Resonant activations for a fluctuating barrier system driven by dichotomous noise and Gaussian white noise

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We consider the escape over a fluctuating barrier in the presence of a dichotomous noise and a Gaussian white noise. It is shown that the mean first passage time (MFPT) over the fluctuating barrier displays two resonant activations. One is the resonant activation of the MFPT as a function of the flipping rate of the fluctuating potential barrier; the other is the resonant activation of the MFPT as a function of the transition rate of the dichotomous noise. In addition, we find that the dichotomous noise can weaken the former resonant activation, but enhance the latter one. By further study, we find that, when the fluctuating potential barrier is driven by two or more dichotomous noises, there are three or more resonant activations. [S1063-651X(99)00608-X]

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Noise-induced nonequilibrium phenomena in nonlinear systems have recently attracted a great deal of attention in a variety of contexts [1]. In general, these phenomena involve a response of the system that is not only produced or enhanced by the presence of noise, but that is optimized for certain values of the parameters of the noise. One example is the phenomenon of stochastic resonance [2], wherein the response of a nonlinear system to a signal is enhanced by the presence of noise and maximized for certain values of the noise parameters. Another is the ''Brownian motors,'' wherein intrinsically unbiased Brownian motion in stochastic spatial periodic potentials with spatial asymmetry or noise asymmetry leads to a systematic drift motion whose magnitude and even direction can be tuned by parameters of the noise [3]. A third is the recent discovery of a reentrant noise-induced phase transition that is only observed for certain finite ranges of noise parameters [4]. A fourth such phenomenon, the one of interest to us in this paper, has been called ''resonant activation'' and was first identified by Doering and Gadoua [5] and further studied by a number of other authors [6–16]. Here the mean first passage time (MFPT) of a particle driven by (usually white) noise over a fluctuating potential barrier exhibits a minimum as a function of the parameter of the fluctuating potential barrier (usually the flipping rate of the fluctuating potential barrier).

Earlier studies of the activation of the MFPT over fluctuating potentials were restricted to limiting cases, i.e., slow [17] or fast [17,18] barrier fluctuations, or small fluctuations [19]. Owing to using approximate treatments in Refs. [17–19], the resonant activation cannot be observed. Recently, in Refs. [5–16], the authors reported results concerning the escape time (i.e., MFPT) over a fluctuating potential in the absence of approximate treatments as in Refs. [17–19]. They revealed a resonant activation (RA) of the MFPT as a function of the potential fluctuation flipping rate.

All of the above work for the RA of the MFPT over a fluctuating potential barrier has been focused on systems driven only by a Gaussian white noise. One unavoidably wants to ask the question, i.e., if the systems are driven by a Gaussian white noise and a dichotomous noise simultaneously, how is the situation? In this paper, we shall study the escape time over a fluctuating potential barrier in the presence of a Gaussian white noise and a dichotomous noise. For simplicity and convenience, we take the fluctuating potential barrier as a piecewise linear one depicted in Fig. 1. In our study, we will find a new phenomenon; i.e., there are two RA's, which are, respectively, for the MFPT a function of the flipping rate of the fluctuating potential barrier and for the MFPT a function of the transition rate of the driving dichotomous noise.

The Langevin equation of the model studied by us is (in dimensionless form)

$$\dot{x} = -V'(x,t) + \xi(t) + \eta(t), \tag{1}$$

where V' denotes the derivative of the potential V (depicted in Fig. 1) with respect to x. $\xi(t)$ is a Gaussian white noise with zero mean and correlation function $\langle \xi(t)\xi(t')\rangle = 2D\delta(t-t')$. $\eta(t)$ is a dichotomous noise with zero mean and correlation function $\langle \eta(t)\eta(t')\rangle = (D/\tau)\exp[-|t-t'|/\tau] = a^2\exp[-\lambda|t-t'|]$. Here $\eta(t)$ has two values $\pm a$ $(a>0)$, and $1/(2\tau)=\lambda/2$ is the transition rate of $\eta(t)$ from a to $-a$ or vice versa. The potential $V(x,t)$ is piecewise linear and fluctuating (see Fig. 1).

