PHYSICAL REVIEW E VOLUME 59, NUMBER 5 MAY 1999

Dark incoherent soliton splitting and ''phase-memory'' effects: Theory and experiment

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(Received 3 December 1998)

We report on an experimental observation of dark incoherent soliton Y splitting. The effects of incoherence on the evolution of incoherent dark soliton doublets are investigated both theoretically and experimentally. We show that the dynamics of these incoherent self-trapped entities are associated with strong ''phase-memory'' effects that are otherwise absent in the linear regime. [S1063-651X(99)50205-5]

PACS number(s): 42.65.Tg, 42.65.Hw, 42.25.Kb

Recent experimental and theoretical studies have shown that incoherent spatial solitons are fundamentally different from their coherent counterparts [1–13]. Unlike coherent solitons [14], these newly discovered incoherent self-trapped entities are multimoded and are known to exist in noninstantaneous nonlinear media [5,6]. Bright spatial incoherent solitons were the first to be observed experimentally in strontium barium niobate (SBN) photorefractive crystals [1,2]. In order to explain their behavior, two complimentary methods have been developed [3–6]. The first one is the so-called coherent density approach, which describes incoherent beam dynamics via a nonlinear Schrödinger-like integro-differential equation [3,4]. The second method is a self-consistent multimode description which is capable of identifying multimode incoherent soliton solutions and their range of existence [5,6,11–13,15]. Finally, a ray approach has been suggested in the limit of ''big incoherent'' bright beams [7,8]. This latter transport approach is to some extent relevant to that taken in the theory of random-phase solitons, previously considered within the context of plasma physics [16,17]. Lately, a numerical study based on the coherent density approach, has revealed that incoherent dark solitons may be possible in biased photorefractives [9]. Subsequently, incoherent dark planar and two-dimensional dark solitons (''vortices'') were observed in a SBN:60 crystal [10]. This was achieved by employing the photorefractive self-defocusing nonlinearity associated with screening solitons [18,19]. As predicted in Ref. [9], the incoherent dark solitons were found to be gray. Moreover, these dark incoherent solitons were efficiently excited provided that an initial π-phase flip was imposed on the incoherent wave front. Following the experimental observation, the modal structure of these incoherent dark solitons was analyzed [11] using the self-consistent multimode method [5,6]. In this study, it was shown that these soliton states involve a belt (a continuum) of odd and even radiation modes and possibly bound states. The π-phase shift required for their excitation was explained by considering the radiation mode distribution within the dark soliton notch [11]. Incoherent Y-soliton splitting was also predicted in Ref. [9]. The behavior of these incoherent soliton doublets, for different degrees of coherence, is presented here.

In this Rapid Communication, we report an experimental observation of dark incoherent soliton Y-splitting in a noninstantaneous self-defocusing nonlinear medium as predicted in Ref. [9]. The evolution of these incoherent soliton doublets is then systematically investigated as a function of their coherence, both theoretically and experimentally. Surprisingly, we find that over a wide range of parameters, the Y-splitting is approximately the same, irrespective of coherence. Moreover, we show that the dynamical behavior of this incoherent Y-splitting process is associated with strong ''phase-memory'' effects which are otherwise absent in the linear regime. In other words, we show that dark incoherent self-trapped entities (dark incoherent solitons) are characterized by a strong memory effect that lasts throughout propagation and governs their propagation behavior (single soliton versus Y-soliton splitting, etc.). This is in sharp contrast to all known so far about linear propagation of incoherent beams, in which all phase information is fully washed out after a finite distance [20].

