). The full scheme of equations reads (with $\Phi=\{++,--\};\{+-,-+\};\{-+,+-\};\{--,++\}$):

$$g_{ab}^{\Phi}=\mathcal{G}_{ab}^{\Phi}+\mathcal{G}_{ab}^{R}[V_{ab}+\Sigma_{ab}^{R}]g_{ab}^{\Phi}+\mathcal{G}_{ab}^{R}\sigma_{ab}^{\Phi}G_{ab}^{A}+\mathcal{G}_{ab}^{\Phi}[V_{ab}+\Sigma_{ab}^{A}]G_{ab}^{A}, \tag{34}$$

$$G_{ab}^{R/A}=\mathcal{G}_{ab}^{R/A}+\mathcal{G}_{ab}^{R/A}[V_{ab}+\Sigma_{ab}^{R/A}]G_{ab}^{R/A}. \tag{35}$$

These equations are not all independent. G_{ab}^{R} and G_{ab}^{A} are connected by Hermitian conjugation. Further they are linear combinations of the preceding four functions g_{ab}^{Φ} according to Eq. (17). The system of equations is consistent, i.e., combining the equations for g_{ab}^{Φ} according to Eqs. (17) or (18), respectively, one obtains the Dyson equation (35).

σ^{Φ} are given by (cf. Fig. 3)

$$\sigma_{ab}^{\alpha,\beta,\gamma,\delta}=i[V_{aa}^{\alpha,\gamma}(\mathbf{r}_1t,\mathbf{r}_1't')+V_{bb}^{\beta,\delta}(\mathbf{r}_2t,\mathbf{r}_2't')+V_{ab}^{\alpha,\delta}(\mathbf{r}_1t,\mathbf{r}_2't')+V_{ba}^{\beta,\gamma}(\mathbf{r}_2t,\mathbf{r}_1't')]g_{a}^{\alpha,\gamma}(\mathbf{r}_1t,\mathbf{r}_1't')g_{b}^{\beta,\delta}(\mathbf{r}_2t,\mathbf{r}_2't'). \tag{36}$$

The retarded two-particle self-energy is defined in analogy to G_{ab}^{R} by

$$\Sigma_{ab}^{R}(\mathbf{r}_1\mathbf{r}_2t,\mathbf{r}_1'\mathbf{r}_2't')=\Sigma_{ab}^{0}(\mathbf{r}_1\mathbf{r}_2,\mathbf{r}_1'\mathbf{r}_2't)\,\delta(t-t')+\Theta(t-t')i[\sigma_{ab}^{++,--}-\sigma_{ab}^{+-,-+}-\sigma_{ab}^{-+,+-}+\sigma_{ab}^{--,++}]. \tag{37}$$

The term which is local in time consists of the single-particle Hartree and Hartree-Fock (HF) self-energies as well as the Pauli-blocking contribution

$$\Sigma_{ab}^{0}(\mathbf{r}_1\mathbf{r}_2t,\mathbf{r}_1'\mathbf{r}_2't')=\Sigma_{a}^{\mathrm{HF}}(\mathbf{r}_1\mathbf{r}_1't)\,\delta(\mathbf{r}_2-\mathbf{r}_2')+\Sigma_{b}^{\mathrm{HF}}(\mathbf{r}_2\mathbf{r}_2't)\,\delta(\mathbf{r}_1-\mathbf{r}_1')+[ig_{a}^{<}(\mathbf{r}_1t,\mathbf{r}_1't)\,\delta(\mathbf{r}_2-\mathbf{r}_2')+ig_{b}^{<}(\mathbf{r}_2t,\mathbf{r}_2't)\,\delta(\mathbf{r}_1-\mathbf{r}_1')]V_{ab}(\mathbf{r}_1'-\mathbf{r}_2'). \tag{38}$$

Inserting the expression for the σ_{ab}^{Φ}, [Eq. (36)], into Eq. (37), one indeed obtains Eq. (31).

The advanced quantity Σ_{ab}^{A} is given by $\Sigma_{ab}^{A}(\mathbf{r}_1\mathbf{r}_2t,\mathbf{r}_1'\mathbf{r}_2't')=[\Sigma_{ab}^{R}(\mathbf{r}_1'\mathbf{r}_2't',\mathbf{r}_1\mathbf{r}_2t)]^{\dagger}$. The analytic properties of the two-particle self-energy are more involved than those of the single-particle self-energies [36], because off-diagonal matrix elements can already occur in the spatially homogeneous case.

Often it is more useful to consider, instead of Eqs. (34) and (35), the differential equations

$$\left[i\frac{\partial}{\partial t}-\hat{H}_{ab}^{0}-V_{ab}\right]g_{ab}^{\Phi}(t,t')=\int d\bar{t}[\Sigma_{ab}^{R}(t,\bar{t})g_{ab}^{\Phi}(\bar{t},t')+\sigma_{ab}^{\Phi}(t,\bar{t})G_{ab}^{A}(\bar{t},t')]. \tag{39}$$

There are additional equations for the propagator functions G_{ab}^{R} and G_{ab}^{A}. As these two functions are connected by Hermitian conjugation, only the respective equation for G_{ab}^{R} is written down:

$$\left[i\frac{\partial}{\partial t}-\hat{H}_{ab}^{0}-V_{ab}\right]G_{ab}^{R}(t,t')=\delta(t-t')+\int d\bar{t}\,\Sigma_{ab}^{R}(t,\bar{t})G_{ab}^{R}(\bar{t},t'). \tag{40}$$

Equations (34) and (35) and (39) and (40), respectively, can be considered as the most important result of the present paper. The latter equations are the two-particle counterpart to the Kadanoff-Baym equations in the single-particle case. Thus these equations are the proper basis for the description of two-particle properties. From Eq. (39) there follow generalized kinetic equations, whereas Eq. (40) gives information on the spectral properties.