The master equation of Eq. (1) is [20,21]

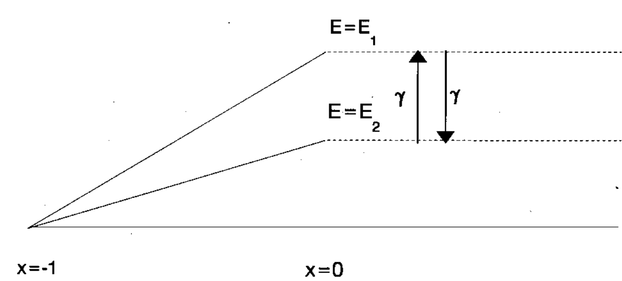

FIG. 1. The fluctuating barrier $V(x,t)$ in Eq. (1).

$$\partial_t \begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{pmatrix} = \begin{pmatrix} \hat{L}_1 & \lambda & \gamma & 0 \\ \lambda & \hat{L}_2 & 0 & \gamma \\ \gamma & 0 & \hat{L}_3 & \lambda \\ 0 & \gamma & \lambda & \hat{L}_4 \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{pmatrix}, \quad (2)$$

where $\hat{L}_1 = -\gamma - \lambda + \partial_x(E_1 - a) + D\partial_x^2$, $\hat{L}_2 = -\gamma - \lambda + \partial_x(E_1 + a) + D\partial_x^2$, $\hat{L}_3 = -\gamma - \lambda + \partial_x(E_2 - a) + D\partial_x^2$, and $\hat{L}_4 = -\gamma - \lambda + \partial_x(E_2 + a) + D\partial_x^2$, and $P_1 = P(x,t,E_1,a)$, $P_2 = P(x,t,E_1,-a)$, $P_3 = P(x,t,E_2,a)$, and $P_4 = P(x,t,E_2,-a)$. $P_1 = P(x,t,E_1,a)$ represents that the particle is at x, the potential is in $V = E_1$ configuration, and the dichotomous noise is in $\eta(t) = a$ configuration. There is the same understanding for P_2, P_3, and P_4. γ denotes the flipping rate of the fluctuating barrier. We start with the particle at the bottom $(x = -1)$, so the initial condition is $\Sigma_{i=1}^4 P_i(x,0) = \delta(x+1)$. The boundary conditions for the reflecting $(x = -1)$ and absorbing $(x = 0)$ boundaries, respectively, are $\partial_x P_i(x,t)|_{x=-1} = 0$ and $P_i(x,t)|_{x=0} = 0$.

The equations of the MFPT's for Eqs. (2) are (see Appendix A)

$$[-\gamma - \lambda - (E_1 - a)\partial_x + D\partial_x^2]T_1 + \lambda T_2 + \gamma T_3 + 1 = 0,$$

$$[-\gamma - \lambda - (E_1 + a)\partial_x + D\partial_x^2]T_2 + \lambda T_1 + \gamma T_4 + 1 = 0,$$

$$[-\gamma - \lambda - (E_2 - a)\partial_x + D\partial_x^2]T_3 + \lambda T_4 + \gamma T_1 + 1 = 0,$$

$$[-\gamma - \lambda - (E_2 + a)\partial_x + D\partial_x^2]T_4 + \lambda T_3 + \gamma T_2 + 1 = 0, \quad (3)$$

where T_i ($i = 1$, 2, 3, and 4) is the MFPT corresponding to the probability density P_i. The absorbing boundary condition and the reflecting boundary condition of Eq. (3) are $T_i(0) = 0$ and $\partial_x T_i(-1) = 0$ ($i = 1$, 2, 3, and 4), respectively. The MFPT for a particle over the fluctuating barrier that starts at the bottom $(x = -1)$ is $T = \Sigma_{i=1}^4 T_i(-1)$. Taking $\partial_x T_i = s_i$ ($i = 1$, 2, 3, and 4), we derive

