Resistivity-independent dissipation of magnetohydrodynamic waves in an inhomogeneous plasma

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The heating of high temperature plasmas by magnetohydrodynamic (MHD) waves is one of the most interesting and challenging problems of plasma physics especially when the energy is injected into the system at length scales much larger than the dissipative ones. It has been conjectured that in two-dimensional MHD systems the possibility exists of establishing a state in which energy is dissipated at a rate that is independent of the Ohmic resistivity and that the time needed to reach such a state is finite and independent of resistivity as well. In this paper we prove that this is actually possible as a result of the nonlinear interaction of long-wavelength, ''small'' amplitude perturbations with a constant, inhomogeneous magnetic field, at least in the relatively moderate Lundquist number (magnetic Reynolds) range $100 \leqslant S \leqslant 3200$.
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I. INTRODUCTION

Plasmas, both in laboratories and in cosmic environments, are quasi-ideal systems, given the extremely large values of the corresponding magnetic Reynolds numbers. This would seemingly preclude any possibility of heating such plasmas via Ohmic dissipation. It must be remembered, however, that those values are computed by assuming that the spatial scale entering their definition is the ''large'' energy injection scale. However, the cascade of energy to smaller and smaller scales, down to the tiny ''dissipation'' scale, where the local Reynolds numbers are of the order of unity, is made possible by nonlinear processes, whose presence thus opens the possibility of actually heating plasmas to high temperatures. The investigation of the physical mechanisms capable of producing such a cascade and the determination of their dissipative efficiency has been an active subject of research for a number of years. Most of the literature deals with the energy cascade in the context of magnetic reconnection and magnetoydrodynamic (MHD) turbulence. By analogy with hydrodynamic turbulence theory, it is quite natural to assume that, in the presence of a three-dimensional (3D) MHD strongly nonlinear dynamics, the small scale formation is so rapid, that the dissipation rate remains constant when smaller and smaller values of the dissipative coefficients are considered, i.e., the dissipation rate is independent of the Reynolds and Lundquist numbers. On this ground, phenomenological models have been developed in different contexts, from space plasmas to laboratory plasmas [1–9]. Recently, a number of these simple models have been examined by the help of numerical simulations [10]. In this idealized (full periodic boundaries) 3D framework, the main conclusion is that it is reasonable to assume that the energy transfer is rapid enough to justify a dissipation rate independent of the dissipative coefficients. However, those models cannot replace detailed investigation of the MHD systems and in real systems, with different energy injecting mechanisms and nonperiodic boundaries, the above conclusions must be essentially considered an interesting conjecture.

Much of the effort of the numerical investigation of the nonlinear dynamics of the MHD equations has been performed in the 2D limit. Only recently, due to the impressive development of super-computers, it has been possible to start to simulate the full 3D MHD equations of a ''realistic'' model [39] where, moreover, the turbulence is quasi-two-dimensional.

The energy transfer and dissipative properties of a realistic 2D MHD system represent an interesting and still unresolved problem, since 2D MHD cannot be directly connected to 2D or 3D hydrodynamic results. Given the smallness of the dissipative coefficients in practically all cases of interest, the possibility that the energy dissipation tends to a finite limit when the Reynolds numbers tend to infinity is, of course, of capital importance. The existence of such a limit has been conjectured almost 20 years ago [11], but so far has not been explicitly proven, even if the possibility of generating a direct cascade with a corresponding extended inertial spectrum has been demonstrated by numerical simulations in the homogeneous case with large amplitude initial perturbations [12–14]. In the magnetic reconnection context, when the system is driven by the development of the tearing instability, i.e., by initial long-wavelength perturbations, a turbulence capable to strongly speed up dissipative effects at high Lundquist numbers can be generated by the tearing mode only in the presence of a 3D sheared magnetic field, while in 2D the tearing mode develops at large scales and does not generate turbulence [15]. Furthermore, numerical studies of the evolution of the instability of a sheet pinch have shown that nonlinear interactions are more likely to produce the coalescence of magnetic islands rather than efficiently excite small scale dissipative structures [16]. However, when a low level of turbulence is introduced into the system at the initial time, an inertial spectrum reminiscent of the homogeneous MHD turbulent case develops [17] similar to the driven reconnection case [18].