VI. THERMODYNAMIC EQUILIBRIUM

A. Two-particle Dyson equation

In thermodynamic equilibrium only the spectral properties have to be determined, i.e., only Eq. (40) for the two-particle propagator has to be considered. Its Fourier transform is

$$[\Omega-H_{ab}^{0}-V_{ab}-\Sigma_{ab}^{0}-\Sigma_{ab}^{R\,\mathrm{corr}}(\Omega)]G_{ab}^{R}(\Omega)=1. \tag{41}$$

This equation can be called two-particle Dyson equation or, likewise, the Bethe-Salpeter equation. We want to draw the reader's attention to the fact that this equation is given here for the function G_{ab}^{R}, whereas in earlier attempts it was tried

to formulate such an equation for the causal two-particle Green's function [12,21], or a function $g^R_{ab}=\Theta(t-t')(g^>_{ab}-g^<_{ab})$ [25].

The static part of the two-particle self-energy in Eq. (41) is given by $\Sigma^0_{ab}=\Sigma^{\rm HF}_a+\Sigma^{\rm HF}_b+(N_{ab}-1)V_{ab}$. The self-energy consists of two different types of terms: some do not contain an interaction between the particles a and b, whereas others do. The first terms are due to single-particle self-energies.

For the correlation part of the two-particle self-energy, there holds the same distinction: $\Sigma^{R\,\rm corr}_{ab}(\Omega)$ consists of two contributions according to

$$\Sigma^{R\,\rm corr}_{ab}(\Omega)=\Delta^R_{ab}(\Omega)+V^{{\rm eff}\,R}_{ab}(\Omega), \tag{42}$$

where the first one is due to the one-particle self-energies [cf. the first two terms in Eq. (31)], whereas the second one describes an effective interaction between particles a and b in the many-particle system [the last four terms in Eq. (31)]. These terms can be further evaluated using a quasiparticle approximation

$$\pm ig^<_a(\mathbf{p},\omega)=2\pi\delta(\omega-\epsilon_a(p))f_a(\omega), \tag{43}$$

with $f_a(\omega)=\{\exp[\beta(\omega-\mu_a)]\mp 1\}^{-1}$. The correlation functions $V^{\lessgtr}_{ab}$ can be expressed in terms of the dielectric function $\varepsilon(\omega)$ and Bose functions $n_B(\omega)=[\exp(\beta\omega)-1]^{-1}$ according to [37,10]

$$\begin{aligned} iV^{S<}_{ab}(\mathbf{q},\omega)&=-2V_{ab}(q)\,{\rm Im}\,\varepsilon^{R-1}(\mathbf{q},\omega)n_B(\omega),\\ iV^{S>}_{ab}(\mathbf{q},\omega)&=-2V_{ab}(q)\,{\rm Im}\,\varepsilon^{R-1}(\mathbf{q},\omega)[1+n_B(\omega)]. \end{aligned} \tag{44}$$

For the function $\Delta^R_{ab}(\Omega)$, there follows

$$\begin{aligned} \Delta^R_{ab}(\mathbf{p}_1\mathbf{p}_2,\mathbf{p}_1'\mathbf{p}_2',\Omega)&=(2\pi)^6\delta_{\mathbf{p}_1,\mathbf{p}_1'}\delta_{\mathbf{p}_2,\mathbf{p}_2'}\int\frac{d^3q}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{d\omega_1}{\pi}[-{\rm Im}\,\varepsilon^{R-1}(\mathbf{q},\omega_1)]\\ &\quad\times\left\{V_{aa}(\mathbf{q})\frac{1\pm f_a(\mathbf{p}_1+\mathbf{q})+n_B(\omega_1)}{\Omega-\omega_1-\epsilon_a(\mathbf{p}_1+\mathbf{q})-\epsilon_b(\mathbf{p}_2)+i0}+(a\leftrightarrow b,1\leftrightarrow 2)\right\}\\ &=(2\pi)^6\delta_{\mathbf{p}_1,\mathbf{p}_1'}\delta_{\mathbf{p}_2,\mathbf{p}_2'}[\Sigma^R_a(\mathbf{p}_1',\Omega-\epsilon_b(\mathbf{p}_2'))+\Sigma^R_b(\mathbf{p}_2',\Omega-\epsilon_a(\mathbf{p}_1'))]. \end{aligned} \tag{45}$$

Thus this function is just the sum of the single-particle self-energies (in the V^S approximation) to be taken off-shell.