$$\partial_x \begin{pmatrix} T_1 \\ s_1 \\ T_2 \\ s_2 \\ T_3 \\ s_3 \\ T_4 \\ s_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\gamma+\lambda}{D} & \frac{E_1 - a}{D} & -\frac{\lambda}{D} & 0 & -\frac{\gamma}{D} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{\lambda}{D} & 0 & \frac{\gamma+\lambda}{D} & \frac{E_1 + a}{D} & 0 & 0 & -\frac{\gamma}{D} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -\frac{\gamma}{D} & 0 & 0 & 0 & \frac{\gamma+\lambda}{D} & \frac{E_2 - a}{D} & -\frac{\lambda}{D} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{\gamma}{D} & 0 & -\frac{\lambda}{D} & 0 & \frac{\gamma+\lambda}{D} & \frac{E_2 + a}{D} \end{pmatrix} \begin{pmatrix} T_1 \\ s_1 \\ T_2 \\ s_2 \\ T_3 \\ s_3 \\ T_4 \\ s_4 \end{pmatrix} + \begin{pmatrix} 0 \\ -\frac{1}{D} \\ 0 \\ -\frac{1}{D} \\ 0 \\ -\frac{1}{D} \\ 0 \\ -\frac{1}{D} \end{pmatrix}. \quad (4)$$

When $E_1 + E_2 \neq 0$, the solution of Eq. (4) is (see Appendix B)

$$s_i = \sum_{j=1}^{7} k_j^{(i)} A_j^{(1)} \exp(\lambda_j x) + \frac{4}{E_1 + E_2},$$

$$T_i = \sum_{j=1}^{7} \frac{k_j^{(i)} A_j^{(1)}}{\lambda_j} \exp(\lambda_j x) + B_8^{(1)} + F_i + \frac{4}{E_1 + E_2} x, \quad (5)$$

where $i = 1$, 2, 3, and 4, λ_j ($i = 1, 2, \ldots, 7$) are seven independent nonzero eigenvalues of the matrix of the homogeneous part about T_i and s_i ($i = 1$, 2, 3, and 4) in Eq. (4). The coefficients $k_j^{(i)}$ and F_i ($i = 1$, 2, 3, and 4) have been given in Appendix B. Substituting Eq. (5) into the boundary conditions $T_i(0) = 0$ and $s_i(-1) = 0$ ($i = 1$, 2, 3, and 4), one can obtain eight linear algebraic equations for $A_j^{(1)}$ ($j = 1, 2, \ldots, 7$) and $B_8^{(1)}$. From these linear algebraic equations, one can derive $A_j^{(1)}$ and $B_8^{(1)}$. The MFPT for a particle over the fluctuating barrier is

$$T = \sum_{i=1}^{4} T_i(-1)$$

$$= \sum_{i=1}^{4} \sum_{j=1}^{7} \frac{k_j^{(i)}}{\lambda_j} A_j^{(1)} e^{-\lambda_j} + 4B_8^{(1)} + \sum_{i=1}^{4} F_i - \frac{8}{E_1 + E_2}. \quad (6)$$

Here, the condition for validity of formula (6) is $E_1 + E_2 \neq 0$.

By numerical calculus based on Eq. (6) we find that for Eq. (1) there are two RA's for the MFPT. One is the RA of the MFPT as a function of the flipping rate γ of the fluctuating potential barrier $V(x,t)$ (the first RA); the other is the RA of the MFPT as a function of the transition rate λ of the

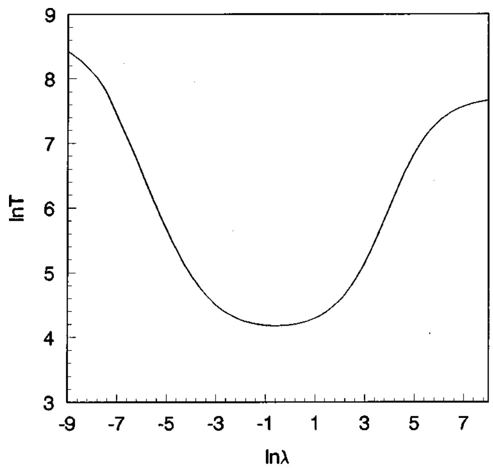

FIG. 2. The ln of the MFPT over the fluctuation barrier versus the ln of the dichotomous noise transition rate with $\gamma=10$, $E_1=10$, $E_2=14$, $a=6$, and $D=1$.

