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Measuring the charge on single particles by laser-excited resonances in plasma crystals

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Experiments on the excitation of vertical particle resonances by laser radiation pressure are presented. The vertical resonance is used for the measurement of the charge on particles forming plasma crystals. The laser excitation technique allows us to excite the vertical resonance of a single particle in the plasma trap, whereas in earlier experiments the entire plasma crystal was excited to vertical oscillations. From the close agreement of both resonance techniques the earlier values of the dust charge are reconfirmed. [S1063-651X(99)51304-4]

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Plasma crystals represent an ideal system for the investigation of dusty plasmas and strongly coupled systems. Plasma crystals are regular arrangements of micrometer size monodisperse spherical particles which usually acquire negative charges in a plasma environment due to the collection of plasma electrons and ions. In typical experimental situations [1–3], the particles are trapped in the sheath above the lower electrode of parallel plate rf discharges, where the electric field force on the particles balances their weight. There, the particles interact by means of their Coulomb repulsion and form an ordered lattice, the plasma crystal. Hence, the charge on the particles is a crucial parameter for particle trapping as well as for the formation of ordered plasma crystals.

Plasma crystals have been widely studied with respect to lattice structure [2,4–6] and defects [7,8], phase transitions [5,9], and waves [10–13]. In all these fields a reasonable understanding of the underlying mechanisms has been achieved. On the other hand, there have been only a few experiments on the determination of the dust charge, which is a key parameter in plasma crystals.

Melzer and co-workers [3,14] had proposed a method in which the resonance frequency of vertical oscillations of dust particles trapped in the potential well was used to determine the dust charge. In the sheath the dust particles are trapped in a potential well resulting from the inhomogeneous electric field and the force of gravity. Therefore the rf voltage was modulated by a small-amplitude, low-frequency voltage to excite the vertical oscillations. This method is easy to apply and can be used under plasma crystal conditions, but the influence of the low-frequency voltages on the sheath environment is not known.

Direct Coulomb collisions of two dust particles for charge determination at very low gas pressures have been used by Konopka *et al.* [15]. The accuracy of this method, however, is poor and the applicability to the many-body plasma crystal is limited.

Wave dispersion in plasma crystals provides another possibility for the determination of the dust charge [10,12,13]. For this method, however, the whole wave dispersion has to be measured. The dust charge is then obtained from a fit by theoretical dispersion relations. The obtained dust charge is sensitive to the particular wave model chosen. Two different wave types have been identified experimentally [10,12,13].

In this Rapid Communication, the vertical resonance method [3,14] is investigated with respect to the influence of the rf voltage modulation on the sheath environment and charge measurement results. Therefore, this resonance method is compared with a noninvasive technique to excite single particle oscillations. Lasers have proved to be a very versatile tool for the manipulation of dust particles in plasmas [12,13,16,17]; hence, the radiation pressure of a laser is used for excitation of the resonances in the potential well. The results obtained by laser excitation are compared with those derived from the modulation of the rf voltage.

The dust particles in the plasma crystals are trapped in the sheath above the lower electrode due to a balance of their weight $m_d g$ and the electric field force acting on them,

$$m_d g = ZeE(z_0), \tag{1}$$

where Z is the charge number and $E(z_0)$ is the electric field in the sheath at a height z_0 above the electrode. It is a reasonable assumption for the ion sheath that the electric field increases linearly from the sheath boundary to the electrode with a constant slope $E' = \partial E/\partial z$. Therefore, the particles are trapped in a harmonic potential well,

$$V(z) = \frac{1}{2} m_d \omega_0^2 (z - z_0)^2, \tag{2}$$

where

$$\omega_0 = \sqrt{\frac{ZeE'}{m_d}} \tag{3}$$

is the natural frequency of the potential well. The slope of the electric field E' is, via Poisson's equation, mainly related to the ion density n_i in the sheath. In the experiment, the ion density is measured in the plasma volume by Langmuir probes and is then extrapolated into the sheath [14]. This natural frequency then allows one to determine the dust charge with an acceptable error of 40% [14].