The other contribution to the two-particle self-energy, $V^{\rm eff}_{ab}$, is given by

$$\begin{aligned} V^{{\rm eff}\,R}_{ab}(\mathbf{p}_1\mathbf{p}_2,\mathbf{p}_1'\mathbf{p}_2',\Omega)&=(2\pi)^3\delta_{\mathbf{p}_1+\mathbf{p}_2,\mathbf{p}_1'+\mathbf{p}_2'}\int_{-\infty}^{\infty}\frac{d\omega_1}{\pi}[-{\rm Im}\,\varepsilon^{R-1}(\mathbf{p}_1-\mathbf{p}_1',\omega_1)]\\ &\quad\times\left\{V_{ab}(\mathbf{p}_1-\mathbf{p}_1')\frac{1\pm f_a(\mathbf{p}_1)+n_B(\omega_1)}{\Omega-\omega_1-\epsilon_a(\mathbf{p}_1)-\epsilon_b(\mathbf{p}_2')+i0}+(a\leftrightarrow b,1\leftrightarrow 2)\right\}. \end{aligned} \tag{46}$$

These two contributions to the two-particle self-energy look very similar. Replacing the Coulomb potential V_{ab} by z_az_bV, with z_a and z_b being the charge numbers, one can see that for particles attracting each other, there is a compensation between these two functions. This is especially to be seen considering the functions integrated with respect to $\mathbf{p}_1$ and $\mathbf{p}_2$:

$$\begin{aligned} &\int\frac{d^3p_1}{(2\pi)^3}\int\frac{d^3p_2}{(2\pi)^3}[\Delta^R_{ab}(\mathbf{p}_1\mathbf{p}_2,\mathbf{p}_1'\mathbf{p}_2',\Omega)+V^{{\rm eff}\,R}_{ab}(\mathbf{p}_1\mathbf{p}_2,\mathbf{p}_1'\mathbf{p}_2',\Omega)]\\ &\quad=(z_a+z_b)\int\frac{d^3q}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{d\omega_1}{\pi}[-{\rm Im}\,\varepsilon^{R-1}(\mathbf{q},\omega_1)]\left\{z_aV(\mathbf{q})\frac{1\pm f_a(\mathbf{p}_1'+\mathbf{q})+n_B(\omega_1)}{\Omega-\omega_1-\epsilon_a(\mathbf{p}_1'+\mathbf{q})-\epsilon_b(\mathbf{p}_2')+i0}+(a\leftrightarrow b,1\leftrightarrow 2)\right\}. \end{aligned} \tag{47}$$

In the case of a symmetrical plasma, $z_a=-z_b$, the right hand side of the above equation vanishes, and it follows that

$$\begin{aligned} &\Sigma^R_a(\mathbf{p}_1',\Omega-\epsilon_b(\mathbf{p}_2'))+\Sigma^R_b(\mathbf{p}_2',\Omega-\epsilon_a(\mathbf{p}_1'))\\ &\quad=-\int\frac{d^3p_1}{(2\pi)^3}\int\frac{d^3p_2}{(2\pi)^3}V^{{\rm eff}\,R}_{ab}(\mathbf{p}_1\mathbf{p}_2,\mathbf{p}_1'\mathbf{p}_2',\Omega). \end{aligned} \tag{48}$$

These expressions for the two-particle self-energy have to be compared with the results of former papers [12,15]. The notations are slightly different in comparison with ours, so one should compare the expressions of the effective Hamiltonians. The total Hamiltonian is $H^0_{ab}+V_{ab}+H^{\rm pl}_{ab}(\Omega)$, with $H^0_{ab}+V_{ab}$ being the Hamiltonian of the isolated pair of particles whereas the medium-dependent part of the Hamiltonian is denoted by $H^{\rm pl}_{ab}(\Omega)$. In the present paper this latter quantity is given by

$$H_{ab}^{\rm pl}(\Omega)=\Sigma_{ab}^{0}+\Sigma_{ab}^{\rm corr}(\Omega)=\Sigma_{a}^{\rm HF}+\Sigma_{b}^{\rm HF}+N_{ab}V_{ab}-V_{ab}+\Delta_{ab}^{R}(\Omega)+V_{ab}^{{\rm eff}\,R}(\Omega), \tag{49}$$

where Ω is to be understood as a parameter.

The differences consist in the following: (i) now there are no additional static parts beyond the Hartree-Fock level, and (ii) no division by Pauli-blocking terms occurs. Both things seem to be produced artificially in the former attempts by adopting a closed equation for the wrong quantity.

B. Limiting cases

It is interesting to study some limiting cases of our expressions. First, in the nondegenerate case the one-particle distribution functions in Eqs. (45) and (46) can be neglected. Thus the result is in agreement with the nondegenerate limit of the former approaches using the Shindo approximation [12,15,10,20].

The second important limiting case is that of statical screening. Following Zimmermann [20], we consider the case that the excitation energy into a pair of two free particles, $\epsilon_a(\mathbf{p}_1)+\epsilon_b(\mathbf{p}_2')-\Omega$, is small in comparison with the energy ω_1 occurring in the dielectric function. This could be a reasonable approximation for excited states. Then, from Eqs. (45) and (46), we obtain

$$\begin{aligned}\Delta_{ab}^{R}(\mathbf{p}_1\mathbf{p}_2,\mathbf{p}_1'\mathbf{p}_2',\Omega)&=(2\pi)^6\delta(\mathbf{p}_1-\mathbf{p}_1')\,\delta(\mathbf{p}_2-\mathbf{p}_2')\\&\times\int\frac{d^3q}{(2\pi)^3}\Bigg\{[V_{aa}^{S}(\mathbf{q},\Omega=0)-V_{aa}(\mathbf{q})]\\&\times\left[\pm f_a(\mathbf{p}_1+\mathbf{q})+\frac{1}{2}\right]\\&+[V_{bb}^{S}(\mathbf{q},\Omega=0)-V_{bb}(\mathbf{q})]\left[\pm f_b(\mathbf{p}_2+\mathbf{q})+\frac{1}{2}\right]\Bigg\},\end{aligned} \tag{50}$$

and, for the effective interaction term,

$$\begin{aligned}V_{ab}^{{\rm eff}\,R}(\mathbf{p}_1\mathbf{p}_2,\mathbf{p}_1'\mathbf{p}_2',\Omega)&=(2\pi)^3\delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_1'-\mathbf{p}_2')[V_{ab}^{S}(\mathbf{p}_1-\mathbf{p}_1',\Omega=0)\\&-V_{ab}(\mathbf{p}_1-\mathbf{p}_1')][1\pm f_a(\mathbf{p}_1)\pm f_b(\mathbf{p}_2)].\end{aligned} \tag{51}$$