dichotomous noise $\eta(t)$ (the second RA). The first RA is the same as that proposed by Doering and Gadoua [5], and studied in Refs. [6–16]. In order to avoid unnecessary repetition we do not present a figure that is basically similar to the one in Refs. [5–16]. In Fig. 2 we plot the behavior of the ln of the MFPT numerically based on Eq. (6) with respect to the ln of the dichotomous noise transition rate λ. The figure shows that there is a RA (the second RA) in the dynamics of the MFPT with an increase of the transition rate λ of the dichotomous noise. A reason for this RA happening here is given below (the reason for the first RA was given in Ref. [5]). The resonance in Fig. 2 occurs when the crossing takes place with the dichotomous noise most likely in the $-a$ configuration. Now the MFPT has a local minimum for the dichotomous noise transition rate on the order of the inverse of the time required to cross the fluctuation barrier with the dichotomous noise in $-a$ configuration. A point (1) marked in Fig. 2 is the point where the transition time equals the MFPT over the fluctuating barrier with the dichotomous noise in the $-a$ configuration, which accords with the above reason for the RA happening in Fig. 2.

In Figs. 3(a) and 3(b), for different values of dichotomous noise strength we represent the ln of the MFPT as a function of the ln of the potential barrier fluctuation flipping rate and as a function of the dichotomous noise transition rate, respectively. The figures show that as the dichotomous noise strength increases, the resonant behavior of the MFPT as a function of the barrier fluctuation flipping rate becomes more and more indistinct, but the resonant behavior of the MFPT as a function of the dichotomous noise transition rate becomes more and more distinct. So the dichotomous noise can weaken the first RA, but enhance the second RA. In addition, Figs. 3(a) and 3(b) also show that with increasing the dichotomous noise strength the value of the MFPT over the fluctuating potential barrier decreases, and its minimum value moves toward right.

Below we consider the case when $E_1+E_2=0$. By numerical simulation and analysis we can find that when $E_1+E_2=0$ there are six nonzero real independent eigenvalues and two zero eigenvalues for the matrix of the homogeneous part in Eq. (4). Now we have [note $\partial_x T_i = s_i$ ($i=1$, 2, 3, and 4)]

$$s_i=\sum_{j=1}^{6} C_j^{(i)}\exp(r_j x)+C_7^{(i)}+C_8^{(i)}x,$$

$$T_i=\sum_{j=1}^{6} \frac{C_j^{(i)}}{r_j}\exp(r_j x)+D_7^{(i)}+C_7^{(i)}x+\frac{1}{2}C_8^{(i)}x^2, \qquad (7)$$

where r_j ($j=1,2,\ldots,6$) are the six nonzero eigenvalues of the matrix of the homogeneous part in Eq. (4). From Eq. (4) and the boundary conditions for T_i and s_i, using a similar method which has been used when calculating formula (6),

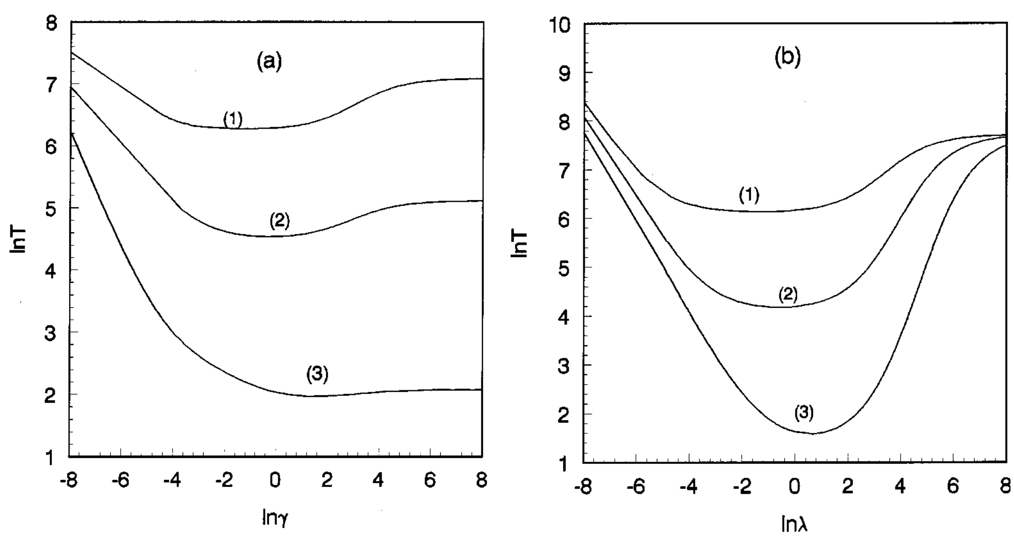

FIG. 3. For different values of the dichotomous noise strength (1) $a=3$, (2) $a=6$, and (3) $a=12$, the ln of the MFPT versus the ln of the barrier fluctuation flipping rate γ with $\lambda=10$ (a) and the ln of the MFPT versus the ln of the dichotomous noise transition rate λ with $\gamma=10$ (b). $E_1=10$, $E_2=14$, and $D=1$.