The equation of motion of a dust particle in the potential well is that of a driven damped harmonic oscillator

$$\ddot{z} + \beta \dot{z} + \omega_0^2 z = F_{\mathrm{ext}}(\omega t). \tag{4}$$

Here, β is the Epstein friction constant due to dust-neutral gas collisions. F_{ext} is a periodical force for the excitation of vertical dust particle oscillations. This equation holds for the motion of the entire plasma crystal as well as for the oscil-

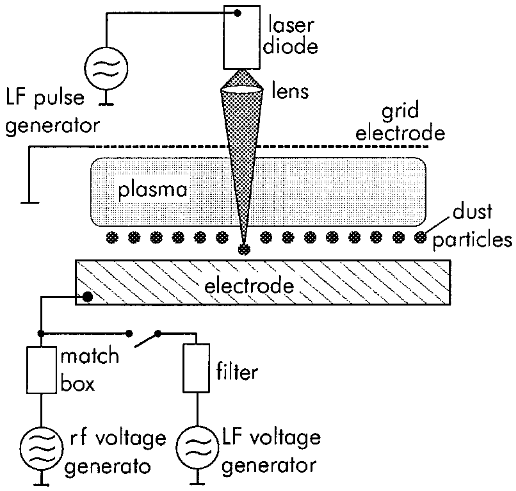

FIG. 1. Scheme of the experimental setup.

lations of a single particle embedded in a one-layer plasma crystal, since the change of interaction with neighboring particles is of second order.

When modulating the rf voltage by a low frequency sinusoidal voltage of small amplitude the external driving force also is a sine wave $F_{\mathrm{ext}}=F_0\sin(\omega t)$. The amplitude of the particle oscillations is then given by the well-known response function

$$R_{\mathrm{sine}}(\omega)=\frac{F_0}{\sqrt{(\omega_0^2-\omega^2)^2+\beta^2\omega^2}}. \tag{5}$$

In the experiment, also the radiation pressure of a laser is used to excite the particle oscillations. The laser beam, however, can only be switched ''on'' and ''off'' periodically, resulting in a square wave excitation force

$$F_{\mathrm{ext}}(\omega t)=\begin{cases} F_0 & \text{for } 0<t<\alpha\dfrac{2\pi}{\omega}, \\ 0 & \text{else,} \end{cases} \tag{6}$$

where α is the fraction of the excitation period with laser ''on.'' From the Fourier transform of F_{ext} the response function is obtained as the superposition of the response functions of each Fourier component of the driving force

$$R_{\mathrm{square}}(\omega)=\sum_k \left|\frac{\sin\alpha\pi}{k\pi}\right| R_{\mathrm{sine}}(k\omega). \tag{7}$$

From the measurement of the resonance curves the natural frequency ω_0 is determined and thus the charge on the dust particle is derived. In the experiment, therefore, the resonance curves obtained from rf voltage modulation and from laser excitation will be compared in order to study the influence on the rf voltage modulation technique on the sheath and the dust charge.

The experiments are performed in a parallel plate rf discharge operated in helium at discharge pressures between 30 and 100 Pa and a discharge power of 5 W. The experimental setup is shown in Fig. 1. The lower electrode is powered by an rf power generator via a matching network. The upper electrode is grounded.

Monodisperse plastic spheres of 9.47 μm diameter are immersed into the rf plasma. The particles are trapped in the sheath above the lower electrode. The amount of dust in the experiment presented here is chosen in such a way that a single-layer dust crystal is produced. The particles are illuminated by a vertically expanded laser beam. The scattered light from the particles is then viewed side-on by a charge-coupled-device camera (the camera and the illumination laser are not shown in Fig. 1 for clarity). The video images are stored in a computer for further processing.

In the usual resonance method [3,14] the vertical oscillations of the dust particles are excited by adding a low-frequency voltage of small amplitude to the rf voltage (see Fig. 1). The voltage modulation can be a sine or a square wave. This additional voltage leads to a periodic change of the sheath width which ''shakes'' the potential well of the dust particles, thus leading to the excitation of vertical oscillations of the entire plasma crystal.

As a second technique, which does not perturb the sheath environment, a laser beam (690 nm, 40 mW) entering the discharge from top is used for the excitation of the dust particles. Hereby, the beam is focused onto a single dust particle in the sheath. The radiation pressure of the laser light then pushes the particle downward. The laser is switched ''on'' and ''off'' periodically, thus leading to a periodic square wave excitation of the particle. It should be noted, that with laser excitation only a single particle is excited, whereas the electrode voltage modulation leads to the excitation of the entire dust crystal.

Figure 2 shows the trapping and manipulating of a single dust particle by the laser beam. In Fig. 2(a) two dust particles located in the sheath are shown before the laser is switched ''on.'' In Fig. 2(b) one of the particles is hit by the laser beam, which is easily recognized by the enhanced scattering cone. It is clearly visible that only a single particle is affected by the laser. Furthermore, in the horizontal plane, this particle is confined by the laser light due to net forces on the dielectric material and the inhomogeneous laser profile [18]. Due to the radiation pressure the particle then is pushed downward without leaving the laser focus. The resulting oscillatory motion is shown in Fig. 2(c) by a series of video images taken at a frequency of about 7 Hz. The oscillatory motion is presented in Fig. 2(d) in a longer sequence from which the oscillation amplitude is readily obtained. For frequencies higher than 12 Hz the oscillation amplitude is derived directly from two successive video images since each video frame (of 40 ms) then contains the particle trajectory from at least half an oscillation.