In this paper we propose to concentrate on one specific problem, that of the heating of the solar corona. On one hand this subject is of interest in itself, being one of the big unresolved problems of plasma astrophysics. On the other, it is sufficiently ''typical'' to be be a good paradigm for other situations, so that any progress in this particular field would equally well constitute an advancement in our knowledge of the behavior of plasmas. In essence the problem is the following. The solar corona, the outermost reach of the solar atmosphere, is a tiny, hot plasma ($T \simeq 10^6$ K) which mainly radiates in the soft x-ray range. High resolution observations have now shown that the solar corona is a rapidly evolving plasma where a large variety of spatial and temporal scales are present. A general consensus has been reached on the fact that the basic physical processes underlying energetic phenomena occur mainly on very small scales, well beyond the present and possibly the future observational capabilities. The first consequence of this situation is the lack of even a ''zero order'' realistic model and the presence of a number of ''conceptual'' models which, starting from the available data, try to give a rough scenario of the small scale dynamics and to derive from it all the possible consequences on mean quantities to fit the observational large scale constraints. All these models have some common ingredients. The primary energy source has been identified in the turbulent motions observed at lower levels in the solar atmosphere which are the direct consequence of the presence of an extended convection zone underneath the visible solar surface. The magnetic field plays a fundamental role both on the large scales, as a link between the photosphere and the corona, and on the small scales where it helps transferring energy by some mechanisms like, for example, resonant absorption and phase mixing of MHD waves, magnetic reconnection, nonlinear interaction, and so on. The strict connection between magnetic-field structure and coronal heating events has been experimentally established by satellite observations combined with magnetograms from space or from the ground (see Ref. [19], and references therein).

Two standard pictures are used in order to understand how photospheric energy is injected on dissipative scales. In the first one, known as quasistatic equilibrium evolution, it is claimed that since the typical time scale of photospheric motions is much longer than the Alfvén time, large scale structures like coronal loops evolve through a series of magnetostatic force-free equilibria since they have enough time to reorganize themselves after any external perturbation. This problem is strictly connected to that of the existence of ideal MHD singularities, [20–24]. During this ''equilibria walking,'' strong current sheets eventually develop near the separatrix and magnetic energy is released via magnetic reconnection. However, when a dynamical rather than quasistatic approach is applied to the evolution of such a configuration it turns out, at least in a 2D slab geometry, that ideal singularities (i.e. current sheets) are in reality not accessible since the Alfvén time becomes infinite near the separatrix [25]. In other words, the intensity of the current sheet cannot be made infinite in a finite time due to resistive effects; in particular, the current sheets grow on a characteristic time which scales as $S^{1/3}$, where $S = \tau_r / \tau_a$, the ratio of the resistive to the Alfvén times, is the Lundquist number.

Wave heating theories, on the contrary, consider the photospheric perturbations as a continuous source of MHD waves which propagate upwards and damp, mainly by transferring their energy to small scales as the result of the interaction with an inhomogeneous background field. In this contest, the most promising mechanism are phase mixing [26] and resonant absorption [27–32]. In the first one, the front wave becomes increasingly more corrugated during propagation due to the spatial variation of the phase speed; as a result, stronger and stronger transverse gradients are generated and the wave energy is dissipated in a characteristic time which scales as $S^{1/3}$. In the second one, energy is concentrated into a critical layer by a resonant mechanism; a normal mode analysis shows that the dissipation rate is then independent from the Lundquist number, but the transient time necessary to set up this dissipative mechanism is exceedingly long if only linear interactions are considered [33–35].

Some preliminary effects of nonlinearity on the evolution of a nonuniform field subject to moderate amplitude, long-wavelength perturbations have been investigated in [35]. In this paper the inhomogeneous background field is perturbed at $t=0$ and the system is kept energetically isolated from the outside. In the subsequent evolution, energy is drawn via nonlinear processes from the large scale field and transferred to smaller scales where it is dissipated and the initial field profile becomes increasingly more uniform. Because of the limited free energy available, the system is prevented from reaching an asymptotic regime where the dissipation remains constant in time. This behavior will be referred to as a dissipative decay. The results of this paper strongly suggest that the formation time of the current sheet, which is the resistive analog of an ideal singularity, could indeed be independent of the actual value of the dissipative coefficients. Although interesting, the validity of this conclusion is weakened by the fact that the values of the Lundquist number used in those simulations were quite low ($S \leqslant 10^3$) and that the predicted substantial restructuring of the background field that accompanies the dissipation does not seem to be supported by the observations.

In the present paper we try to circumvent the latter problem by studying the nonlinear evolution of an inhomogeneous 2D incompressible MHD system perturbed as in the previous case, but where the large scale field is assumed to stay the same at all times, as actually suggested by the observations. This is equivalent to the assumption that an unspecified external mechanism is at work to compensate the changes that would be otherwise imposed on the field profile as a result of the the processes responsible for the energy transfer to smaller scales and for its subsequent dissipation. Contrary to the case previously studied, the system is no longer energetically isolated from the outside and we refer to this case as a driven dissipative evolution. Notice that in this configuration the energy injection mechanism from the large scale equilibrium to the small dissipative scale is still given by the ''resonant'' interaction between the long-wavelength initial perturbation and the nonuniformity of the background field. We underline that the initial magnetic equilibrium is stable with respect to tearing modes, since no neutral lines are present at the initial time.