we can derive $C_k^{(i)}$ and $D_7^{(i)}$ ($i=1$, 2, 3, and 4, $k=1$, $2,\ldots,8$). The MFPT for a particle over the fluctuating barrier is

$$T=\sum_{i=1}^{4}\sum_{j=1}^{6}\left[\frac{C_j^{(i)}}{r_j}e^{-r_j}+D_7^{(i)}-C_7^{(i)}+\frac{1}{2}C_8^{(i)}\right]. \tag{8}$$

Further study shows that when $E_1+E_2=0$ there is the same phenomenon as reported above.

In Eq. (1), the fluctuating potential barrier is only driven by a dichotomous noise. When the fluctuating potential barrier is driven by two or more dichotomous noises, further study shows that there are three or more resonant activations (a detailed theory is under study). In addition, when the fluctuating potential barrier is not piecewise linear, such as $-V'(x,t)=-U'(x)+\eta_0(t)g(x)$ [where $U(x)$ is a bistable potential or a multitable potential, $\eta_0(t)$ is a noise which takes on the values ± 1 (the transition rate between ± 1 is γ), coupled to $g(x)$, and causes the potential barrier to fluctuate], as long as there is the dichotomous noise $\xi(t)$ in Eq. (1) the phenomenon (i.e., there are two resonant activations) reported by us in the paper still exists.

Because the model with a fluctuating potential barrier is of generic interest in chemistry [22], biology [23–25], physics, and other sciences, our results probably apply to a broader class. Furthermore, in our case, the dependence of the RA as a function of the transition rate of the dichotomous noise, which is an external random force, would make control of the RA easier in an experimental search.

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APPENDIX A: THE MFPT EQUATIONS FOR EQ. (2)

The backward master equation for master equation (2) is [20,21]

$$\partial_t\begin{pmatrix}G_1\\G_2\\G_3\\G_4\end{pmatrix}=\begin{pmatrix}\hat{L}_1' & \lambda & \gamma & 0\\ \lambda & \hat{L}_2' & 0 & \gamma\\ \gamma & 0 & \hat{L}_3' & \lambda\\ 0 & \gamma & \lambda & \hat{L}_4'\end{pmatrix}\begin{pmatrix}G_1\\G_2\\G_3\\G_4\end{pmatrix}, \tag{A1}$$

where $\hat{L}_1'=-\gamma-\lambda-\partial_x(E_1-a)+D\partial_x^2$, $\hat{L}_2'=-\gamma-\lambda-\partial_x(E_1+a)+D\partial_x^2$, $\hat{L}_3'=-\gamma-\lambda-\partial_x(E_2-a)+D\partial_x^2$, and $\hat{L}_4'=-\gamma-\lambda-\partial_x(E_2+a)+D\partial_x^2$.

The MFPT is defined as [20]

$$T_i(x)=-\int_0^\infty t\partial_t G_i(x,t)dt=\int_0^\infty G_i(x,t)dt, \tag{A2}$$

where $i=1$, 2, 3, and 4.

From Eqs. (A1) and (A2), one obtains the equations of the MFPT:

$$[-\gamma-\lambda-(E_1-a)\partial_x+D\partial_x^2]T_1+\lambda T_2+\gamma T_3+1=0,$$

$$[-\gamma-\lambda-(E_1+a)\partial_x+D\partial_x^2]T_2+\lambda T_1+\gamma T_4+1=0,$$

$$[-\gamma-\lambda-(E_2-a)\partial_x+D\partial_x^2]T_3+\lambda T_4+\gamma T_1+1=0,$$

$$[-\gamma-\lambda-(E_2+a)\partial_x+D\partial_x^2]T_4+\lambda T_3+\gamma T_2+1=0. \tag{A3}$$