Our experiments were carried out in SBN:60 crystals. For this reason, here we use the $(1+1)$D saturable nonlinearity of the form $1/(1+I)$ [18,19] so as to make direct comparisons with experiment. We emphasize, however, that our results hold for any noninstantaneous nonlinearity that can give rise to dark solitons. In this material system (photorefractives), the normalized intensity $I_N = I/I_d$ (where I_d is the dark irradiance) of the incoherent dark beam evolves according to the following normalized nonlinear integro-differential equation [3,4,9]:

$$i\left(\frac{\partial f}{\partial \zeta} + \alpha \frac{\partial f}{\partial s}\right) + \frac{1}{2}\frac{\partial^2 f}{\partial s^2} + \beta \frac{f}{1+I_N(s,\zeta)} = 0, \tag{1}$$

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where

$$I_N(s,\zeta)=\int_{-\infty}^{+\infty}|f(s,\zeta,\theta)|^2 d\theta \tag{2}$$

and at $\zeta=0$, the coherent density f is given by

$$f(\zeta=0,s,\theta)=\rho^{1/2}G_N^{1/2}(\theta)\phi_0(s). \tag{3}$$

In the above equations, we have used the following normalized coordinates: $\zeta=z/(kx_0^2)$ and $s=x/x_0$, where x_0 is an arbitrary spatial scale associated with the intensity full width at half maximum (FWHM) of the beam. Moreover, $\alpha=kx_0\theta,\beta=(k^2x_0^2/2)n_e^2r_{33}|E_0|(1+\rho)$, where θ represents an angle (in radians) with respect to the z axis, $k=k_0n_e$ is the wave number, $k_0=2\pi/\lambda_0$, n_e is the extraordinary refractive index of the material, and r_{33} is the electrooptic coefficient involved. $E_0=-V/W$ is the value of the space charge field at $x\rightarrow\pm\infty$, where V is the reverse applied bias and W the x-width of the crystal. $G_N(\theta)$ is the normalized angular power spectrum of the incoherent source and $\phi_0(s)$ is the input complex spatial modulation function. In this study, $G_N(\theta)$ is assumed to be Gaussian, i.e., $G_N(\theta)=(\pi^{1/2}\theta_0)^{-1}\exp(-\theta^2/\theta_0^2)$, where θ_0 is associated with the width of angular power spectrum. Finally, ρ is the normalized intensity of the dark beam at $x\rightarrow\pm\infty$.Here, as usual, we assume that the beam at the input obeys a stationary random process. In general, the coherence properties of these beams can be followed using a version of the Van Cittert-Zernike theorem as in Ref. [9]. The coherence length of the beam at $z=0$ can be readily obtained from $G_N(\theta)$ and it is given by $l_c=\sqrt{2\pi}/(k\theta_0)$ [9]. When the beam is fully coherent ($\theta_0=0$), the coherence length of the beam becomes infinite, i.e., $l_c\rightarrow\infty$. In this case, the system of Eqs. (1) and (2) collapse to a standard single differential equation given in Refs. [18] and [19].

Before we present our experimental results, it may prove beneficial to first discuss the behavior of such incoherent dark beams from a theoretical point of view. As in the experiment, let us consider a biased SBN:60 crystal with $n_e=2.3$, $r_{33}=250$ pm/V, $\lambda_0=514$ nm, $W=5.3$ mm [10]. We let the spatial modulation function at the input be $\phi_0(x)=\tanh(x/x_0)$ under odd initial conditions, and $\phi_0(x)=[1-\epsilon^2\mathrm{sech}^2(x/x_0)]^{1/2}$ under even. The quantity ϵ^2 defines the beam's grayness. Throughout this work, we assume that at $z=0$, $\epsilon^2\approx 1$ (almost black even dark beams). The input intensity FWHM of the even and odd dark beams is taken here to be 25 μm, as shown in Fig. 1(a). Moreover, the normalized background intensity is $\rho=3$. First, we consider linear diffraction of coherent and incoherent dark-beams under odd and even initial conditions. Figures 1(b) and 1(c) show the diffracted intensity profiles of coherent odd and even dark beams respectively after ~12 mm of propagation. In this case, the intensity FWHM of the odd dark beam at the output is ~42 μm, whereas that of the even is ~76 μm. It is important to note that after diffraction, the intensity of the odd coherent beam is always zero at the center, whereas that of the even is graylike. Figure 1(d), on the other hand, demonstrates how an odd or even incoherent dark beam will diffract after 12 mm of propagation when at the input $\theta_0\simeq 5$ mrads or $l_c=17$ μm. This latter figure shows that the intensity profiles of the odd and even incoherent dark beams are almost identical with an output FWHM of ~100 μm. Simulations suggest that the same also applies for the $l_c(x)$ curves corresponding to these two cases as shown in Fig. 1(e). Thus, from diffraction data alone, it is extremely difficult to distinguish an odd dark beam from an even one. In other words, the randomly changing speckled structure of an incoherent beam leads to a loss of phase memory. Therefore, as a result of this phase washing effect, a sufficiently incoherent dark beam diffracts approximately the same way regardless of the phase information initially imposed on it. An important distinction between diffraction of a coherent and an incoherent dark beam comes from the structure of their background. Figure 1 clearly demonstrates that a diffracted coherent dark beam involves intensity ripples in its background. These oscillations tend to disappear in the case of an incoherent beam as a result of its speckled structure.