It will be shown in this work that, in typical solar corona conditions, i.e., small amplitude, long-wavelength perturbations interacting with an inhomogeneous magnetic field, it is possible to set up a nonlinear energy cascade capable to dissipate the energy of the equilibrium field at a rate independent of the dissipative coefficients. The characteristic time necessary to set up the cascade is also independent of these coefficients. As already mentioned, the basic physics of the nonlinear energy transfer from large to small scales at a rate sufficiently high to allow a resistivity-independent dissipation has already been investigated in the context of turbulence theory, but, to our knowledge, it has never been proved, in the 2D inhomogeneous case subject to initial small amplitude, coherent, large scale perturbations.

In the next section we introduce the model equations and the initial conditions imposed on the system. These equations are integrated numerically and the results are presented in Sec. III. Finally, conclusions are drawn in Sec. IV.

II. EQUATIONS AND NUMERICAL METHOD

To study the evolution of an initial perturbation in an inhomogeneous time-independent magnetic field, we consider the incompressible magnetohydrodynamic equations in the Elsasser formulation where, for the sake of simplicity, the Reynolds number is equal to the Lundquist number. To reduce the equations to a dimensionless form we choose to measure lengths, velocities, and densities with respect to the spatial scale of variation of the initial magnetic field $\overline{l}$, the average Alfvén speed v_a, and a typical density $\overline{\varrho}$, respectively. Times are then measured in units of $\overline{l}/v_a$. The dimensionless equations then read

$$\frac{\partial \mathbf{z}^{\pm}}{\partial t}=[\pm \mathbf{B}_0(x)-\mathbf{z}^{\mp}]\cdot \boldsymbol{\nabla}[\pm \mathbf{B}_0(x)+\mathbf{z}^{\pm}]-\nabla\Pi+\frac{1}{S}\nabla^2\mathbf{z}^{\pm}, \tag{1}$$

$$\boldsymbol{\nabla}\cdot \mathbf{z}^{\pm}=0, \quad z^{\pm}=\mathbf{v}\pm\mathbf{b}, \tag{2}$$

where $\mathbf{z}^{\pm}$ are the Elsasser variables and $\mathbf{v}$ and $\mathbf{b}$ are the velocity and the magnetic field fluctuations, respectively. In these equations $\mathbf{B}_0$ is the equilibrium field, Π is the total kinetic + magnetic pressure, and S is the Lundquist number. Equations (1) and (2) are integrated in the 2D domain (x,z) of dimension $[-L_x,L_x]\times[0,2\pi/k_{0,z}]$, where $L_x \geqslant 2\pi/k_{0,z}$, and $k_{0,z}$ is the wave number of the initial perturbation (see Ref. [35]).

The unidirectional, inhomogeneous equilibrium magnetic field is given by

$$\mathbf{B}_0(x)=[1+0.5\tanh(x)]\mathbf{e}_z, \tag{3}$$

and, at the initial time, we introduce a perturbation of the form,

$$z_x^{+}=a\sin(k_{0z}z), \quad z_x^{-}=z_z^{\pm}=0, \tag{4}$$

where $a=10^{-2}$ and $k_{0z}=0.03$. In the x inhomogeneous direction we use the following boundary conditions:

$$\frac{\partial z_z}{\partial x}=0, \quad \frac{\partial z_x}{\partial x}=-\frac{\partial z_z}{\partial z}, \tag{5}$$

and periodic boundary conditions are used in the z direction.

The numerical algorithm used to integrate the equations is the explicit *Adams-Bashford III* which is third order accurate in time and does not add numerical dissipation on the short wavelengths. We use fourth order finite differences in the x direction on a nonuniform grid and fast Fourier transform in the z direction. The *ABIII* algorithm is slightly modified since at each time step a Poisson equation must be solved (with particular care) in order to enforce the incompressibility condition. The numerical code is particularly efficient on massively parallel computers. A description of the algorithm can be found in [35]. For further details on the time scheme see also [36]. The typical resolution for the (x,z) numerical grid is (800×2048) points for $S\sim 800$.