Here it was used that $\mathrm{Im}\,\varepsilon^{-1}$ is an odd function, and that the even part of the Bose function $n_B(\omega)$ is $-\frac{1}{2}$. Further, one has

$$V_{ab}^{SR}(\mathbf{q},\Omega)=V_{ab}(\mathbf{q})\left\{1-\int\frac{d\omega_1}{\pi}\frac{\mathrm{Im}\,\varepsilon^{-1}(\mathbf{q},\omega_1)}{\Omega-\omega_1+i0}\right\}. \tag{52}$$

The terms in Eq. (50), containing single-particle distribution functions, and Eq. (51) can be combined with $\Sigma_{ab}^{\rm HF}$ and $N_{ab}V_{ab}$, respectively, to give functionals $\Sigma_{ab}^{\rm HF}\{V^S(\Omega=0\}$ and $N_{ab}V_{ab}^{S}(\Omega=0)$ of the screened potentials in the static limit. The remaining terms in Eq. (50) give a constant term.

The Dyson equation (41) can then be written

$$[\Omega-H_{ab}^{\rm eff}]G_{ab}^{R}(\Omega)=1, \tag{53}$$

with the effective plasma Hamilton operator

$$H_{ab}^{\rm eff}=H_{ab}^{0}+\sum_{c=a,b}\Sigma_{c}^{\rm HF}\{V^S(\Omega=0)\}+N_{ab}V_{ab}^{S}(\Omega=0)+\frac{1}{2}\sum_{c=a,b}\int\frac{d^3q}{(2\pi)^3}[V_{cc}^{S}(q,0)-V_{cc}(q)]. \tag{54}$$

This is in aggreement with the static limit found by Zimmermann [20].

Considering this effective Hamiltonian for a nondegenerate system, one can write (with $V_{ab}\rightarrow z_a z_b V$)

$$H_{ab}^{\rm eff}=H_{ab}^{0}+z_a z_b V(r)+z_a z_b[V^S(r,\Omega=0)-V(r)]+\frac{1}{2}[z_a^2+z_b^2][V^S(0,\Omega=0)-V(0)]. \tag{55}$$

Adopting for $V^S(r,\Omega=0)$ the statically screened Debye potential $V^D(r)=(e^2/r)\exp(-\kappa r)$, for the Hamiltonian in Eq. (55) one obtains

$$H_{ab}^{\rm eff}=H_{ab}^{0}+z_a z_b V^D(r)-\frac{1}{2}(z_a^2+z_b^2)\kappa e^2. \tag{56}$$

The two last terms combined give the well-known effective potential of Ecker-Weizel type [38], which has been used frequently in order to determine energies and wave functions of bound states in a plasma environment [15,10,39,40].

C. Effective wave equation and two-particle energies

Equation (41), which determines the two-particle propagator G_{ab}^{R}, was written down in an operator form. Using a representation one obtains a matrix equation. In order to solve this equation it is favorable to use a representation in which diagonal elements are the main contribution. Here we follow Kilimann *et al.* [15]; however, now it is not possible to achieve symmetric real and imaginary parts of the effective Hamiltonian simply by multiplying with factors $N_{ab}^{\pm 1/2}$. Therefore the Hamiltonian is split into Hermitian and anti-Hermitian parts. This leads to

$$[\Omega-H_{ab}^{\mathcal{H}}(\Omega)]G_{ab}^{R}(\Omega)-H_{ab}^{\mathcal{A}}(\Omega)G_{ab}^{R}(\Omega)=1, \tag{57}$$

with $H_{ab}^{\mathcal{H}}(\Omega)=H_a+H_b+V_{ab}+\Sigma_{ab}^{0\mathcal{H}}+\Sigma_{ab}^{{\rm corr}\,\mathcal{H}}(\Omega)$. The eigenvalue problem of the Hermitian part of the Hamiltonian reads

$$H_{ab}^{\mathcal{H}}(\Omega)|nP,\Omega\rangle=E_{nP}(\Omega)|nP,\Omega\rangle. \tag{58}$$

The eigenstates $|nP,\Omega\rangle$, where nP denote the quantum

numbers and Ω is a real parameter, can be used as a orthonormal basis. The eigenvalues $E_{nP}(\Omega)$ of this effective Schrödinger equation are not yet the spectrum of two-particle excitions [15]. The latter follows from the spectral function A_{ab}.