APPENDIX B: SOLUTION OF EQ. (4)

By numerical simulation and analysis we can find that when $E_1+E_2\neq 0$ the eigenvalues of the matrix of the homogeneous part about T_i and s_i ($i=1$, 2, 3, and 4) in Eq. (4) are real and independent, and there is a zero eigenvalue. The general solutions of Eq. (4) when $E_1+E_2\neq 0$ are

$$s_i=\sum_{j=1}^{7}A_j^{(i)}\exp(\lambda_j x)+A_8^{(i)}+A_9^{(i)}x,$$

$$T_i=\sum_{j=1}^{7}B_j^{(i)}\exp(\lambda_j x)+B_8^{(i)}+B_9^{(i)}x, \tag{B1}$$

where $i=1$, 2, 3, and 4, and λ_j ($i=1,2,\ldots,7$) are the abovementioned nonzero eigenvalues. Substituting s_i and T_i into $\partial_x T_i=s_i$ one can obtain $B_j^{(i)}=A_j^{(i)}/\lambda_j$, $A_9^{(i)}=0$ and $B_9^{(i)}=A_8^{(i)}$. So we have

$$s_i=\sum_{j=1}^{7}A_j^{(i)}\exp(\lambda_j x)+A_8^{(i)},$$

$$T_i=\sum_{j=1}^{7}\frac{A_j^{(i)}}{\lambda_j}\exp(\lambda_j x)+B_8^{(i)}+A_8^{(i)}x. \tag{B2}$$

Substituting Eq. (B2) into Eq. (4) and using the comparing-coefficient method, we get

$$A_8^{(i)}=\frac{4}{E_1+E_2},$$

$$B_8^{(i)}=B_8^{(1)}+F_i,$$

$$A_j^{(i)}=k_j^{(i)}A_j^{(1)},\quad (i=1,\ 2,\ 3,\quad\text{and}\quad 4,\ j=1,2,\ldots,7) \tag{B3}$$

with

$$F_1=0,\quad F_2=-\frac{\gamma}{\lambda(\lambda+\gamma)}-\frac{4a}{\lambda(E_1+E_2)},$$

$$F_3=-\frac{\lambda}{\gamma(\lambda+\gamma)}-\frac{4E_1}{\gamma(E_1+E_2)},\quad F_4=-1/\lambda-1/\gamma-\frac{4a}{\lambda(E_1+E_2)}+\frac{4E_1}{\gamma(E_1+E_2)},$$

$$k_j^{(1)}=1,\quad k_j^{(2)}=\frac{\lambda^2-\gamma^2+(D\lambda_j^2-\gamma-\lambda-E_1+a)(D\lambda_j^2-\gamma-\lambda-E_2+a)}{\lambda(-2D\lambda_j^2+2\gamma+2\lambda+E_1+E_2)},$$

$$k_j^{(3)}=\frac{1}{\gamma}\left[\gamma+\lambda+E_1-a-D\lambda_j^2+\frac{\lambda^2-\gamma^2+(D\lambda_j^2-\gamma-\lambda-E_1+a)(D\lambda_j^2-\gamma-\lambda-E_2+a)}{2D\lambda_j^2-2\gamma-2\lambda-E_1-E_2}\right],$$

$$k_j^{(4)}=\frac{1}{\gamma}\left\{-\lambda+\frac{(\gamma+\lambda+E_1+a-D\lambda_j^2)[\lambda^2-\gamma^2+(D\lambda_j^2-\gamma-\lambda-E_1+a)(D\lambda_j^2-\gamma-\lambda-E_2+a)]}{-2D\lambda_j^2+2\gamma+2\lambda-E_1+E_2}\right\}.$$

Substituting Eq. (B3) into Eq. (B2), one can obtain

$$s_i=\sum_{j=1}^{7}k_j^{(i)}A_j^{(1)}\exp(\lambda_j x)+\frac{4}{E_1+E_2},$$

$$T_i=\sum_{j=1}^{7}\frac{k_j^{(i)}A_j^{(1)}}{\lambda_j}\exp(\lambda_j x)+B_8^{(1)}+F_i+\frac{4}{E_1+E_2}x. \tag{B4}$$

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