We define the energy $E^{\pm}(t)$ and the dissipated power $W^{\pm}(t)$ of the fluctuations as

$$E^{\pm}(t)=\frac{1}{2L_x}\int_{-L_x}^{L_x}\epsilon^{\pm}(x,t)dx, \quad \epsilon^{\pm}=z_x^{2,\pm}+z_z^{2,\pm}, \tag{6}$$

$$z_x^{2,\pm}(x)=\sum_{k=1}^{N}|\zeta_{x,k}^{\pm}|^2, \quad z_z^{2,\pm}(x)=\sum_{k=1}^{N}|\zeta_{z,k}^{\pm}|^2.$$

$$W^{\pm}(t)=\frac{1}{2L_x}\int_{-L_x}^{L_x}w^{\pm}(x,t)dx, \tag{7}$$

$$w^{\pm}=\frac{1}{S}\sum_{k=1}^{N}\left[\left|\frac{\partial\zeta_{x,k}^{\pm}}{\partial x}\right|^2+\left|\frac{\partial\zeta_{z,k}^{\pm}}{\partial x}\right|^2+k^2|\zeta_{x,k}^{\pm}|^2+k^2|\zeta_{z,k}^{\pm}|^2\right].$$

Here $\zeta_{x,k}^{\pm}(x,t)$ and $\zeta_{z,k}^{\pm}(x,t)$ are the Fourier transforms in the z direction of the fluctuations $z_x^{\pm}(x,z,t)$ and $z_z^{\pm}(x,z,t)$, respectively. In the following we will use the quantity $z_{x,k}^{2,\pm}$ and $z_{z,k}^{2,\pm}$ as the averaged values in the inhomogeneous direction of $|\zeta_{x,k}^{\pm}|^2$ and $|\zeta_{z,k}^{\pm}|^2$, respectively, and $\theta_k^{\pm}=z_{x,k}^{2,\pm}+z_{z,k}^{2,\pm}$. Finally, we define the averaged dissipation rate of the perturbation as the ratio of the energy $E^{\pm}(t)$ over the dissipated power $W^{\pm}(t)$ of the fluctuations, $\Gamma^{\pm}=W^{\pm}/E^{\pm}$.

By assuming an exponential decay of the fluctuation energy as $\sim\exp(-k^2t/S)$, where k is the largest wave number (without distinction between the parallel and the transverse one), we estimate the dissipative characteristic time at the scale $\lambda=2\pi/k$ as $t_d[k(t),S]\simeq S/k^2$.

III. NUMERICAL RESULTS

In Fig. 1 we plot the fluctuating energy E^{+}, the dissipated energy W^{+} and Γ^{+} vs time for different values of the Lundquist number, $S=100$, 200, 400, 800, 1600, 3200. These quantities are normalized with $E^{+}(t=0,S=100)$, $W^{+}(t=0,S=100)$, $\Gamma^{+}(t=0,S=100)$, respectively. In the top panel, increasing values of the curves at $t=100$ correspond to larger Lundquist numbers, while in the middle and bottom panels increasing values of the curves at $t=0$ correspond to lower Lundquist numbers. Notice that the curves corresponding to $S=1600,3200$ stop at nearly $t=100$ due to numerical grid resolution problems.

In the top panel two features are worth noticing. The first is that, for $t<100$, the energy of the fluctuations increases with time due to an energy transfer from the free (inhomogeneous) energy of the equilibrium field to the fluctuations (see also [35,37,38]). The second is that the differences between the curves corresponding to different values of S tend to disappear as S increases. This is a clear indication that the energy transferred to the fluctuations will become practically independent of S starting from a value of S not much

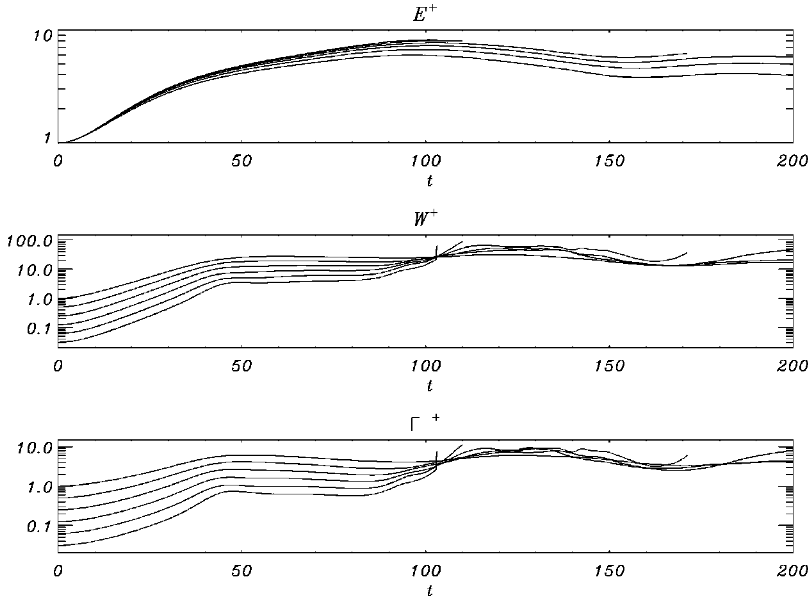

FIG. 1. The time evolution of the energy of the fluctuations E^+, the dissipated power W^+ and their ratio $\Gamma^+ = W^+/E^+$. These quantities are normalized with $E^+(t=0,\ S=100)$, $W^+(t=0,\ S=100)$, $\Gamma^+(t=0,\ S=100)$, respectively.