In the representation with respect to the eigenstates $|nP,\Omega\rangle$, Eq. (57) reads (conservation of center-of-mass momentum already taken into account)

$$[\Omega - E_{nP}(\Omega)]G^R_{nn'}(P,\Omega) - \sum_m H^A_{nm}(P,\Omega)G^R_{mn'}(P,\Omega) = \delta_{nn'}. \tag{59}$$

In the following it is assumed that nondiagonal matrix elements of the anti-Hermitian part of the effective Hamiltonian are small. Then Eq. (59) has the approximate solution

$$G^R_{nn'}(P,\Omega) = \frac{\delta_{nn'}}{\Omega + i0 - E_{nP}(\Omega) + i\Gamma_{nn}(P,\Omega)} + \frac{-i\Gamma_{nn'}(P,\Omega)(1-\delta_{nn'})}{[\Omega + i0 - E_{nP}(\Omega) + i\Gamma_{nn}(P,\Omega)][\Omega + i0 - E_{n'P}(\Omega) + i\Gamma_{n'n'}(P,\Omega)]}, \tag{60}$$

where it was introduced that $\Gamma_{nn'}(P,\Omega) = iH^A_{nn'}(P,\Omega)$. For the coherent part of the spectral function there follows

$$A_{nn}(P,\Omega) = \frac{2\Gamma_{nn}(P,\Omega)}{[\Omega - E_{nP}(\Omega)]^2 + \Gamma^2_{nn}(P,\Omega)}. \tag{61}$$

According to this equation, the spectrum of the two-particle excitations is given by the roots $\tilde{E}_{nP}$ of

$$\Omega = E_{nP}(\Omega), \tag{62}$$

whereas the damping is given by $\Gamma_{nn}(P,\tilde{E}_{nP})$ [15].

VII. SUMMARY AND CONCLUSION

Starting from the nonequilibrium BSE in the dynamically screened ladder approximation, we have derived a set of nonequilibrium Dyson equations for two-time, two-particle correlation functions. The two-time structure of these equations was achieved in an exact way using the semigroup properties of the ideal one-particle Green's functions. The price one has to pay for this simpler structure of the equation is that the two-particle self-energy in the Dyson equation now consists of irreducible diagrams in all orders with respect to the dynamically screened potential (in some sense this is similar to the transition from Feynman diagrams to Goldstone diagrams [31]). Irreducibility means here that a diagram cannot be cut with respect to a pair of single-particle lines which begin at equal times and end at equal times, i.e., two or more interaction potentials have some overlap in time.

For further considerations we have restricted ourselves to a two-particle self-energy in first order with respect to the screened potential. The algebraic structure of the equations is not affected by this approximation. It was shown that there is a set of equations for four two-time correlation functions. This generalizes the pair of Kadanoff-Baym equations for the one-particle correlation functions $g^{\gtrless}$ (g^{+-} and g^{-+}, respectively). In analogy to the single-particle case there is no closed equation for the correlation functions, but always a coupling to other correlation functions. Only for two certain functions, namely, $G^{R/A}_{ab}$, do there exist closed equations. Thus these functions are the two-particle generalization of the retarded (advanced) commutator Green's functions $g^{R/A}_a$ in the single-particle case, and just these functions describe the propagation of a pair of particles in the nonequilibrium many-particle system.

The case of thermodynamic equilibrium was considered in some detail in order to show the differences from former approaches. In former attempts [41,12] closed equations for the causal two-time two-particle Green's function were anticipated. These equations were enforced by the Shindo approximation. The expressions for the effective Hamiltonian were the same as those of the present paper, only for the case of a nondegenerate system. The agreement in this special case is easy to understand, taking into account that the difference between the functions used is of higher order in the density.

For arbitrary degeneracy there are clear differences between the former results and ours. In the present results there is no division by Pauli-blocking terms. The only intrinsic static contributions of the effective Hamiltonian (the two-particle self-energy) are the Hartree-Fock single-particle self-energies and the Pauli-blocked basic potential.

We can conclude that the proper generalization of the Kadanoff-Baym equations for two-particle functions is given by the system of equations (39) and (40). The algebraic structure of these equations was identified starting from a concrete approximation, the dynamically screened ladder equation. More general considerations of how the self-energy functions can be determined in higher approximations will be presented in a subsequent paper [27].

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APPENDIX: EVALUATION OF DYNAMICALLY SCREENED LADDER TERMS

The aim of this appendix is to show the evaluation of the lowest-order terms in the dynamically screened ladder equation. Single-particle self-energy contributions and interaction terms have to be treated on equal footing. Special attention is paid to the transformation into a structure involving two-

particle quantities which depend on two times only. The analysis is made here for the expansion of the function $g^<_{ab} = g^{++,--}_{ab}$. Similar considerations are possible for the other three functions $g^{+-,-+}_{ab}$, $g^{-+,+-}_{ab}$, and $g^{--,++}_{ab}$. This is sketched in Appendix A 3.

1. First-order contributions

There are three diagrams of first order with respect to the dynamically screened interaction V^S_{ab}; see Fig. 2. Two terms have single-particle self-energy insertions of particles *a* and *b*, respectively. The third one is a ladder diagram with one rung.

The first term with a self-energy insertion for particle *a* is given simply by [cf. Eq. 4]

$$I^{(1)}_1(t,t') = \int_{t_0}^{\infty} dt_1\, d\bar{t}_1 [g^<_{a,0}(t,t_1)\Sigma^A_a(t_1,\bar{t}_1)g^A_{a,0}(\bar{t}_1,t') + g^R_{a,0}(t,t_1)\Sigma^<_a(t_1,\bar{t}_1)g^A_{a,0}(\bar{t}_1,t') + g^R_{a,0}(t,t_1)\Sigma^R_a(t_1,\bar{t}_1)g^<_{a,0}(\bar{t}_1,t')]g^<_{b,0}(t,t'). \tag{A1}$$