First, the comparison of the two excitation methods was performed at a gas pressure of 42 Pa. Figure 3 shows the resonance curve obtained from the resonance method with sinusoidal modulation of the electrode voltage. The resonance curve can be described by Eq. (5) with the natural frequency $\omega_0/(2\pi)=16.3\ \mathrm{s}^{-1}$ and the friction coefficient $\beta=13.9\ \mathrm{s}^{-1}$. The theoretical Epstein friction coefficient is in the range between $\beta=11.1\ \mathrm{s}^{-1}$ and $\beta=15.6\ \mathrm{s}^{-1}$ under these conditions, which is in very good agreement with the experimental value.

The resonance curve due to laser excitation is also shown in Fig. 3. The duty cycle of laser-''on'' was chosen as $\alpha=50\%$ here. One can see the main resonance peak at ap-

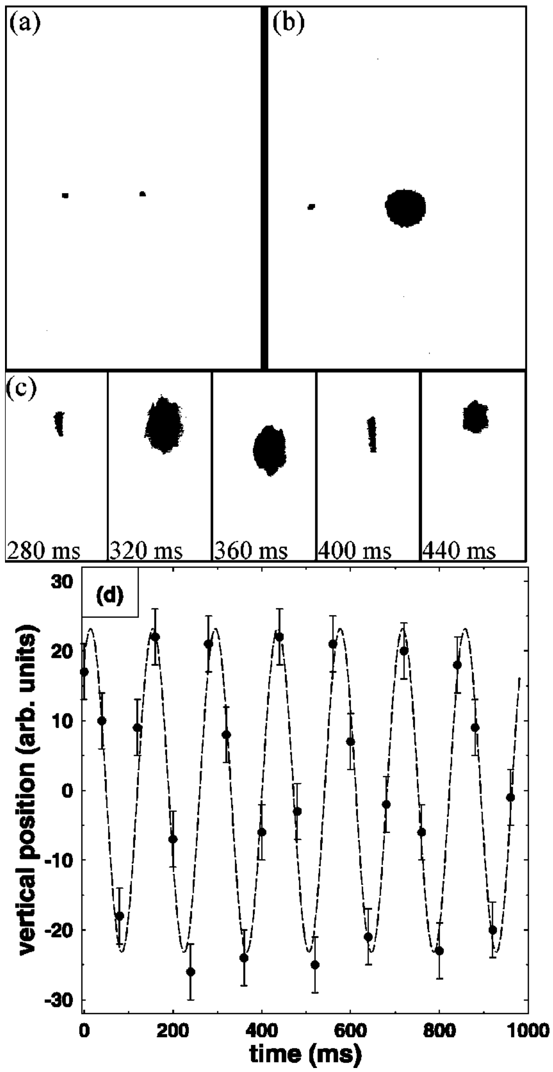

FIG. 2. Trapping and manipulating of a dust particle in the sheath above the lower electrode. (a) Undisturbed particles, the laser is ''off.'' (b) A single particle is trapped in the focus of the laser beam. Note the bright spot of scattered light. (c) Series of (digitally enhanced) video images showing one vertical oscillation at 7 Hz. Note the ''blooming'' of the particle image when hit by the laser. (d) Time series of the particle oscillation. For illustration of the periodical particle motion recorded by the camera at fixed time steps, a sine wave with the same frequency is shown along with the experimental data which corresponds to the first term in the multi-resonance curve. The times given in (c) are with respect to the oscillation in (d). The video images (a) to (c) are negative images.

proximately $\omega_0/(2\pi)=16\ \mathrm{s}^{-1}$. A second peak is observable at $\omega_0/3$ around 5 Hz. A resonance peak at $\omega_0/2$ is not present due to the missing second harmonic in a symmetrical square wave. A least square fit of the multi-resonance curve according to Eq. (7) yields a natural frequency of $\omega_0/(2\pi)=16.0\ \mathrm{s}^{-1}$ and a friction coefficient $\beta=17.4\ \mathrm{s}^{-1}$. The natural frequencies obtained from both methods coincide very well. The friction coefficient by laser excitation is slightly larger than the theoretical one and that by electrode modulation. The least square deviation, however, is not very sensitive to the value of β, but to that of ω_0 which is the more important one.