larger than a few units in 10^3. It is also important to notice that the total amount of energy transferred from the equilibrium to the fluctuations is a small fraction of the energy initially stored in the large scale equilibrium field, $\Delta E/E \sim 10^{-3}$.

In the middle and bottom panels, we observe that the evolution of the system is characterized by three distinct phases, each of them with a characteristic time independent of the Lundquist number. We define these phases as the linear resonant phase $0<t<45$, the nonlinear phase $45<t<90$, and the strongly nonlinear phase $t>90$.

A. Linear regime

In the initial phase $t<45$, the nonlinear interactions are negligible with respect to the linear ones since the amplitude and the gradients of the initial perturbation are ''small'' [see Eq. (4)]. In such conditions, the dynamics of a long-wavelength perturbation, $k \ll 1$, in a inhomogeneous medium is controlled by resonant absorption, while phase mixing is inefficient, at least for times shorter than the period of the wave, $t<2\pi/k_0 \simeq 200$ (see Ref. [35]). In the left frame of Fig. 2 we plot $z_z^{2,+}$ vs x at the end of the linear phase, $t=40$, for $S=800$. This picture shows that the resonant process concentrates the energy inside the inhomogeneous region close to the maximum of the equilibrium magnetic field gradient. As a consequence, the local amplitude of the wave is enhanced by some order of magnitude and larger and larger spatial gradients are generated in the inhomogeneous direction [notice that the initial perturbation is flat in the x direction, see Eq. (4)]. On the other hand, no significant energy transfer towards small scales occurs in the parallel z direction, so that the typical parallel wave number of the fluctuations remains of the order of the initial one, $k_z \simeq k_{0z}$. Therefore, at the end of the resonant linear phase we get $k_x \gg k_z$; this result is not surprising since in the z direction the plasma is assumed to be perfectly homogeneous.

In the right frame of Fig. 2 we show the computed values of Γ^+ at the end of the linear phase, $t=40$ for different values of S (stars), together with a linear best fit (continuous line) whose slope turns out to be $-7/12$. At $t=0$ the same linear fit would have a slope of -1. The $-7/12$ scaling law remains unaltered until the strongly nonlinear phase starts, i.e., up to $t \sim 90$.

B. Nonlinear regime

At the end of the linear resonant phase, the amplitude of the wave in the inhomogeneous region has grown by approximately two orders of magnitude becoming comparable to the equilibrium field, so that nonlinear interactions come into play.

In Fig. 3 we show the energy spectrum averaged along the direction of nonuniformity, θ_k^+, as a function of the parallel wave number k_z, during the resonant, nonlinear, and strongly nonlinear phases for two cases, $S=200$ and $S=800$. In this figure the energy spectrum of the initial perturbation, a single point at $k=k_{0,z}$, see Eq. (4), is indicated by a star. Notice that the dissipative characteristic time of the initial perturbation is $t_d(k=k_{0,z}) \simeq 10^3 S$.

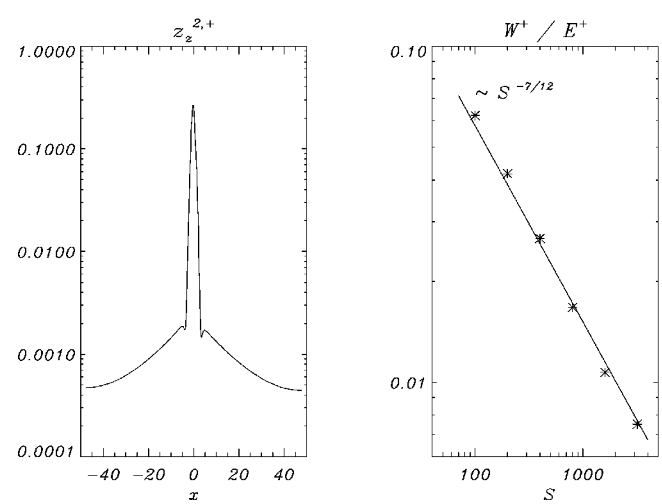

FIG. 2. Left frame: $z_z^{2,+}$ vs x at $t=40$, $S=800$; right frame: Γ^+ vs S (stars) at the end of the linear phase, $t=45$, and a best fit curve (continuous line), $\sim S^{-7/12}$.