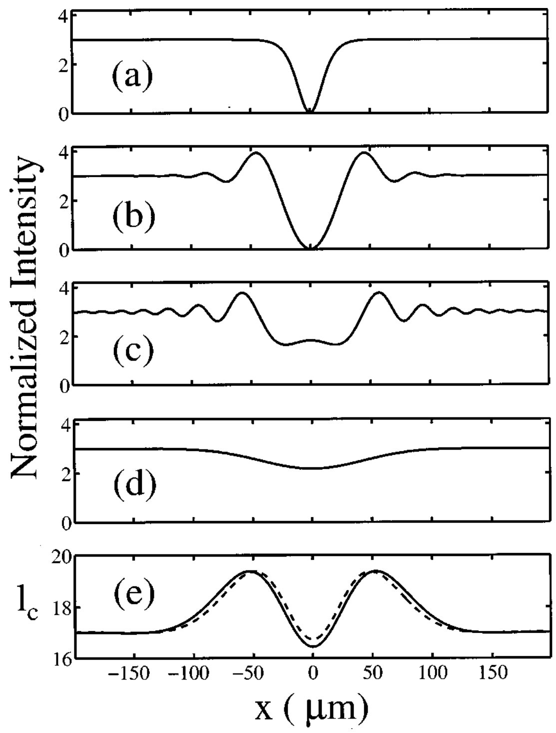

FIG. 1. (a) Intensity profile of a 25 μm odd or even dark beam at the input. Diffraction of (b) an odd coherent dark beam, (c) even coherent dark beam, (d) incoherent odd or even dark beam after 12 mm of propagation. (e) l_c in μm as a function of x for the odd (dashed curve) and even (solid curve) diffracted incoherent dark beam shown in (d).

When on the other hand the nonlinearity is activated, the dynamics of these incoherent beams depend on initial phase information. As previously predicted, generation of a single incoherent dark (which is in reality gray) beam or a higher-order triplet requires a π-phase shift [9–11]. Conversely, starting from even initial conditions, an incoherent gray soliton pair or Y-soliton splitting can be obtained [9]. In other words, in the presence of nonlinearity, an incoherent dark beam tends to remember its origins and identity, i.e., a "phase-memory" effect is established. Thus, the beam starts to behave in a quasi-coherent fashion [21].

Experiments with an amplitude notch (even initial conditions) are performed by using both coherent and spatially

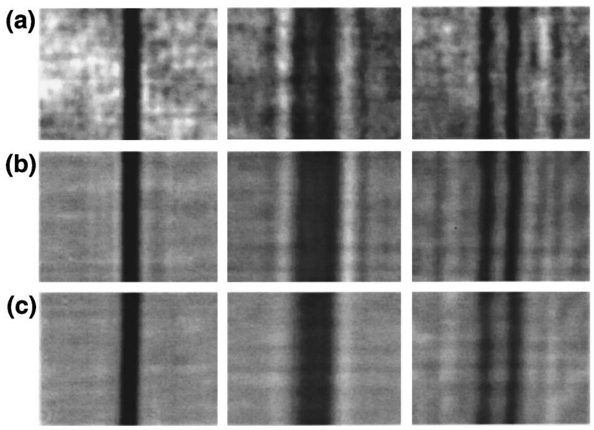

FIG. 2. Experimental observation of coherent and incoherent Y splitting: (a) coherent dark beam; (b) and (c) incoherent dark beam with an average speckle size of 30 and 15 μm, respectively. The first column depicts the input intensity, the second one diffraction data, and the third one Y splitting at -350 V. In all the cases the intensity FWHM of the beam at the input is 25 μm.