In order to achieve the anticipated structure, one can use the semigroup properties for the correlation function $g^<_{b,0}(t,t')$. In the first term on the right hand side, for instance, it holds that $t_1 < \bar{t}_1 < t'$, enforced by the advanced functions $\Sigma^A_a(t_1,\bar{t}_1)\, g^A_{a,0}(\bar{t}_1,t')$. For this case, in Eq. (A1) we can use $g^<_{b,0}(t,t') = g^<_{b,0}(t,t_1)(-i)g^A_{b,0}(t_1,\bar{t}_1)(-i)g^A_{b,0}(\bar{t}_1,t')$. Treating the other two terms in a similar way, one obtains

$$\begin{aligned} I^{(1)}_1(t,t') = \int_{t_0}^{\infty} dt_1\, d\bar{t}_1 \{ & g^<_{a,0}(t,t_1)g^<_{b,0}(t,t_1)[(-i)\Sigma^A_a(t_1,\bar{t}_1)g^A_{b,0}(t_1,\bar{t}_1)](-i)g^A_{a,0}(\bar{t}_1,t')g^A_{b,0}(\bar{t}_1,t') \\ & + ig^R_{a,0}(t,t_1)g^R_{b,0}(t,t_1)[\Sigma^<_a(t_1,\bar{t}_1)g^<_{b,0}(t_1,\bar{t}_1)](-i)g^A_{a,0}(\bar{t}_1,t')g^A_{b,0}(\bar{t}_1,t') \\ & + ig^R_{a,0}(t,t_1)g^R_{b,0}(t,t_1)[i\Sigma^R_a(t_1,\bar{t}_1)g^R_{b,0}(t_1,\bar{t}_1)]g^<_{a,0}(\bar{t}_1,t')g^<_{b,0}(\bar{t}_1,t')\}. \end{aligned} \tag{A2}$$

This fits into the structure $\mathcal{G}^<_{ab}\Sigma^A_{ab}\mathcal{G}^A_{ab} + \mathcal{G}^R_{ab}\sigma^<_{ab}\mathcal{G}^A_{ab} + \mathcal{G}^R_{ab}\Sigma^R_{ab}\mathcal{G}^<_{ab}$. The term $I^{(2)}_1$ containing a self-energy insertion for the other particle of species *b* has a similar shape.

The third first-order term in the perturbation expansion is the ladder term. Each of the two vertices can have the Keldysh indices + and −. Thus one obtains the four terms

$$\begin{aligned} I^{(3)}_1(t,t') = i \int d\bar{t}_1\, d\bar{t}_2 [& g_{a,0}(t,\bar{t}_1)g_{b,0}(t,\bar{t}_2)V^S_{ab}(\bar{t}_1,\bar{t}_2)g^<_{a,0}(\bar{t}_1,t')g^<_{b,0}(\bar{t}_2,t') \\ & - g^<_{a,0}g_{b,0}V^{S>}_{ab}\bar{g}_{a,0}g^<_{b,0} - g_{a,0}g^<_{b,0}V^{S<}_{ab}g^<_{a,0}\bar{g}_{b,0} + g^<_{a,0}g^<_{b,0}\bar{V}^S_{ab}\bar{g}_{a,0}\bar{g}_{b,0}]. \end{aligned} \tag{A3}$$

The causal and anticausal Green's functions can be eliminated in favor of retarded and advanced Green's functions; see Eq. (3):

$$\begin{aligned} I^{(3)}_1 = {} & g^R_{a,0}g^R_{b,0}iV^S_{ab}g^<_{a,0}g^<_{b,0} + g^R_{a,0}g^<_{b,0}iV^{SR}_{ab}g^<_{a,0}g^<_{b,0} + g^<_{a,0}g^R_{b,0}iV^{SA}_{ab}g^<_{a,0}g^<_{b,0} + g^R_{a,0}g^<_{b,0}iV^{S<}_{ab}g^<_{a,0}g^A_{b,0} + g^<_{a,0}g^R_{b,0}iV^{S>}_{ab}g^A_{a,0}g^<_{b,0} \\ & + g^<_{a,0}g^<_{b,0}iV^{SR}_{ab}g^A_{a,0}g^<_{b,0} + g^<_{a,0}g^<_{b,0}iV^{SA}_{ab}g^<_{a,0}g^A_{b,0} + g^<_{a,0}g^<_{b,0}i\bar{V}^S_{ab}g^A_{a,0}g^A_{b,0}. \end{aligned} \tag{A4}$$

There are three classes of terms in the above equation: (i) terms ending with a product $g^<_{a,0}g^<_{b,0}$, (ii) terms beginning with one retarded function and ending with one advanced function, and (iii) terms beginning with $g^<_{a,0}g^<_{b,0}$.

The further procedure is presented in detail for the first term on the right-hand side of Eq. (A4). The causal screened potential V^S is expressed according to Eq. (7). The occurring Heaviside functions allow it to use the semigroup property in certain functions g^R and $g^<$, respectively, in the following manner:

$$\begin{aligned} & \int d\bar{t}_1\, d\bar{t}_2\, g^R_{a,0}(t,\bar{t}_1)g^R_{b,0}(t,\bar{t}_2)iV^S_{ab}(\bar{t}_1,\bar{t}_2)g^<_{a,0}(\bar{t}_1,t')g^<_{b,0}(\bar{t}_2,t') \\ & = \int d\bar{t}\, d\tilde{t}[ig^R_{a,0}(t,\bar{t})g^R_{b,0}(t,\bar{t})]\{V_{ab}\delta(\bar{t}-\tilde{t}) + [ig^R_{b,0}(\bar{t},\tilde{t})iV^{S>}_{ab}(\bar{t},\tilde{t})g^R_{a,0}(\bar{t},\tilde{t})] \\ & \quad + [ig^R_{a,0}(\bar{t},\tilde{t})iV^{S<}_{ab}(\tilde{t},\bar{t})g^R_{b,0}(\bar{t},\tilde{t})]\}[g^<_{a,0}(\tilde{t},t')g^<_{b,0}(\tilde{t},t')]. \end{aligned} \tag{A5}$$

Thus this term belongs to the anticipated structure $\mathcal{G}^R_{ab}\Sigma^R_{ab}\mathcal{G}^<_{ab}$.