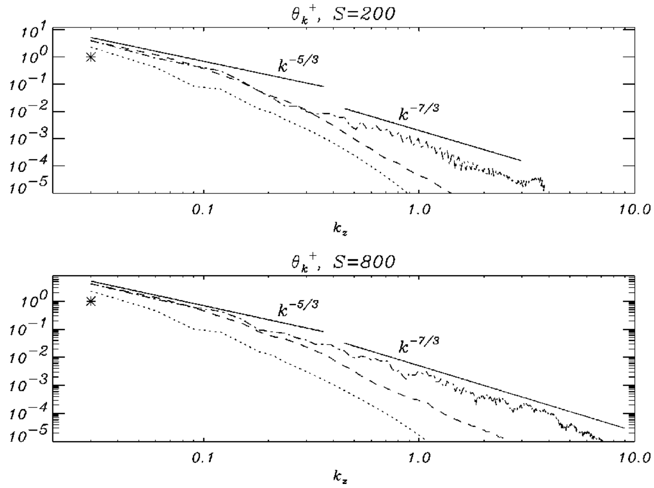

FIG. 3. The energy spectrum θ_k^+ vs the parallel wave number k_z at $t=45$ (points), $t=80$ (dashed), and $t=120$ (dotted-dashed) for $S=200$, first frame, and $S=800$, second frame. The star represents the energy spectrum of the initial perturbation in both pictures. All these curves have been normalized with $\theta_k^+(t=0)$.

At the end of the resonant phase (dotted line), the energy spectrum in terms of k_z is an exponential rather than a power law, which means that during the linear phase no energy is efficiently transferred on small scales along the homogeneous direction. In the subsequent evolution $t\geqslant 50$, due to the resonant amplification of the amplitude of the perturbation, nonlinear interactions become efficient and the energy starts to cascade also in the homogeneous direction. This is shown by the $-5/3$ power law for $0.03\leqslant k_z\leqslant 0.2$. In this regime, $t\simeq 80$ (dashed line), the energy is dissipated at wave numbers $k_d\leqslant S^{1/2}$. In fact, as discussed in Ref. [35], the presence of an inhomogeneity in the background magnetic field is responsible for a speed up of the small scale formation process in the inhomogeneous direction as soon as phase mixing becomes ''efficient,'' i.e., for $k\sim 1\gg k_{0,z}$. In this situation, the characteristic time necessary to transfer the energy on the scale $k\sim 1$ equals the characteristic dissipative time by phase mixing [37]. Therefore, the sink in the homogeneous direction is moved at lower values of k_z with respect to a pure homogeneous 2D case, where the energy is dissipated at $k_d^{hom}\simeq S^{1/2}\simeq 10-30$. At higher values of k_z the slope of the spectrum becomes slightly steeper, $\propto k_z^{-7/3}$. We cannot offer a physical explanation for this particular value of the spectral index, even if we think that it should be connected with the existence of another cascade along the direction of nonuniformity.

To summarize the evolution of the system up to $t\simeq 90$, we notice that the setting up of the ''anisotropic'' energy cascade is able (i) to transfer and to dissipate at a constant rate (as shown in Fig. 1) the energy stored in the large scale field and (ii) to substantially speed up the dissipation rate that now scales as $S^{-7/12}$, to be compared with an initial S^{-1} dependence. The characteristic time needed to reach the nonlinear regime, $t\simeq 50$, is also independent of S. This result is obtained in spite of the very long initial wavelength of the perturbation with respect to the equilibrium gradient and of the smallness of the initial amplitude of the perturbation with respect to the equilibrium. However, even if weaker than before, the dependence of the dissipation rate from the Lundquist number is still too strong to consider this as a possible heating mechanism for astrophysical plasmas whose typical Lundquist numbers are of the order of $10^{12}-10^{14}$.

TABLE I. From left to right: the value of the Lundquist number used in the simulation, the minimum and mean value of the energy of the fluctuations, the minimum and mean value of the dissipated power, and the minimum and mean value of their ratio. The minimum values of these quantities correspond to their respective values at $t=0$, while the mean values $\langle E^+\rangle$, $\langle W^+\rangle$, and $\langle\Gamma^+\rangle$ are time averaged in the interval $120\leqslant t\leqslant 160$ (see Fig. 1) of $E^+(t)$, $W^+(t)$, and $\Gamma^+(t)$, respectively.