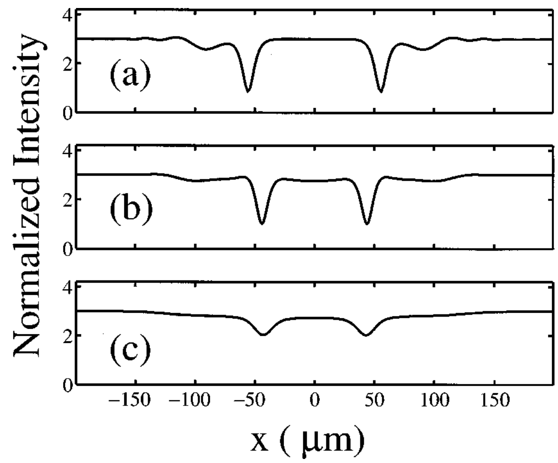

FIG. 3. Intensity profile of a soliton doublet at $z=12$ mm when the external bias is -450 V and the beam is (a) coherent ($l_c=\infty$) or incoherent with (b) $l_c=25$ μm, (c) $l_c=17$ μm. In all cases the initial intensity FWHM of the beam is 25 μm.

incoherent light sources for comparison. Details regarding the coherent dark soliton experiments can be found in Ref. [21]. The laser used is an Ar ion laser ($\lambda_0=514$ nm). An artificial background dark irradiance is provided by uniformly illuminating the entrance face of the crystal (SBN:60) along the ordinary axis. The maximum intensity ratio (at the tails) of the dark beam with respect to dark irradiance is approximately 1.5. The dark beam is also broad enough to cover the entire input face of the crystal. For the incoherent case, a rotating diffuser is employed to provide random phase fluctuations across the beam [1,2,10]. In this case, we generate a dark notch on a broad partially spatially incoherent beam with controllable degree of coherence. The experimental arrangement is the same as that in Ref. [10], except that the phase mask is now replaced by an amplitude mask which involves reflection from a metallic wire as done in Ref. [21]. Incoherent Y-junction solitons are generated and then compared with the coherent ones. Figure 2 shows typical experimental results. When the dark beam is coherent, it diffracts from a FWHM of 25 μm (left) to about 58 μm after $\sim$12 mm of propagation (middle) when no nonlinearity is present. Note that, with the exception of the dark notch FWHM (which from simulations is expected to be $\sim$76 μm), its intensity structure is in agreement with Fig. 1(c). The discrepancy in FWHM is attributed to the fact that the reflection from the metallic wire introduces a quadratic phase [21], which is not accounted for in our simulations. After applying a voltage of -350 V (negative relative to the c axis), the dark amplitude notch evolves into a pair of gray solitons (right). The second and third rows of this figure depict the same data when the dark beam is incoherent. The right column of the figure was obtained at $V=-350$ V and with an input FWHM of 25 μm. As seen in Fig. 2, the grayness of the soliton pair increases as the incoherence of the beam increases. Nevertheless, the spacing of these two solitons at the crystal output face is about the same for a varying degree of coherence.

These experimental results are now compared with numerical simulations. By keeping in mind that in the experiment, the input speckle size of the incoherent beams is 30 μm for Fig. 2(b) and 15 μm for Fig. 2(c) and by considering their diffraction behavior, we estimate that the width of the angular power spectrum in these two cases is $\sim$3.5 and 5.2 mrads, respectively. The simulation shown in Fig. 3(a) demonstrates how a coherent soliton doublet forms from a 25 μm even dark beam after 12 mm of propagation when $V=-450$ V. For the same bias voltage and initial beam width, the intensity profile of an incoherent doublet after 12 mm of propagation is shown in Fig. 3(b) when $\theta_0=3.5$ mrads or $l_c=25$ μm. Figure 3(c) depicts similar data when $\theta_0=5.2$ mrads ($l_c=17$ μm) and again $V=-450$ V. Both figures, 3(b) and 3(c), were obtained by numerically solving Eqs. (1)–(3) as done in Ref. [9]. In agreement with the experiment, Fig. 3 demonstrates that the doublet becomes grayer as