In order to investigate possible differences between sine

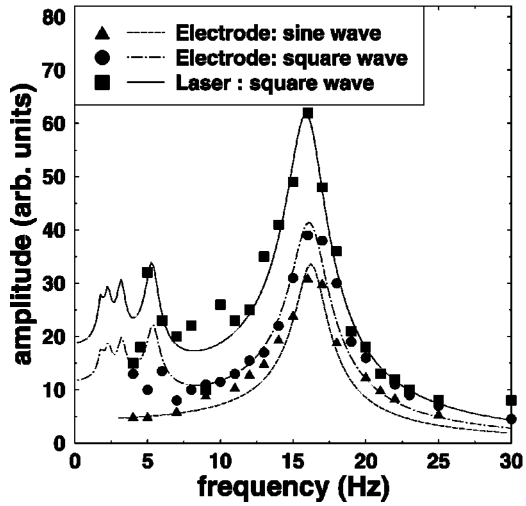

FIG. 3. Resonance curves of dust particles by electrode voltage modulation and laser excitation at 42 Pa. The symbols denote the experimental data and the lines are least square fits of the theoretical response functions. The electrode voltage was modulated with sine and square waves, and the laser excitation can be done by square wave form, only.

wave and square wave modulation, the LF electrode voltage was also switched ''on'' and ''off'' periodically. The resulting resonance curve shows the peak at ω_0 and the peak at $\omega_0/3$ as expected. From the best approximation according to the multi-resonance curve [Eq. (7)] $\omega_0/(2\pi)=16.2\ \mathrm{s}^{-1}$ and $\beta=16.5\ \mathrm{s}^{-1}$ were obtained which are also very close to the other values. Finally, from these three methods the natural frequency is determined to be $\omega_0/(2\pi)=(16.2\pm0.15)\ \mathrm{s}^{-1}$.

In a second experiment, the discharge pressure was increased to 70 Pa. The laser ''on'' fraction was increased to $\alpha=75\%$. Figure 4 shows the resonance curves obtained from laser excitation together with the sinusoidal electrode voltage modulation. The mean resonance is found at approximately

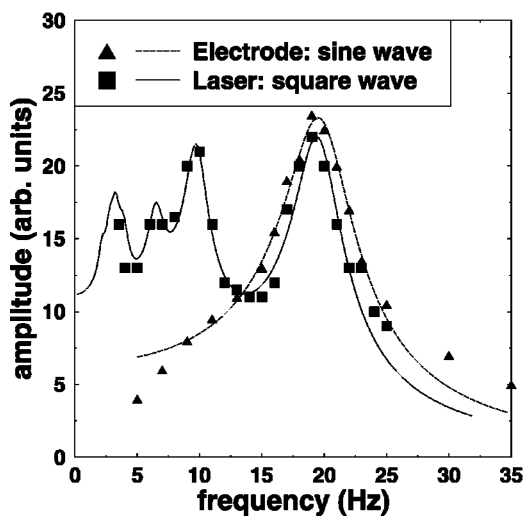

FIG. 4. Resonance curves of dust particles by electrode voltage modulation and laser excitation at 70 Pa. The symbols denote the experimental data and the lines are least square fits of the theoretical response functions. The electrode voltage was modulated with a sine wave, and the laser excitation can be done by square wave form, only.

20 Hz. Now, resonances at $\omega_0/2$ and $\omega_0/3$ are observable, and even a weak peak near $\omega_0/4$ can be identified. From the least square fit the values $\omega_0/(2\pi)=19.6\ \mathrm{s}^{-1}$ and $\beta=24.4\ \mathrm{s}^{-1}$ are obtained (the Epstein coefficient is $\beta=19.0\ \mathrm{s}^{-1}$ up to $\beta=26.0\ \mathrm{s}^{-1}$). From the sine wave rf voltage modulation the natural frequency is determined to be $\omega_0/(2\pi)=19.9\ \mathrm{s}^{-1}$ and the friction coefficient to be $\beta=35\ \mathrm{s}^{-1}$, which is larger than that by laser excitation. But here again the least square approximation is very insensitive to changes in β. So, the natural frequency at 70 Pa is determined to be $\omega_0/(2\pi)=(19.75\pm0.15)\ \mathrm{s}^{-1}$.

From ω_0 the dust charge is then $Z=8320\pm120$ (the ion density measured at the sheath boundary extrapolated into the sheath is $n_i=4.3\times10^8\ \mathrm{cm}^{-3}$). The error range given here is only due to the error in the measurement of the natural frequency. The error in the determination of the ion density is of the order of 40% [14].

Summarizing, we have shown that the global resonance of the plasma crystal excited by electrode voltage modulation and the single particle resonance excited by a laser result in the same natural frequencies for the vertical oscillations. This close agreement confirms the earlier reported values for the charge on dust grains. In addition, the electrode modulation technique, which is much easier to apply than the laser excitation, does not result in a measurable sheath disturbance. At last, it has also been demonstrated that the laser is a very useful tool for manipulating single particles in the plasma crystal.

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