S	E^+_{min}	$\langle E^+\rangle$	W^+_{min}	$\langle W^+\rangle$	Γ^+_{min}	$\langle\Gamma^+\rangle$
200	1.0	5.2	0.5	34	0.5	6.5
400	1.0	5.7	0.25	36	0.25	6.2
800	1.0	6.3	0.125	48	0.125	7.4

C. Strongly nonlinear regime

The existence of a strong nonlinear phase is the feature of this paper with respect to our previous work [35]. We recall that in that paper we have followed the dissipative decay of the same background field including the feedback action induced on the equilibrium field by the nonlinear interactions; the field gradient was progressively smoothed out with a corresponding reduction of the energy injection rate from the equilibrium to the fluctuations (see also [37]). Here, as already explained in the Introduction, we study the case of a driven dissipative evolution of a nonuniform MHD state by assuming that the large scale x-dependent magnetic field is forced by an external energy source to keep the same gradient for a time much longer than that needed to reach a stationary state where the energy injected into the system is dissipated by Ohmic resistivity (see Fig. 1, $t>100$). As already remarked, the total energy necessary to sustain the equilibrium field is only a small fraction of the initial energy. As a result, a new regime appears as clearly shown in the middle and bottom panels of Fig. 1. It starts after a characteristic ''charging'' time, $t_{charg}\simeq 90$, independent of S, and it is characterized by having the dissipation power and dissipation both independent of S.

In order to allow a better appreciation of this result, in Table 1 we list the value of the Lundquist number used in the simulation, the minimum and mean value of the energy of the fluctuations, the minimum and mean value of the dissipated power, and the minimum and mean value of their ratio. The minimum values of these quantities correspond to their respective values at $t=0$, while the mean values $\langle E^+\rangle$, $\langle W^+\rangle$ and $\langle\Gamma^+\rangle$ are time averages in the interval $120\leqslant t\leqslant 160$ (see Fig. 1) of $E^+(t)$, $W^+(t)$ and $\Gamma^+(t)$, respectively. As discussed in Sec. III, for ''low'' Lundquist numbers, part of the energy injected towards small scales is dissipated before entering the strongly nonlinear regime. As a result, also the values of $\langle W^+\rangle$ slightly depend on the Lundquist number, while this effect is less evident on the values of $\langle\Gamma^+\rangle$.

The values of $\langle\Gamma^+\rangle$ reported in Table I together with the results of Fig. 1 constitute, to our knowledge, the first direct evidence of a resistivity-independent dissipation rate in a 2D MHD system, although this feature had been already anticipated as a conjecture [11]. This result is obtained in spite of

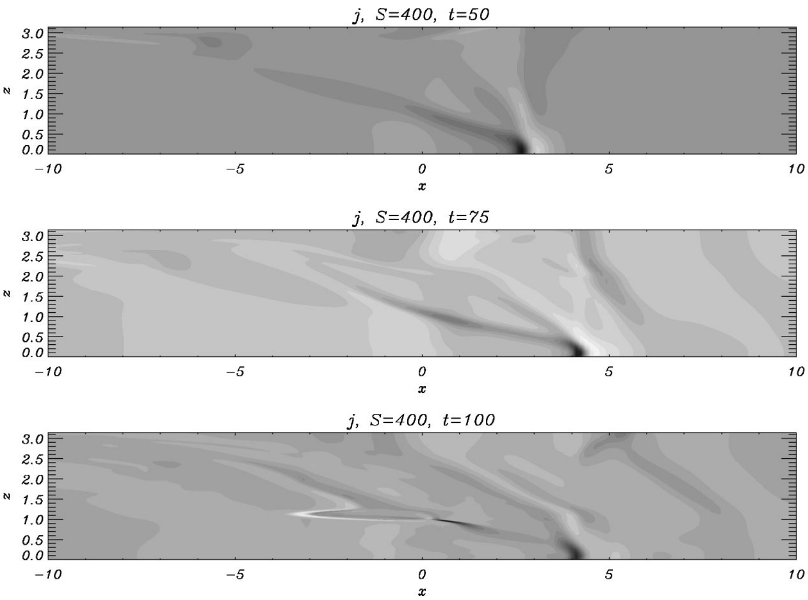

FIG. 4. The current (shaded contours) at $t=50$, 75, 100, $S=400$. Lighter/darker regions correspond to positive/negative values.

the fact that the amplitude of the initial perturbation is very small with respect to the background magnetic field. A confirmation of our findings for Lundquist numbers higher than those used here, $S_{max}\leqslant 1000$, would be highly desirable.