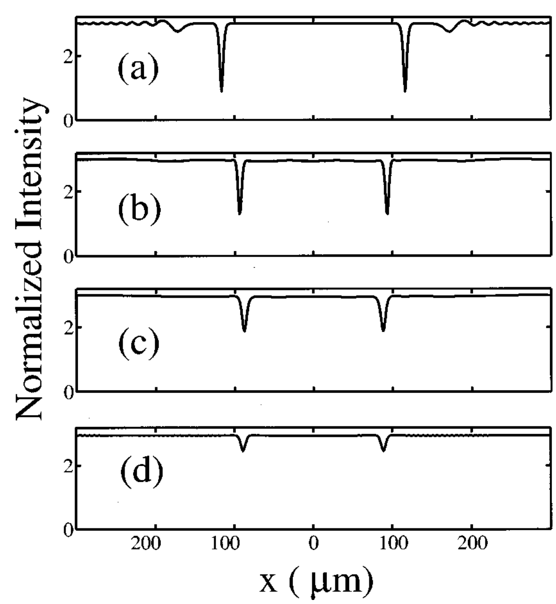

FIG. 4. Intensity profile of a soliton doublet at $z=12$ mm when the external bias is -2400 V and the beam is (a) coherent ($l_c=\infty$) or incoherent with (b) $l_c=9.3$ μm, (c) $l_c=7.3$ μm. (d) Same information when the external bias is -4000 V and $l_c=3.4$ μm. In all cases the initial intensity FWHM of the beam is 10 μm.

the incoherency increases. Surprisingly, for this range of parameters, both theory (Fig. 3) and experiment (Fig. 2) suggest that the Y-splitting angle or the doublet separation does not depend strongly on the degree of coherence. To further understand this Y-splitting process, we carried out another set of simulations. In this latter set, the intensity FWHM of the even dark beam was chosen to be 10 μm (in order to accelerate splitting process) and l_c varied from ∞ down to 3.4 μm. Figures 4(a)–4(c) were obtained for the same initial conditions and bias voltage ($V=-2400$ V) after 12 mm of propagation for different degrees of coherence. Even in this case, the splitting is relatively insensitive to θ_0. This is by itself very interesting considering the range in which l_c varies. This is another manifestation of the ''phase memory'' effect discussed earlier. As the incoherency of the dark beam increases, a higher bias voltage is required to establish a doublet. Figure 4(d) shows Y-splitting of a 10 μm even incoherent dark beam after 12 mm, when $V=-4000$ V and $l_c=3.4$ μm. Finally, at lower l_c's, the doublet practically disappears (because of its grayness) and the splitting angle is further reduced.

In conclusion, incoherent dark soliton Y splitting has been demonstrated experimentally. Using the coherent density approach we have shown that the evolution of incoherent dark solitons in noninstantaneous nonlinear media is associated with strong ''phase-memory'' effects that are otherwise absent in the linear regime. The higher-order behavior of these dark beams have been compared under the same initial conditions but for different degrees of coherence. It was found that over a wide range of parameters, the Y-splitting is approximately the same irrespective of spatial coherence. Experimental observations are in good agreement with theoretical predictions.

This research was supported by the AFOSR, NSF, and ARO.

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[15] The rapid progress in two related topics, incoherent solitons and multimode (vector) solitons, during the last two years, has led to several misconceptions regarding these two fundamentally different types of solitons. For incoherent solitons, there is no quantity that is constant with time apart from the sum of the time averages of the model intensities. For multimode solitons (as for any vector solitons), the modal intensities are constant with time. This is why multimode solitons can be constructed (engineered) from a superposition of laser beams, each fully spatially coherent but mutually incoherent with respect to each other, as was done by M. Mitchell, M. Segev, and D. N. Christodoulides, Phys. Rev. Lett. **80**, 4657 (1998). On the other hand, to generate incoherent solitons, one must use a spatially incoherent source. The only way to generate incoherent solitons from laser light is to use a fast randomizing element (e.g., a rotating diffuser [1,10]) that ''scrambles'' the modal phases and amplitudes.

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