Making the same analysis for all terms of $I_1^{(3)}$, one obtains

$$
\begin{aligned}
I_1^{(3)}(t,t') = & \int d\bar{t}\, d\tilde{t}\; i g^R_{a,0}(t,\bar{t}) g^R_{b,0}(t,\bar{t}) \{ V_{ab}\delta(\bar{t}-\tilde{t}) + i g^R_{b,0}(\bar{t},\tilde{t}) i V^{S>}_{ab}(\bar{t},\tilde{t}) g^R_{a,0}(\bar{t},\tilde{t}) + i g^<_{b,0}(\bar{t},\tilde{t}) i V^{SR}_{ab}(\bar{t},\tilde{t}) g^R_{a,0}(\bar{t},\tilde{t}) \\
& + i g^R_{a,0}(\bar{t},\tilde{t}) i V^{S<}_{ab}(\tilde{t},\bar{t}) g^R_{b,0}(\bar{t},\tilde{t}) + i g^<_{a,0}(\bar{t},\tilde{t}) i V^{SA}_{ab}(\tilde{t},\bar{t}) g^R_{b,0}(\bar{t},\tilde{t}) \} g^<_{a,0}(\tilde{t},t') g^<_{b,0}(\tilde{t},t') \\
& + \int d\bar{t}\, d\tilde{t}\; i g^R_{a,0}(t,\bar{t}) g^R_{b,0}(t,\bar{t}) \{ g^<_{b,0}(\bar{t},\tilde{t}) i V^{S<}_{ab}(\bar{t},\tilde{t}) g^<_{a,0}(\bar{t},\tilde{t}) + g^<_{a,0}(\bar{t},\tilde{t}) i V^{S>}_{ab}(\tilde{t},\bar{t}) g^<_{b,0}(\bar{t},\tilde{t}) \} (-i) g^A_{a,0}(\tilde{t},t') g^A_{b,0}(\tilde{t},t') \\
& + \int d\bar{t}\, d\tilde{t}\; g^<_{a,0}(t,\bar{t}) g^<_{b,0}(t,\bar{t}) \{ V_{ab}\delta(\bar{t}-\tilde{t}) + (-i) g^A_{a,0}(\bar{t},\tilde{t}) i V^{S<}_{ab}(\tilde{t},\bar{t}) g^A_{b,0}(\bar{t},\tilde{t}) + (-i) g^A_{a,0}(\bar{t},\tilde{t}) V^{SR}_{ab}(\tilde{t},\bar{t}) i g^<_{b,0}(\bar{t},\tilde{t}) \\
& + (-i) g^A_{b,0}(\bar{t},\tilde{t}) i V^{S>}_{ab}(\bar{t},\tilde{t}) g^A_{a,0}(\bar{t},\tilde{t}) + (-i) g^A_{b,0}(\bar{t},\tilde{t}) V^{SA}_{ab}(\bar{t},\tilde{t}) i g^<_{a,0}(\bar{t},\tilde{t}) \} (-i) g^A_{a,0}(\tilde{t},t') g^A_{b,0}(\tilde{t},t'). \qquad \text{(A6)}
\end{aligned}
$$

A comparison with structure (27) gives four additional terms for Σ^R_{ab} (Σ^A_{ab}), and two further terms of $\sigma^<_{ab}$. Altogether we obtain the expression (31) for Σ^R_{ab} and Eq. (33) for $\sigma^<_{ab}$.

2. Second-order contributions

According to Eqs. (27) and (28) the second-order terms should lead to diagrams with two self-energy insertions of first order with respect to V^S (reducible diagrams), as well as to diagrams with one self-energy insertion which is of second order. We will demonstrate this here for one typical term. The analysis of the ladder term with two rungs (cf. Fig. 2) leads, among many other terms, to the following contribution ($t_1, t_2, \bar{t}_1, \bar{t}_2$ are integration variables):

$$
I_2 = \int g^R_{a,0}(t,t_1) g^R_{b,0}(t,t_2) i V^S_{ab}(t_1,t_2) g^R_{a,0}(t_1,\bar{t}_1) g^R_{b,0}(t_2,\bar{t}_2) i V^S_{ab}(\bar{t}_1,\bar{t}_2) g^<_{a,0}(\bar{t}_1,t') g^<_{b,0}(\bar{t}_2,t'). \qquad \text{(A7)}
$$