In Fig. 4 we show the shaded contours of the current in the $(-10,10)\times(0,\pi)$ domain at three different time instants, $t=50,75,100$, corresponding to the three distinct physical regimes, the linear, the nonlinear, and the strongly nonlinear one, respectively. In this figure $S=400$, but the qualitative main features and current structures are the same in all runs. In the first frame we observe close to $x=3$ the generation of a resonant dissipative layer. Then, in the next phase (second frame) a current sheet is formed which eventually develops into a very thin, very strong current layer (third frame) located at $-4<x<-2$, $0.8<z<1.2$, characterized by the presence of a neutral line where strong dissipation occurs. These findings demonstrate that the establishment of a strong nonlinear dissipative regime corresponds physically to the formation of current sheets and neutral lines.

The presence of a magnetic neutral line is clearly seen in Fig. 5 (with $S=800$) where the structure of the magnetic field is superimposed on the plot of the contours of the current density. Notice that the initial magnetic configuration had no neutral lines, see Eq. (3), but only acted as a support for the wave motions.

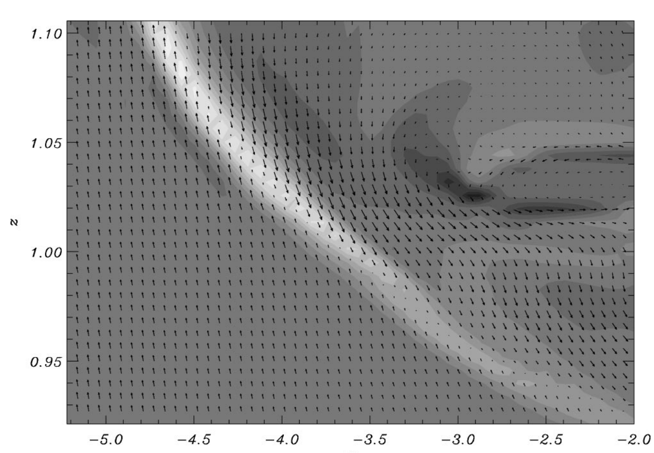

FIG. 5. The current (shaded contours) and the magnetic field (arrows) at $t=120$, $S=800$. Lighter/darker regions correspond to positive/negative values.

IV. CONCLUSION

In this paper we studied the energy transfer in a dissipative 2D MHD system. The initial setup is intended to mimic situations typical of the solar atmosphere, where energy is continuously fed into the system by some turbulent velocity field. Admittedly, the degree of realism of the proposed model is rather poor, but a detailed description of the heating of the actual solar corona was outside of the scope of the paper. Our aim was to present a simple, but hopefully instructive, example of the dynamics of energy transfer in a driven, inhomogeneous 2D MHD system.

The initial configuration represents a typical large scale magnetic field fed by some unspecified external energy source which maintains the equilibrium for times longer than the relatively few Alfvèn times necessary to set up the strongly dissipative regime. This configuration is subject to a long-wavelength perturbation whose amplitude is much smaller than the value of the equilibrium field, as it is the case for typical perturbations induced by photospheric motions in the solar atmosphere. The capability of 2D MHD systems of developing an extended inertial spectrum responsible for a ‘‘rapid’’ energy transfer towards the small dissipative scales was already known and had already been demonstrated by a number of other MHD simulations [13,14,17]. On the other hand, our initial configuration differs from those usually adopted in MHD simulations, where the energy is injected either by the development of resistive instabilities, or by using ‘‘large’’ amplitude initial perturbations, which rapidly evolve toward a turbulent state.

The most interesting result presented here concerns the possibility of tapping the energy contained in the equilibrium field. This is made possible by the development of a resonant mechanism capable of extracting the energy from the large scale background field and of injecting it on smaller and smaller scales, with the wave (i.e., the initial perturbation) acting mostly as a catalyst. As soon as the resonance did concentrate enough energy in the resonant layer, nonlinear interactions come into play and the energy starts to cascade also in the homogeneous direction. Eventually, current sheets are formed in a finite time. The corresponding magnetic field is characterized by the presence of a neutral line. In these current sheets the energy is dissipated at a rate independent of the Lundquist number, at least for the relatively moderate values of S used here. The characteristic time necessary to reach this resistivity independent regime, turns out to be independent of the Lundquist number as well.

It is worth noting that the resonant mechanism observed in our simulations is driven by the interaction of a coherent large scale wave mode with the equilibrium inhomogeneity, a situation reminiscent of the case of the tearing mode where, in order to generate a well developed inertial spectrum, an initial turbulent ''background'' is needed [17]. On the other hand, since no noise is introduced at the initial time in our simulations, we must conclude that the resonant process in the nonlinear stage is much more efficient in injecting the energy towards smaller and smaller scales than the tearing mode.

As this property was already been suggested by our previous simulations [35] in a different situation, it is quite apparent that the independence of the formation time of current sheets on the actual value of the dissipative coefficients is not related to the details of the energy injection mechanism, at least for the values of S used in these simulations.

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