The procedure to achieve a two-time structure is similar to that in Appendix A 1. According to Eq. (7), each causal function V^S consists of three terms, which leads to nine terms in Eq. (A7). All contributions containing at least one time-diagonal part are easily shown to be reducible. Therefore we concentrate on the others:

$$
\begin{aligned}
I_2' = & \int g^R_{a,0}(t,t_1) g^R_{b,0}(t,t_2) i [\Theta(t_1-t_2) V^{S>}_{ab}(t_1,t_2) + \Theta(t_2-t_1) V^{S<}_{ab}(t_1,t_2)] g^R_{a,0}(t_1,\bar{t}_1) g^R_{b,0}(t_2,\bar{t}_2) \\
& \times i [\Theta(\bar{t}_1-\bar{t}_2) V^{S>}_{ab}(\bar{t}_1,\bar{t}_2) + \Theta(\bar{t}_2-\bar{t}_1) V^{S<}_{ab}(\bar{t}_1,\bar{t}_2)] g^<_{a,0}(\bar{t}_1,t') g^<_{b,0}(\bar{t}_2,t'). \qquad \text{(A8)}
\end{aligned}
$$

These terms should fit into the structure $\mathcal{G}^R_{ab}\Sigma^R_{ab(2)}\mathcal{G}^<_{ab} + \mathcal{G}^R_{ab}\Sigma^R_{ab(1)}\mathcal{G}^R_{ab}\Sigma^R_{ab(1)}\mathcal{G}^<_{ab}$, where $\Sigma^R_{ab(2)}$ denotes the two-particle self-energy in second order, and $\Sigma^R_{ab(1)}$ are the first-order quantities identified in Appendix A 1.

Analyzing the expressions in Eq. (A8), we find that the ''mixed'' terms (with one $V^<$ and one $V^>$) are reducible. The terms containing two functions $V^>$ (or two functions $V^<$) lead to a reducible term for $t_2 > \bar{t}_1$ ($t_1 > \bar{t}_2$) and to an irreducible part for $t_2 < \bar{t}_1$ ($t_1 < \bar{t}_2$). This is shown in Fig. 4 in the form of diagrams. The reducible terms contain two-particle self-energy insertions of first order with respect to V^S. The second-order terms contributing to $\Sigma^R_{ab(2)}$ are given by

$$
\Sigma^R_{ab(2)}(t,t') = \int dt_1\, dt_2\, \Theta(t_1-t_2) g^R_{b,0}(t,t_2) V^{S>}_{ab}(t,t_2) g^R_{a,0}(t,t_1) g^R_{b,0}(t_2,t') V^{S>}_{ab}(t_1,t') g^R_{a,0}(t_1,t') + [11'a \leftrightarrow 22'b]. \qquad \text{(A9)}
$$

This term can be shown to be a vertex correction to the two-particle vertex.

The other second-order diagrams in Fig. 2 can be discussed in a similar way. The terms with two single-particle self-energy insertions for the same particle are reducible in any case. For the other two types of diagrams, there are reducible as well as irreducible parts. This is shown in Fig. 5. The first diagram in each row is a reducible one. The second diagram is not reducible, and it corresponds to a vertex correction term. The third diagram is not reducible as well, but is the first self-energy correction to the diagrams of the two-particle self-energy of first order (cf. Fig. 3).

Because we started from a ladder equation, we do not find all possible second-order terms contributing to the two-particle self-energy. Therefore we will restrict ourselves to the self-energy in first order with respect to the dynamically screened interaction.

3. Analysis for the other correlation functions

The analysis for the functions $g^{+-,-+}_{ab}$, $g^{-+,+-}_{ab}$, and $g^{--,++}_{ab}$ can be made in the same way as above. It is sketched here for $g^{+-,-+}_{ab}$. Consider the first rung diagram. Evaluation on the Keldysh contour gives, in analogy to Eq. (A4),

$$I_1^{(3)} = g^R_{a,0} g^R_{b,0} i V^S_{ab} g^<_{a,0} g^>_{b,0} + g^R_{a,0} g^<_{b,0} i V^{SR}_{ab} g^<_{a,0} g^>_{b,0} + g^<_{a,0} g^R_{b,0} i V^{SA}_{ab} g^<_{a,0} g^>_{b,0} + g^R_{a,0} g^>_{b,0} i V^S_{ab} g^<_{a,0} g^A_{b,0} + g^<_{a,0} g^R_{b,0} i \overline{V}^S_{ab} g^A_{a,0} g^>_{b,0} + g^<_{a,0} g^>_{b,0} i V^{SR}_{ab} g^A_{a,0} g^<_{b,0} + g^<_{a,0} g^>_{b,0} i V^{SA}_{ab} g^<_{a,0} g^A_{b,0} + g^<_{a,0} g^>_{b,0} i \overline{V}^S_{ab} g^A_{a,0} g^A_{b,0}. \quad \text{(A10)}$$

Again the two-time structure can be achieved, and the structure is (cf. Eq. 27)

$$g^{+-,-+(1)}_{ab} = \mathcal{G}^R_{ab}[V_{ab} + \Sigma^R_{ab}]\mathcal{G}^{+-,-+}_{ab} + \mathcal{G}^R_{ab}\sigma^{+-,-+}_{ab}\mathcal{G}^A_{ab} + \mathcal{G}^{+-,-+}_{ab}[V_{ab} + \Sigma^A_{ab}]\mathcal{G}^A_{ab}, \quad \text{(A11)}$$

with $\mathcal{G}^{+-,-+}_{ab} = g^<_{a,0} g^>_{b,0}$. The fourth and the fifth terms in Eq. (A10) contribute $g^>_{b,0} V^S_{ab} g^<_{a,0} + g^<_{a,0} \overline{V}^S_{ab} g^>_{b,0}$ to $\sigma^{+-,-+}_{ab}$. Together with the respective contributions from the diagrams involving single-particle self-energies, $\sigma^{+-,-+}_{ab}$ is then given by Eq. (36) with $\{\alpha,\beta,\gamma,\delta\} = \{+--+\}$.

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